ETH Zürich
Mathematical Foundations of
Computer Graphics and Vision
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EXERCISE 4

SAMPLING PATTERNS AND GRAPH CUTS

For running the codes seamlessly, the directory structure must not be changed since paths are hardcoded relative to each other.

1. ANALYZING SAMPLING PATTERNS

For Task 1.1 (Periodograms), running ../PART I/code/main_task11.py produces the plot of input data points taken from 1.txt files and the plot of averaged periodograms of each dataset.

For Task 1.2 (PCF), running ../PART I/code/main_task12.py produces the plots of estimated PCF's.

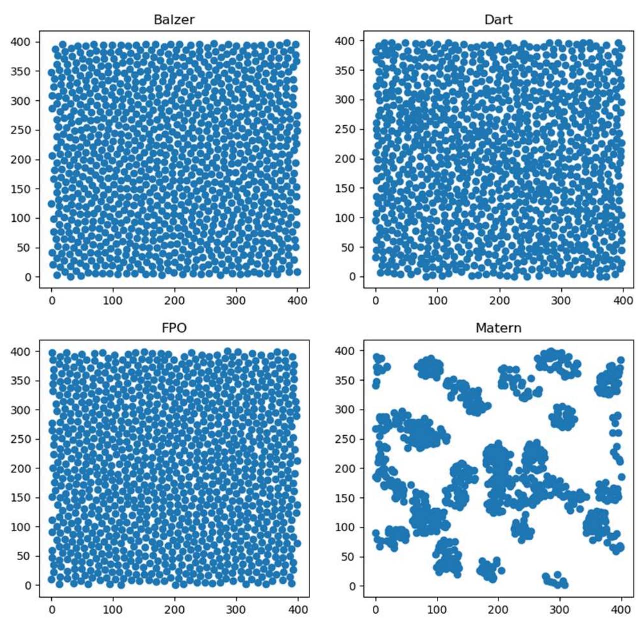
- The periodograms and the PCF's for the provided algorithms.
    - For computing the periodograms of sampling patterns, the steps explained in the exercise sheet is implemented.
    - For PCF's the estimation function is:

$$\hat{g}(r) = \frac{|V|}{|\partial V_d| r^{d-1} n^2} \sum_{i \neq j} k_\sigma \left(r - d(\mathbf{x}_i, \mathbf{x}_j) \right)$$

The r_{max} for normalization (to be able to utilize a generic range for $r_a$and r_b) is computed according to [1] ([2] can also be used since it is the generalized expression for high dimensional spaces and yields the following expression for $d = 2$):

$$r_{max} = \sqrt{\frac{1}{2\sqrt{3}N}}$$

While the data points are normalized with r_{max} , the size of the volume $|V|$, that the data points lie inside is normalized by r_{max}^2 since the volume in our case is 2 dimensional.



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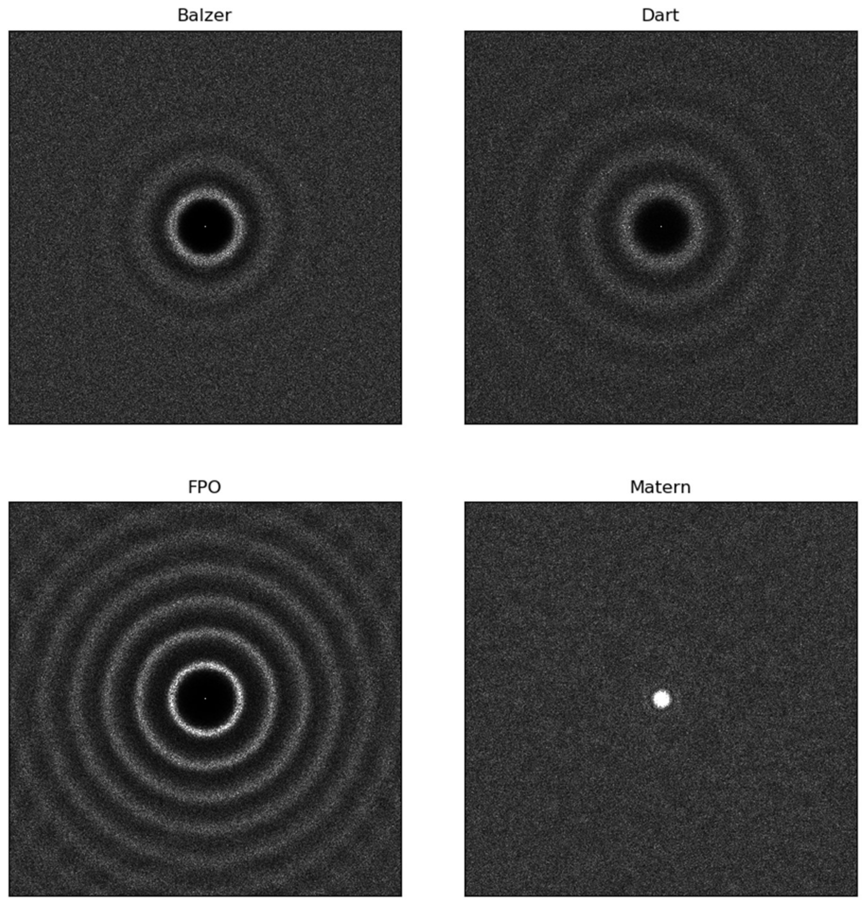
Periodograms:



PCFs:

The recommended r_a and r_b values do not provide visually nice results. Also, all functions should converge to 1, but they keep decreasing as the r value increases. Both artifacts are possibly caused by a small bug in the code that I am not able to find.

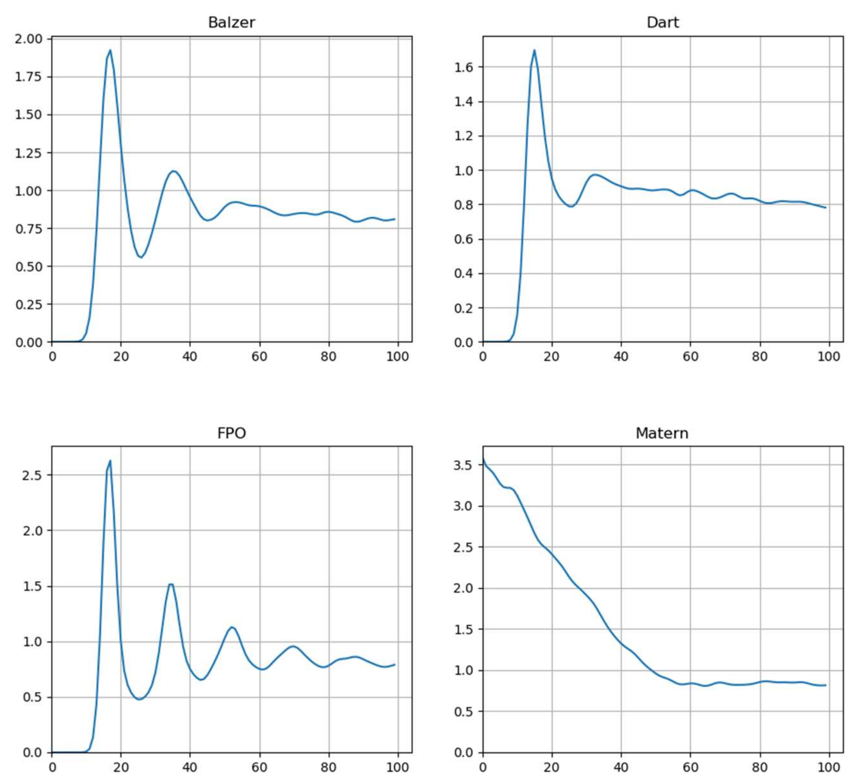
Used parameters:

$$|\delta V d| = 2\pi$$

$$\sigma = 0.25$$

$$r_a = \sigma$$

$$r_b = 10$$



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- Why do we have to avereage over multiple point sets for each algorithm when computing the periodograms, but not when computing the PCF's?

  As [3] suggests, unlike the spectral measures (e.g. periodograms), the smoothing level(σ) makes the PCF estimates smooth and indistinguishable for different instances of the same distribution. Therefore, while still depending on the smoothing level, the PCF generalizes well to the different samples of the same point process, giving the algorithm an inherent statistical power. In essence, smoothing effect comes from the Gaussian kernel which performs weighted averaging on the pairwise differences while σ fundamentally determines the contributions of different pairwise differences.

  Whereas periodograms are calculated without any smoothing and each periodogram only reflects the characteristics of only one point distribution. The generalization as well as statistical significance is achieved by simple averaging here.

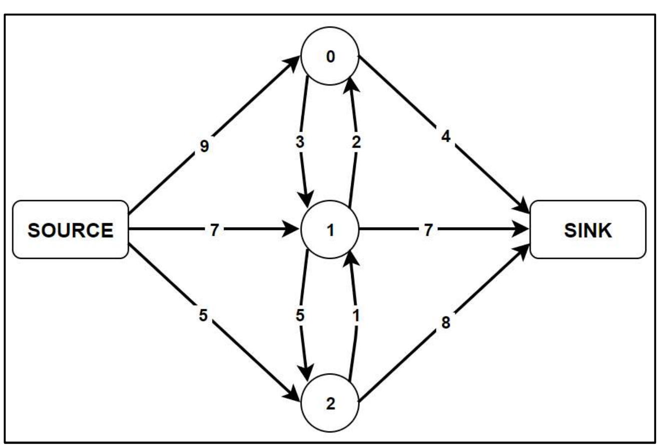
  Also, [3] states that small values of σ in PCF analysis will cause fluctuations in the density estimation and make the estimator change from one instance of a point distribution to another, becoming similar to the problem of periodograms.

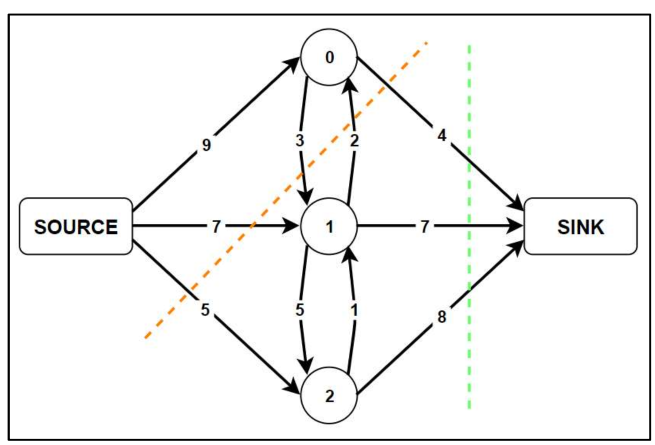
- Are PCF's sufficient to describe the provided point patterns, as they are only one dimensional, while the periodograms are two dimensional. Do the periodograms contain more information for the provided patterns?

  In general, for any point distribution, second order product density measures the joint probability $p(\boldsymbol{x}, \boldsymbol{y})$ of having points at locations $\boldsymbol{x}$ and $\boldsymbol{y}$ at the same time. For isotropic cases, any $\boldsymbol{x}$ and $\boldsymbol{y}$ locations boils down to distance $\boldsymbol{r}$ because of the translation and rotation invariance. Consequently, as [3] points out, interpretation of PCF is directly linked to the distribution of the distances between pairs of points. Therefore, with the provided isotropic patterns, the amount of information that both methods provide is same, in fact, since PCF generalizes well and easier to interpret, it may be more advantageous to utilize. However, in general if there are multiple datasets from the same point process is available (for statistical significance) and isotropy information is not known, periodograms can provide more information thanks to preserving information on different dimensions.

2. INTERACTIVE SEGMENTATION WITH GRAPH CUT

For Task 2.1 (Handling Max Flow), running ../PART II/code/main_task21.py constructs the flow graph, calculates the maxflow and outputs the node assignments and maxflow value to the console.





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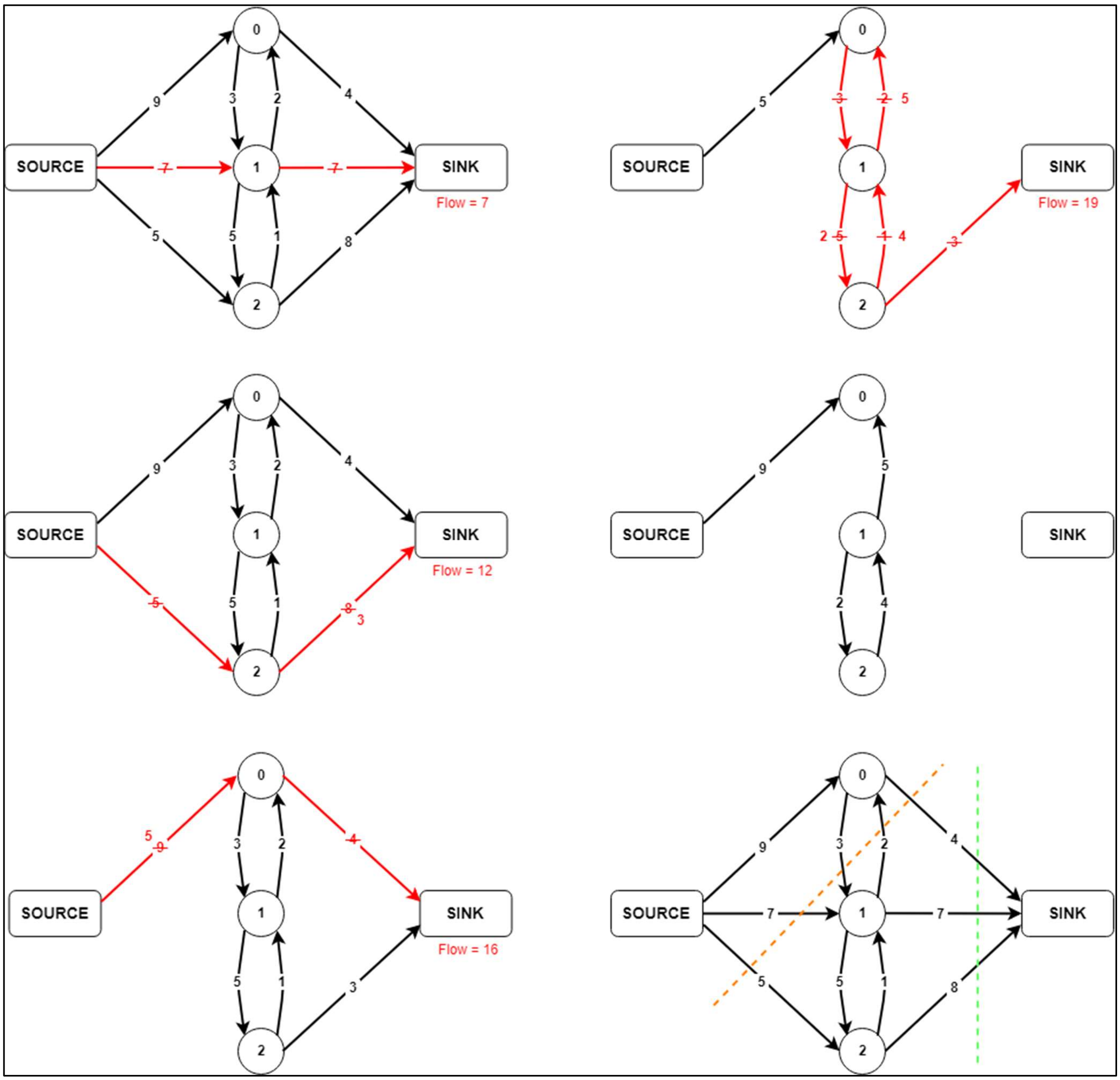
With the given graph, there are two different graph-cuts achieving min-cut(maxflow) value of 19. The code returns the cut indicated with green above which results in the optimal labelling of:

Node 0: SOURCE CLUSTER

Node 1: SOURCE CLUSTER

Node 2: SOURCE CLUSTER

Note that the cut indicated with orange is also an optimal cut results in [SOURCE, SINK, SINK] labelling. The steps of the calculation by hand is:



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For Task 2.2 (Segmentation), running ../PART II/code/main_task22.py opens up the interactive GUI and the rest is straightforward. I am not able reproduce the same results with the same parameters given as example, but the algorithm [4] and the pipeline work flawlessly.

- **Van Damme segmentation**

Input and Seeds

$\lambda = 1.0$

$\lambda = 10^{-4}$ $(BEST)$

$\lambda = 10^{-8}$

- **Batman segmentation**

Input and Seeds

$\lambda = 1.0$

$\lambda = 0.002$

$\lambda = 10^{-8}$ $(BEST)$

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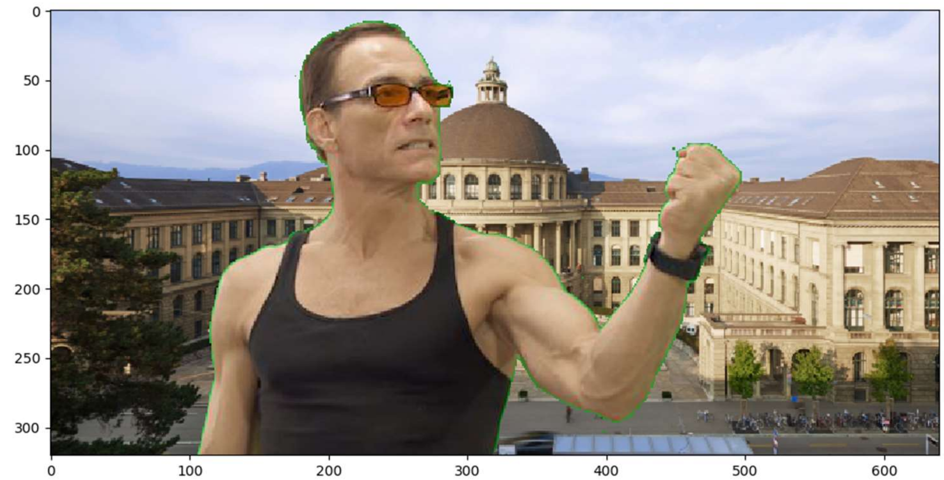
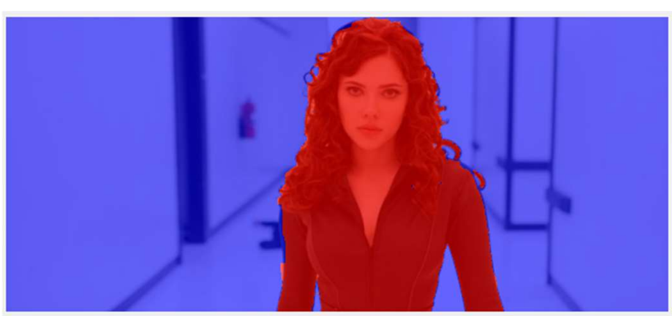


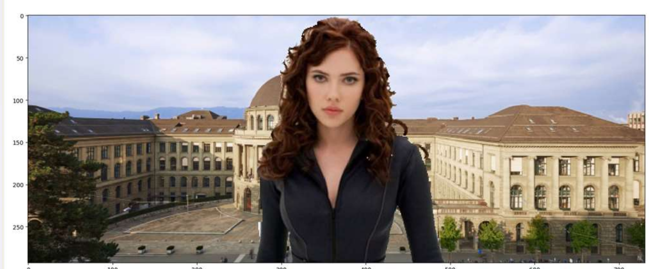
- **Scarlett segmentation**

$Input$

$Seeds$



$\lambda = 1.0$



$Poor\ Black\ Widow: ($

References

[1] A. Lagae and P. Dutre. A Comparison of Methods for Generating Poisson Disk Distributions. Computer Graphics Forum. Volume 27(1), pp. 114-129, 2008.

[2] M. Gamito and S. Maddock. Accurate multidimensional poisson-disk sampling. ACM Trans. Graph. 2, 8:1–8:19. December 2009.

[3] A. C. Oztireli and M. Gross. Analysis and synthesis of point distributions based on pair correlation. *ACM Trans. Graph. (Proc. of ACM SIGGRAPH ASIA)*, 31(6), 2012.

[4] Y. Y. Boykov and M. P. Jolly. Interactive graph cuts for optimal boundary amp; region segmentation of objects in n-d images. *IEEE International Conference on Computer Vision (ICCV)*, volume 1, pages 105–112 vol.1, 2001.