7 Image Analysis –Edge Linking & Boundary Detection

- Edge detection algorithm are followed by linking procedures to assemble edge pixels into meaningful edges.

- Basic approaches

  Local Processing

  Global Processing via the Hough Transform

7 Image Analysis –Local Processing

- Analyze the characteristics of pixels in a small neighborhood (say, 3x3, 5x5) about every edge pixels (x,y) in an image.

- All points that are similar according to a set of predefined criteria are linked, forming an edge of pixels that share those criteria.

7 Image Analysis –Local Processing

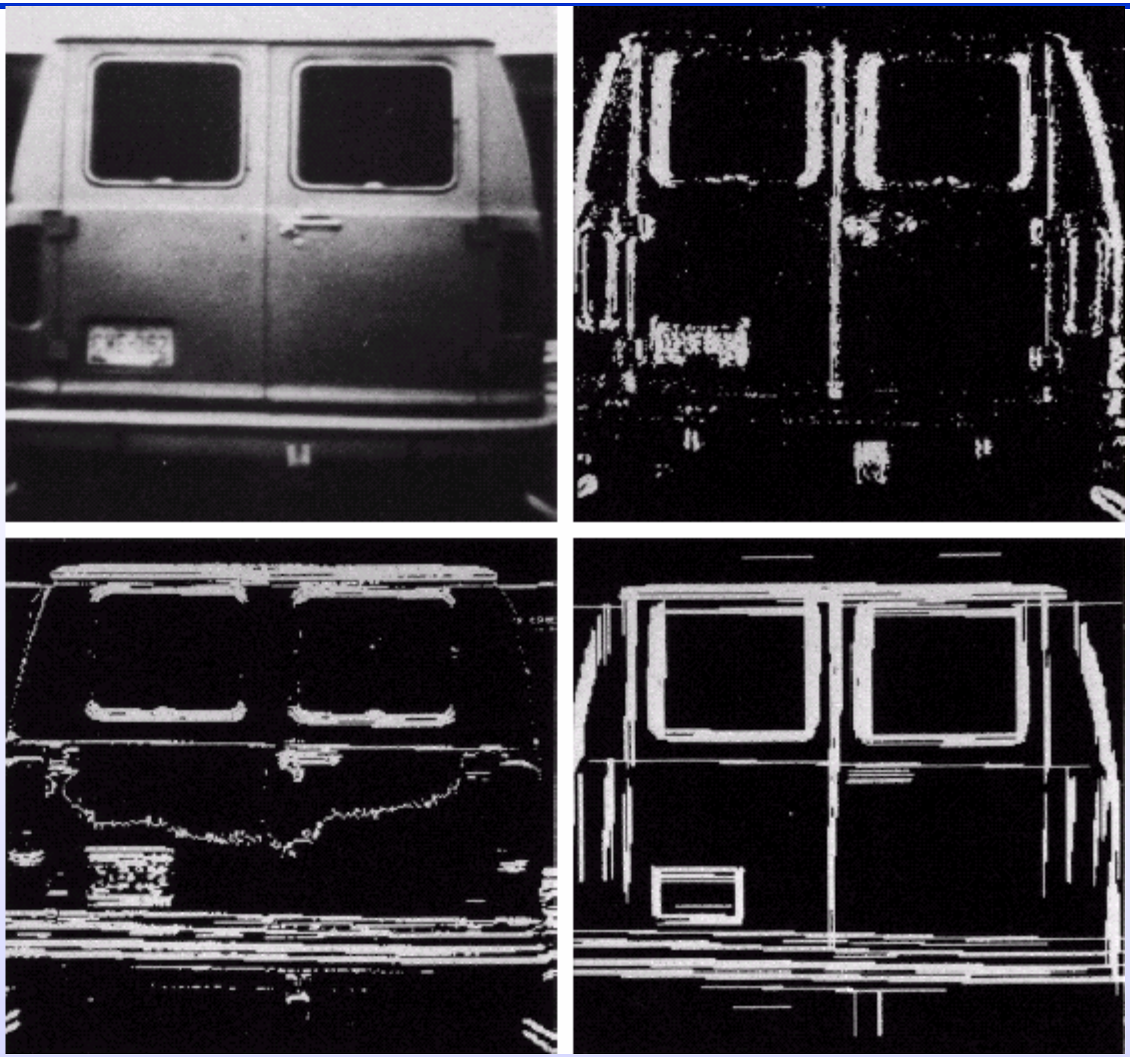
1. The strength of the gradient vector
   An edge pixel with coordinates (x_0,y_0) in a predefined neighborhood of (x,y) is similar in magnitude to the pixel at (x,y) if $|\nabla f(x,y) - \nabla f(x_0,y_0)| \leq E$

2. The direction of the gradient vector
   An edge pixel with coordinates (x_0,y_0) in a predefined neighborhood of (x,y) is similar in angle to the pixel at (x,y) if $|\theta(x,y) - \theta(x_0,y_0)| < A$

A point in the predefined neighborhood of (x_0,y_0) is linked to the pixel at (x,y) if both magnitude and direction criteria are satisfied.

7 Image Analysis –Local Processing

Example



use horizontal and vertical Sobel operators

eliminate isolated short segments

link conditions:
gradient value > 25
gradient direction difference < 15°

7 Image Analysis –Hough Transform

- Hough transform is a technique that can be used to detect (link) regular curves such as lines, circles, and ellipses in an image.

  Line segment in spatial space: $y = ax + b$

  If the line passes through a point (pixel) (x_i, y_i), we obtain:

$$y_i = ax_i + b$$

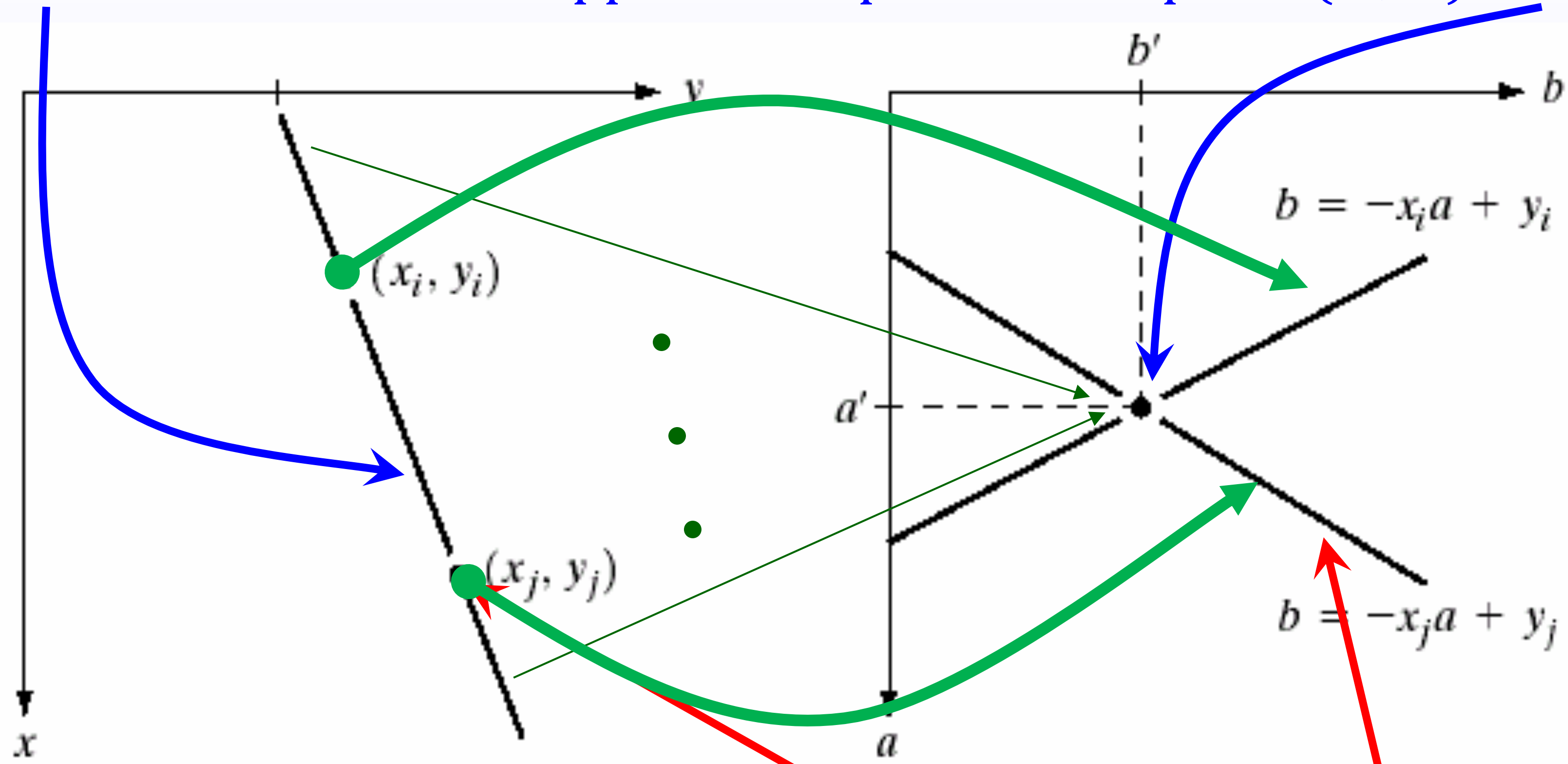
- Rewrite it in ab-plane or parametric space:

$$y_i = ax_i + b$$

$$b = y_i - ax_i$$

7 Image Analysis –Hough Transform

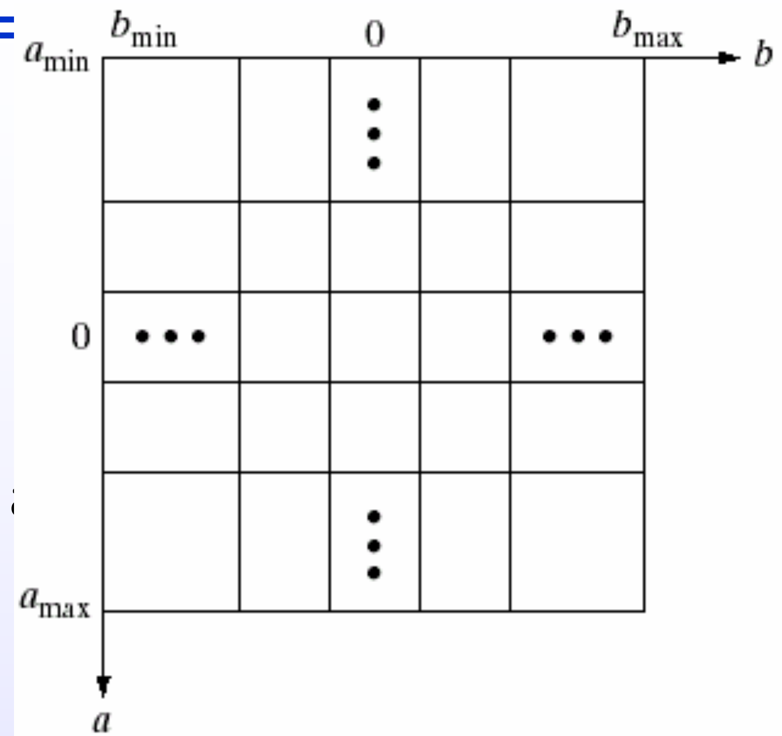
- A line in xy-plane is a set of points (x, y) that satisfy equation: $y = a'x + b'$ which is mapped into a point in ab−plane (a', b')



- A point in xy-plane may have many lines go through it, which is mapped into a line in ab-plane. $(x_j, y_j) \quad \Leftrightarrow \quad b = y_j - a x_j$

a point will be satisfied as a line in ab−plane, vice versa

7 Image Analysis –Hough Transform



- All points (x_i, y_i) on a same line in the image must fall into a same point (a_i, b_i) in the parametric space.

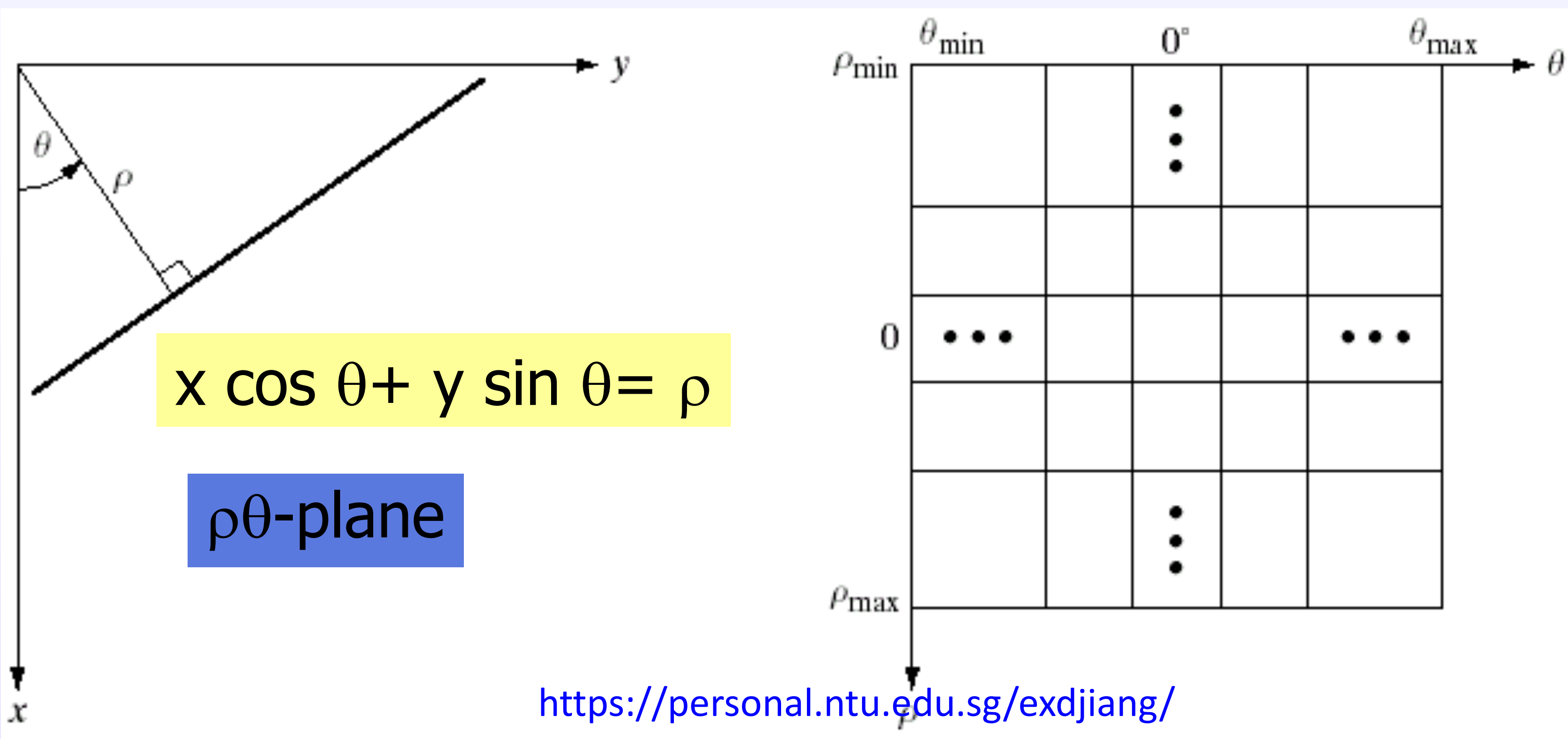
  - Hough transform:
    1. Ddivision of parameter space into cells (a, b).
    2. All cells are initialized to zero,
       $$A(a, b) = 0$$
    3. For each detected point (x_i, y_i) in the image:

$$A(a, b) + 1 \Rightarrow A(a, b) \text{ for all } a \text{ and } b \text{ satisfying } b = y_i - ax_i$$

- At the end of the procedure, value $A(a, b)$ corresponds to the number of points in image lying on the line $y = ax+b$

7 Image Analysis –Hough Transform

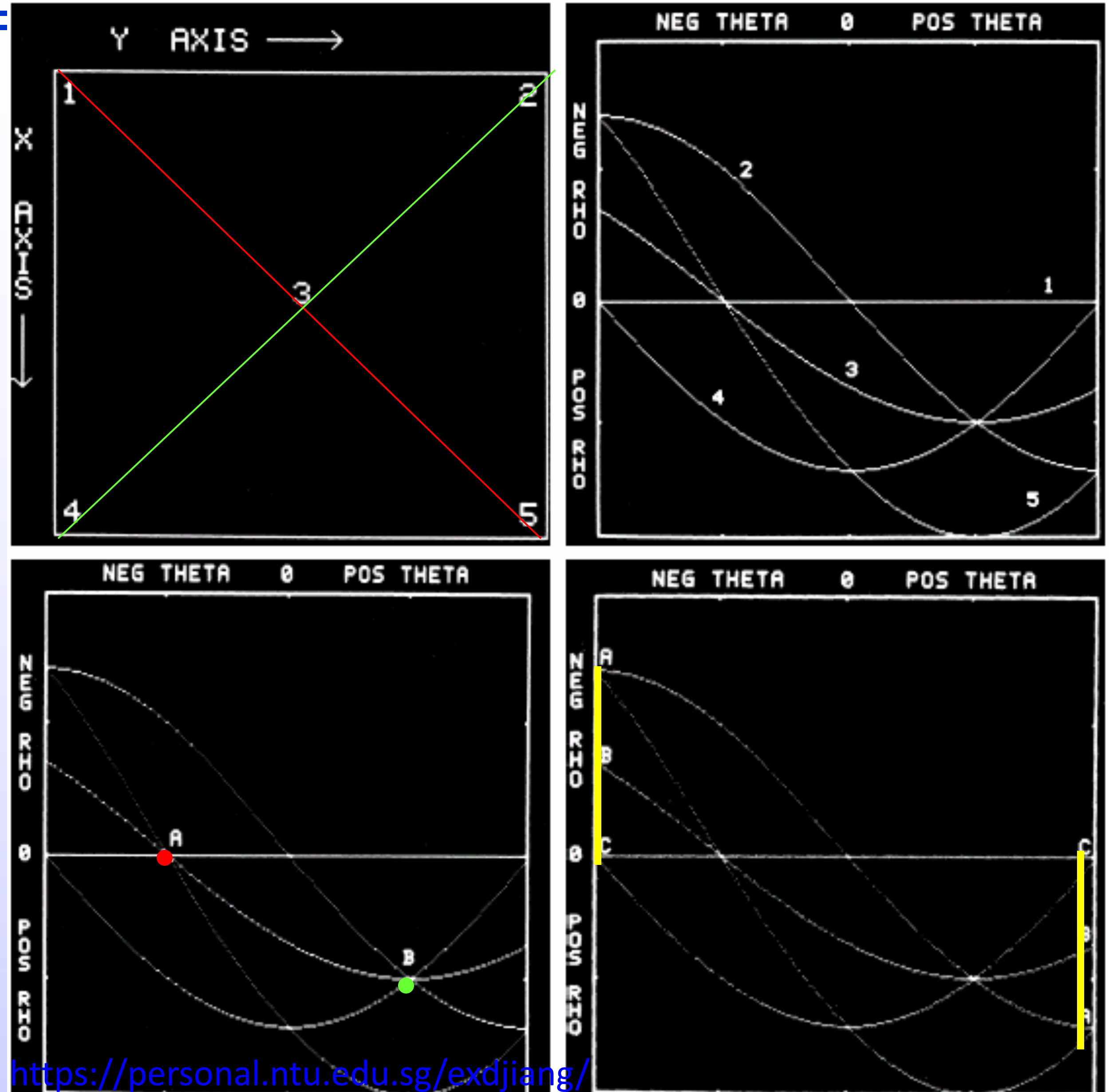
- Problem of using $y=ax+b$ is that a is infinite for a vertical line.
- To avoid the problem, use equation $x\cos\theta + y\sin\theta = \rho$ to represent a line instead.
- Vertical line has θ = 90° with ρ equals to the positive y-intercept or θ = -90° with ρ equals to the negative y-intercept.



7 Image Analysis –Hough Transform

- Example

5 points in the image



ρθ-plane

$$x\cos\theta + y\sin\theta = \rho$$

7 Image Analysis –Hough Transform

- Generalized Hough transform can be used for any function of the form

$$g(v,c) = 0$$

v is a vector of coordinates, c is a vector of coefficients

- For example a circle is represented by equation:

$$(x-c_1)^2 + (y-c_2)^2 = c_3^2$$

- three parameters (c_1, c_2, c_3)
- cube like cells, accumulators of the form $A(c_1, c_2, c_3)$
- For each point in the image, update the value of $A(c_1, c_2, c_3)$ $\{A(c_1, c_2, c_3) + 1 \rightarrow A(c_1, c_2, c_3)\}$ that satisfies the equation $(x-c_1)^2 + (y-c_2)^2 = c_3^2$.

7 Image Analysis –Edge Detection by Hough Transform

1. Compute the gradient of an image and threshold it to obtain a binary image.

2. Specify subdivisions in the $\rho\theta$-plane.

3. Examine the counts of the accumulator cells for high pixel concentrations.

4. Examine the relation (principally for continuity) between pixels in a chosen cell.

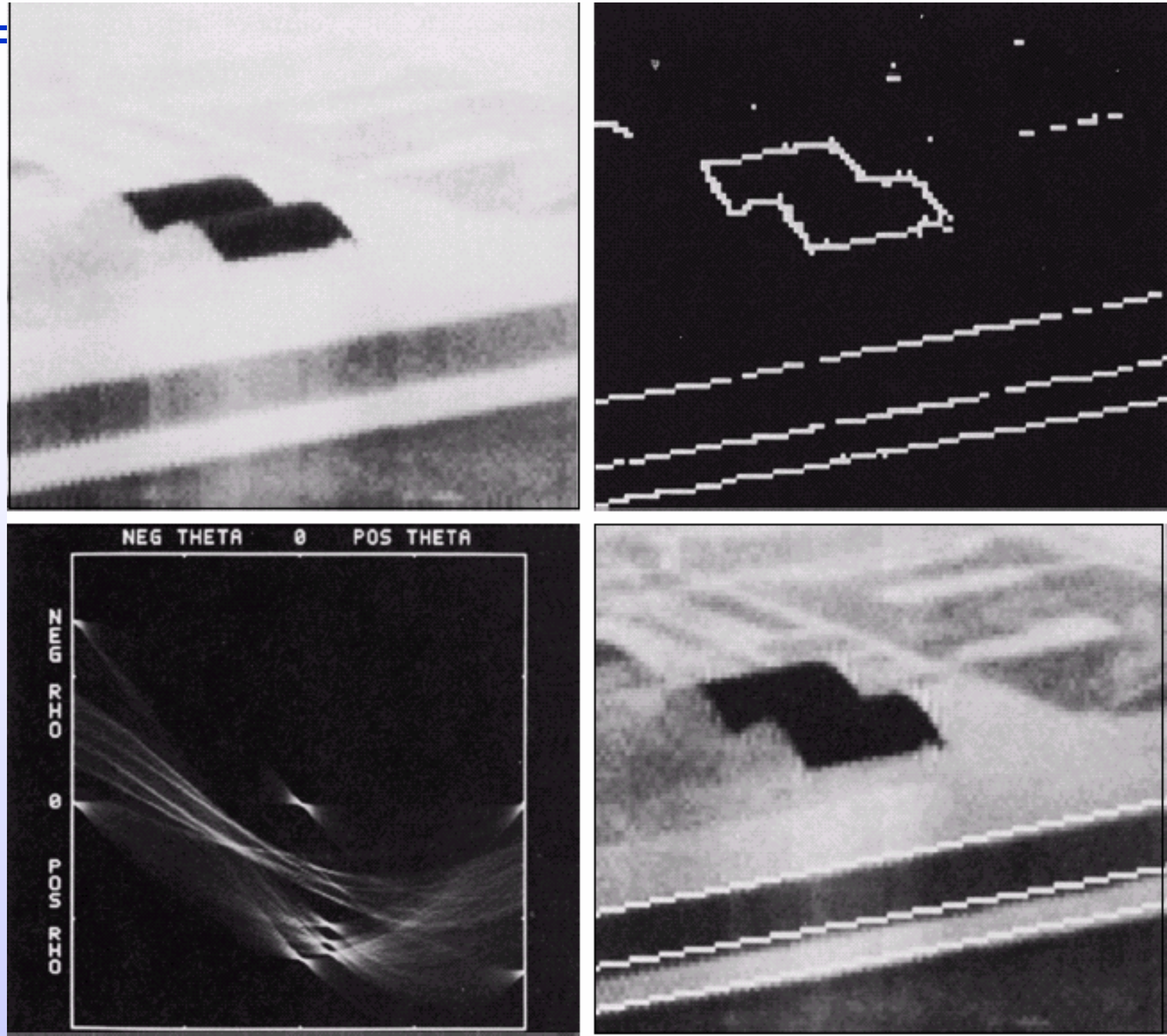
5. A gap at any point is linked if the distance between that point and its closet neighbor below a certain threshold.

7 Image Analysis –Edge Detection by Hough Transform

Example

link criteria: pixels belonged to a set is linked according to the highest count.

no gaps were longer than 5 pixels



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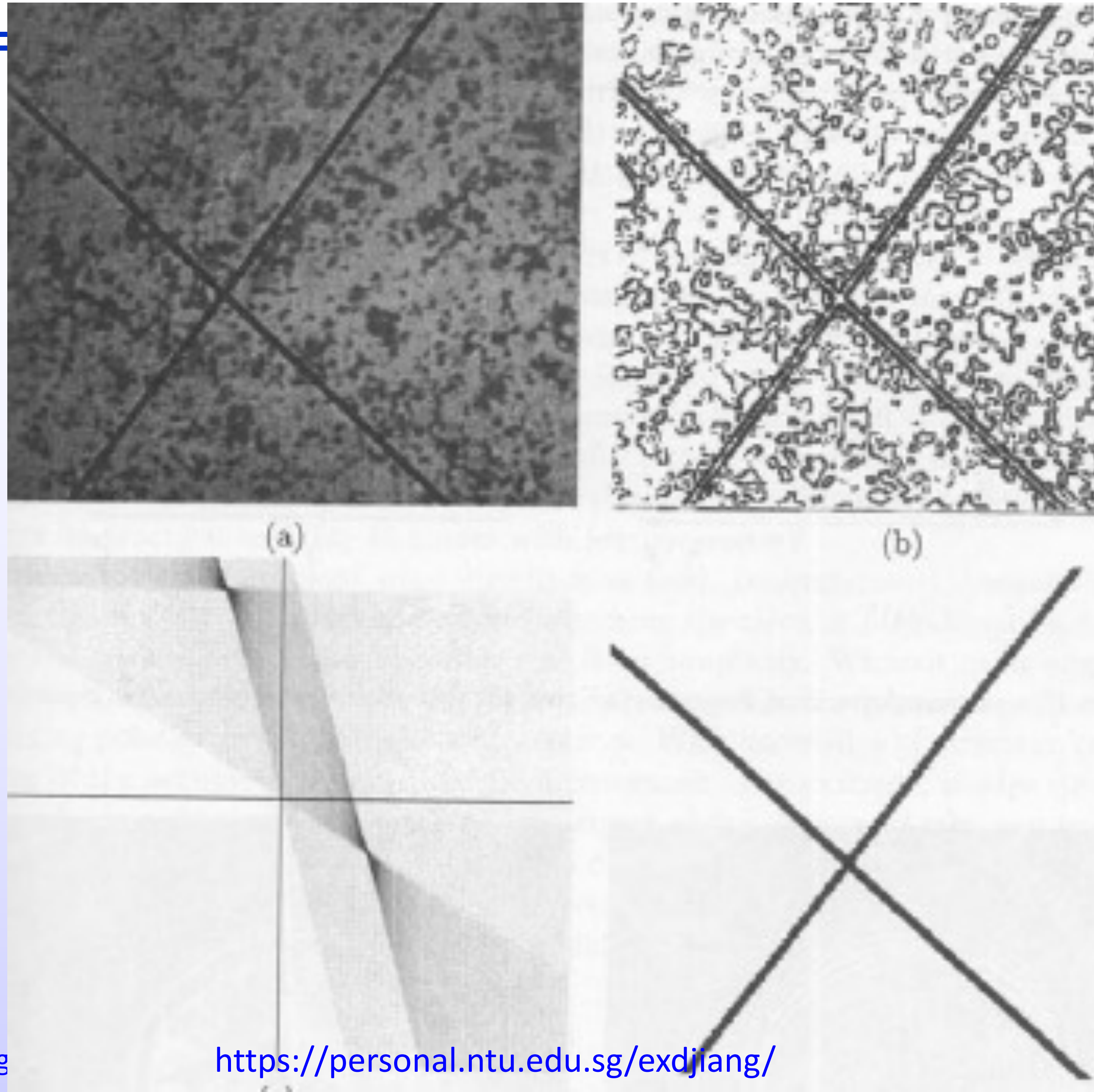
7 Image Analysis –Edge Detection by Hough Transform

Further example



7 Image Analysis–Local Dominant Orientation

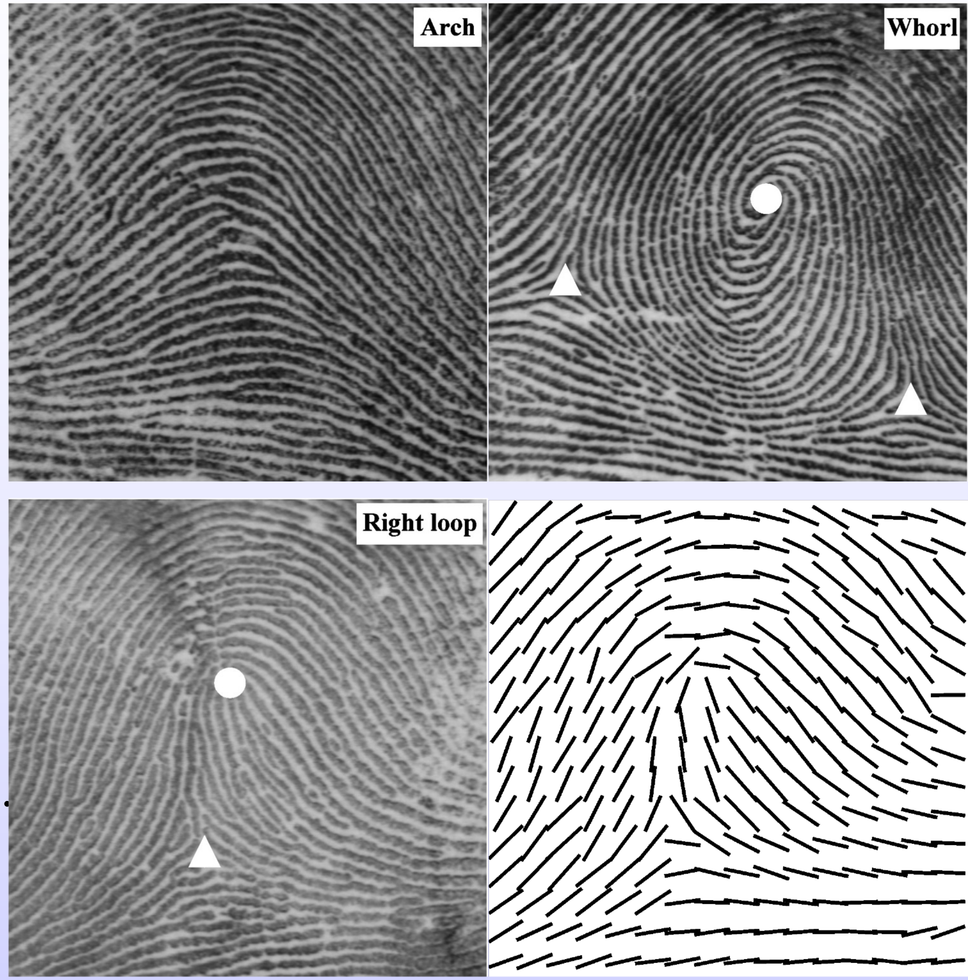
Fingerprint classification: for example, arch, whorl and loop

- Local orientations are intrinsic features for such task.

Orientations denoted by short lines.

X.D. Jiang, M. Liu and A. Kot, Fingerprint Retrieval for Identification, *IEEE Transactions on Information Forensics and Security*, vol. 1, no. 4, pp. 532-542, December 2006.

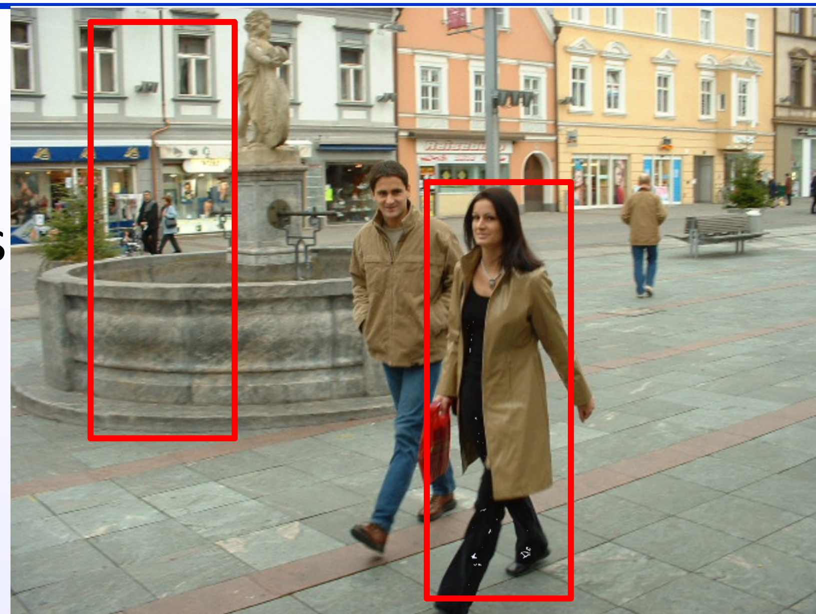
M. Liu, X.D. Jiang and A. Kot, Efficient Fingerprint Search Based on Database Clustering, *Pattern Recognition*, vol. 40, no. 6, pp. 1793-1803, June 2007.

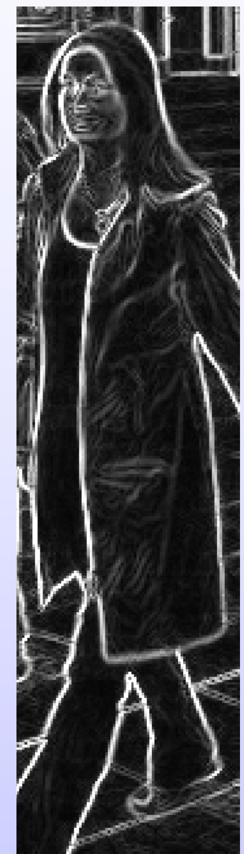


7 Image Analysis–Local Dominant Orientation

Human Detection: Local orientations are intrinsic features for such task.

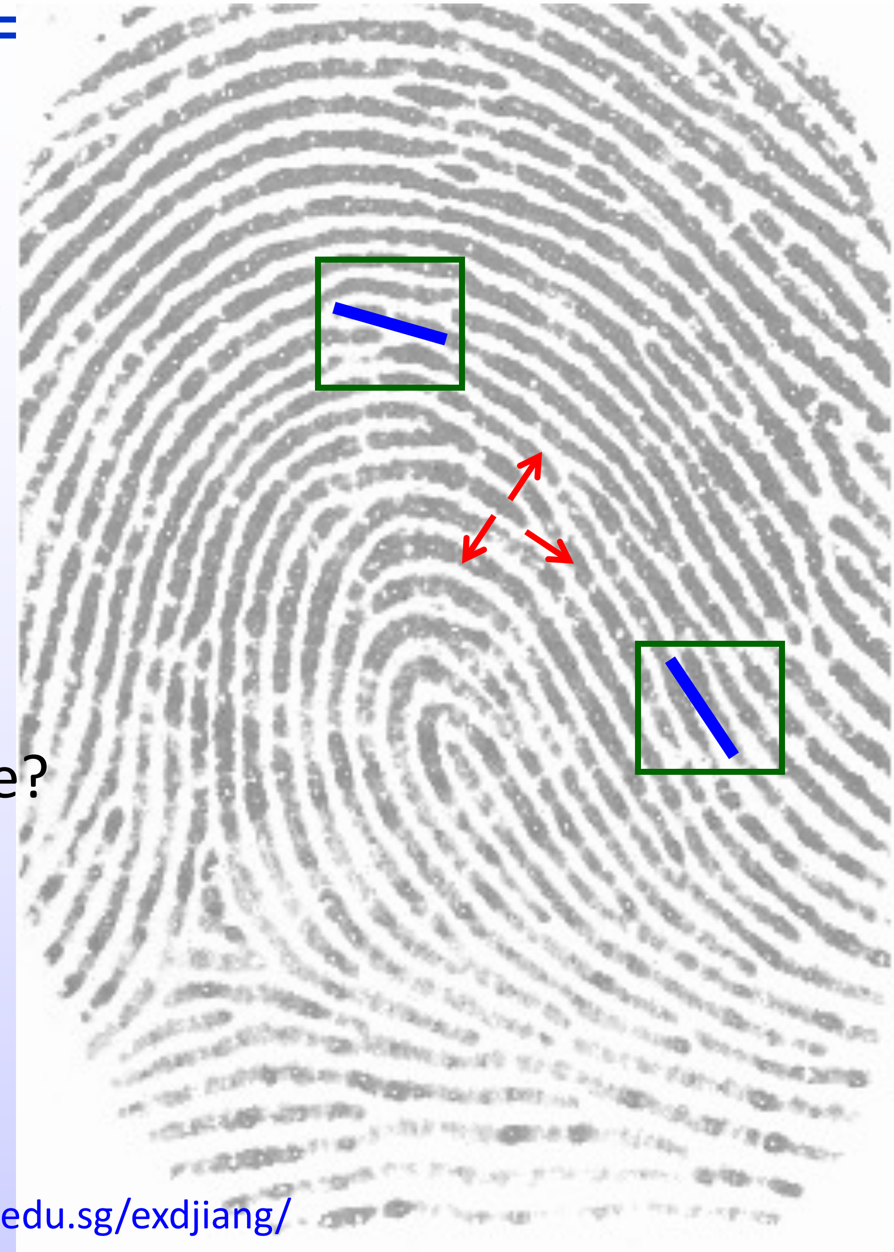
A. Satpathy, X.D. Jiang and H. Eng, "Human Detection by Quadratic Classification on Subspace of Extended Histogram of Gradients," ***IEEE Transactions on Image Processing***, vol. 23, no. 1, pp. 287-297, January, 2014.

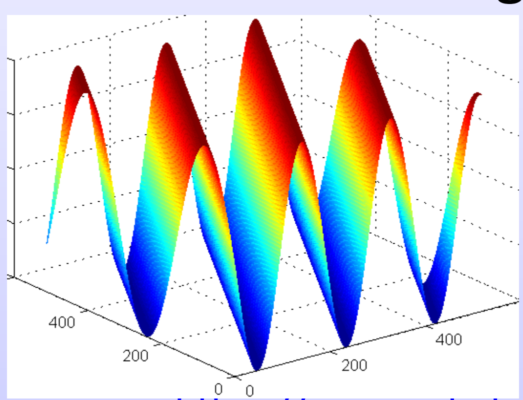




7 Image Analysis–Local Dominant Orientation

- Local orientations $\varphi(x,y)$, $0° < \varphi(x,y) \leq 180°$ of edges and lines are important image features.
- The gradient vectors are very noisy and a same orientation may be represented by gradients with opposite directions.
- How to extract stable and robust dominant (main) orientation information of a local area of image?



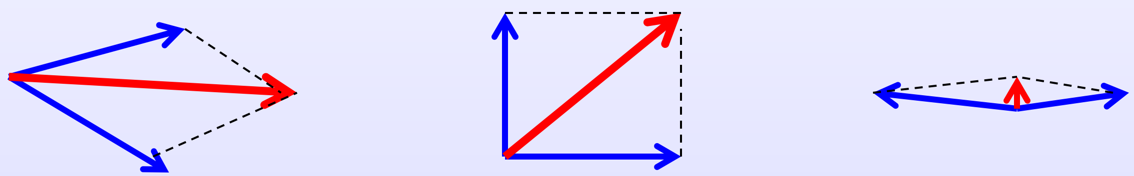


7 Image Analysis–Local Dominant Orientation

- Should we smooth the image first then take gradient or smooth the gradient image?

$$\nabla(h(x,y) * f(x,y)) = h(x,y) * \nabla f(x,y) = \sum_{(s,t)\in S_{xy}} w(s,t)\nabla f(s,t)$$

- The output of any linear filter is a weighted average of all inputs. What is the problem of average the gradient vector?



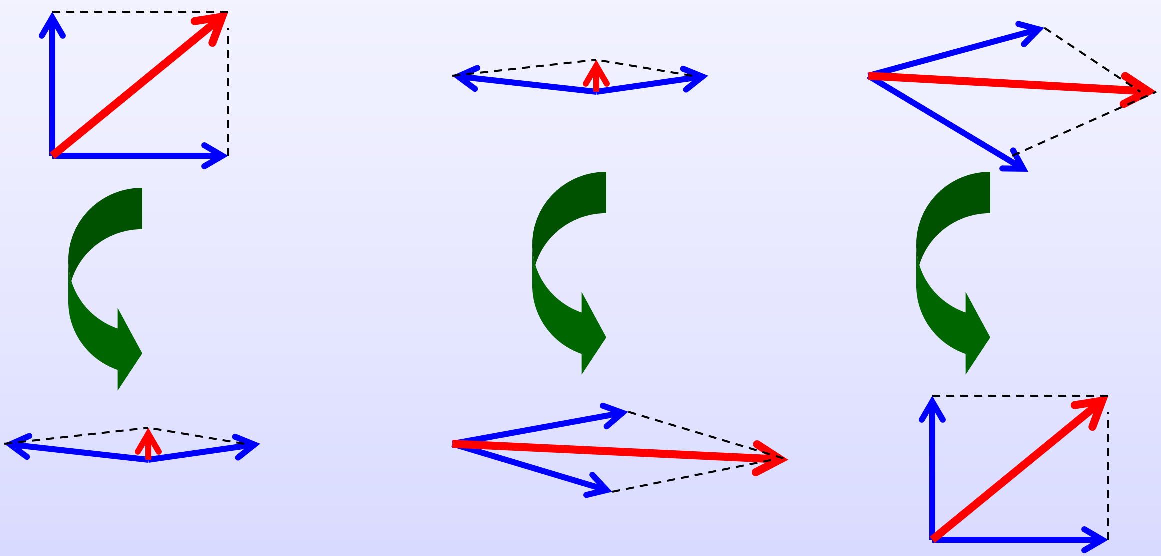
- Problem is the directions of gradient $\theta(x,y)$, $0° < \theta(x,y) \leq 360°$ but the orientations of edges or lines $\varphi(x,y)$, $0° < \varphi(x,y) \leq 180°$.

7 Image Analysis–Local Dominant Orientation

- One solution is to smooth or average the squared gradient vectors.
  What is the square of a vector?
- The vector direction angle will be doubled. Thus:

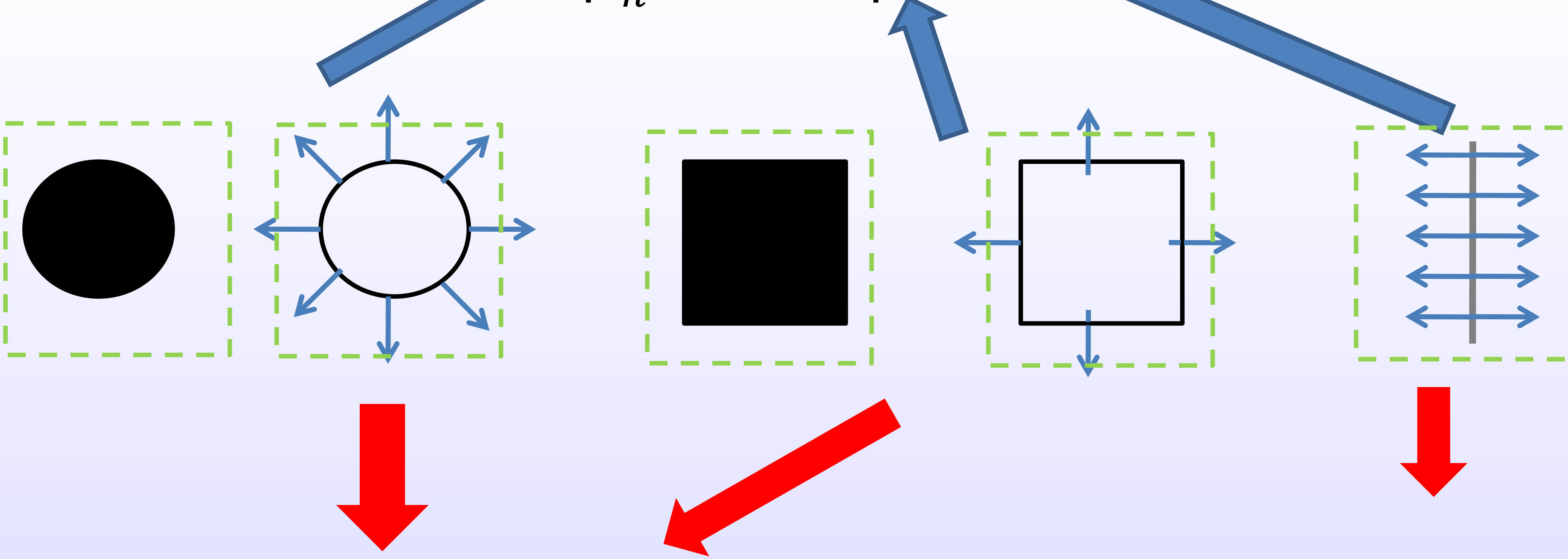
Average:

Squared average:



7 Image Analysis–Local Dominant Orientation

$$\left|\sum_{n} \nabla f(x,y)\right| \Rightarrow 0$$



$$\left|\sum_{n} [\nabla f(x,y)]^2\right| \Rightarrow 0$$

$$\left|\sum_{n} [\nabla f(x,y)]^2\right| \Rightarrow n[\nabla f(x,y)]^2$$

7 Image Analysis–Local Dominant Orientation

- Mathematically, let's represent the gradient vector by a complex variable.

$$\nabla f(x,y) = G_x(x,y) + jG_y(x,y)$$
$$= |\nabla f(x,y)|e^{j\theta(x,y)}, \qquad \text{where } j = \sqrt{-1}$$

- Its square is then

$$[\nabla f(x,y)]^2 = G_x^2(x,y) - G_y^2(x,y) + j2G_x(x,y)G_y(x,y)$$
$$= |\nabla f(x,y)|^2 e^{j2\theta(x,y)}$$

- Obviously: $\theta(x,y) = \frac{1}{2}\tan^{-1}\left(\frac{2G_x(x,y)G_y(x,y)}{G_x^2(x,y) - G_{y\ y}^2(x,y)}\right)$

7 Image Analysis–Local Dominant Orientation

- Now, the average of the squared gradient is

$$\overline{[\nabla f(x,y)]^2} = \sum_{(s,t)\in S_{xy}} [G_x^2(s,t) - G_y^2(s,t) + j2G_x(s,t)G_y(s,t)]$$

$$= A - B + j2C$$

Therefore, the dominant orientation is determined by

$$\overline{\theta(x,y)} = \frac{1}{2}\tan^{-1}\left(\frac{2C}{A-B}\right)$$

where

$$A = \sum_{(s,t)\in S_{xy}} G_x^2(s,t)\,,\ B = \sum_{(s,t)\in S_{xy}} G_y^2(s,t)\,,\ C = \sum_{(s,t)\in S_{xy}} G_x(s,t)G_y(s,t)$$

7 Image Analysis–Local Dominant Orientation

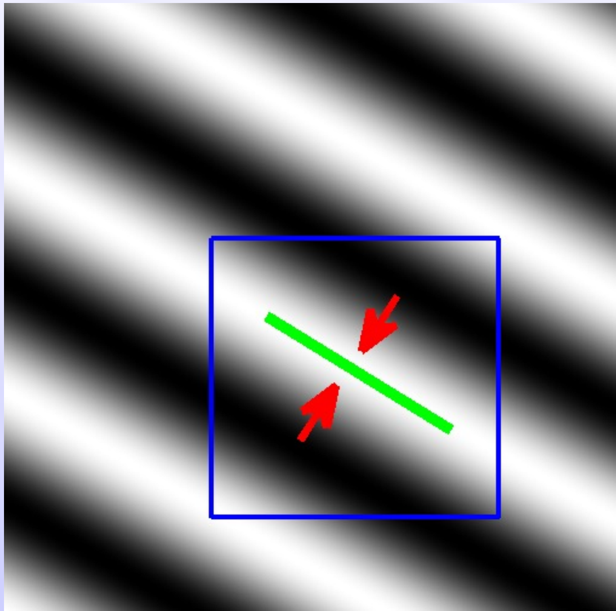
An continuous image is modeled locally by $f(x, y) = \sin(ax + by)$

$$\nabla f(x, y) = \frac{\partial f(x, y)}{\partial x} + j\frac{\partial f(x, y)}{\partial y}$$
$$= a\cos(ax + by) + jb\cos(ax + by)$$

$$\theta(x, y) = \arctan\frac{\mathrm{Im}\{\nabla f(x, y)\}}{\mathrm{Re}\{\nabla f(x, y)\}}$$
$$= \arctan\frac{b}{a} = \varphi + 90°$$

$$\frac{\mathrm{Im}\left\{\int_y^{y+w}\int_x^{x+w}\nabla f(u, v)dudv\right\}}{\mathrm{Re}\left\{\int_y^{y+w}\int_x^{x+w}\nabla f(u, v)dudv\right\}}$$
$$= \frac{0}{0}$$

$$\frac{\mathrm{Im}\left\{\int_y^{y+w}\int_x^{x+w}[\nabla f(u, v)]^2dudv\right\}}{\mathrm{Re}\left\{\int_y^{y+w}\int_x^{x+w}[\nabla f(u, v)]^2dudv\right\}} = \frac{2ab}{a^2 - b^2}$$
$$= \tan 2\theta(x, y)$$



7 Image Analysis–Local Dominant Orientation

- Obviously, the magnitude of the average squared gradient is

$$\left|\overline{[\nabla f(x,y)]^2}\right| = \sqrt{(A-B)^2 + 4C^2}$$

- And the average magnitude of squared gradients is

$$\overline{|\nabla f(x,y)|^2} = A + B$$

- The following amount indicates how all gradients in a local area point to a same orientation, called anisotropy or coherence of an image local area.

$$coh(x,y) = \frac{\left|\overline{[\nabla f(x,y)]^2}\right|}{\overline{|\nabla f(x,y)|^2}} = \frac{\sqrt{(A-B)^2 + 4C^2}}{A+B}$$

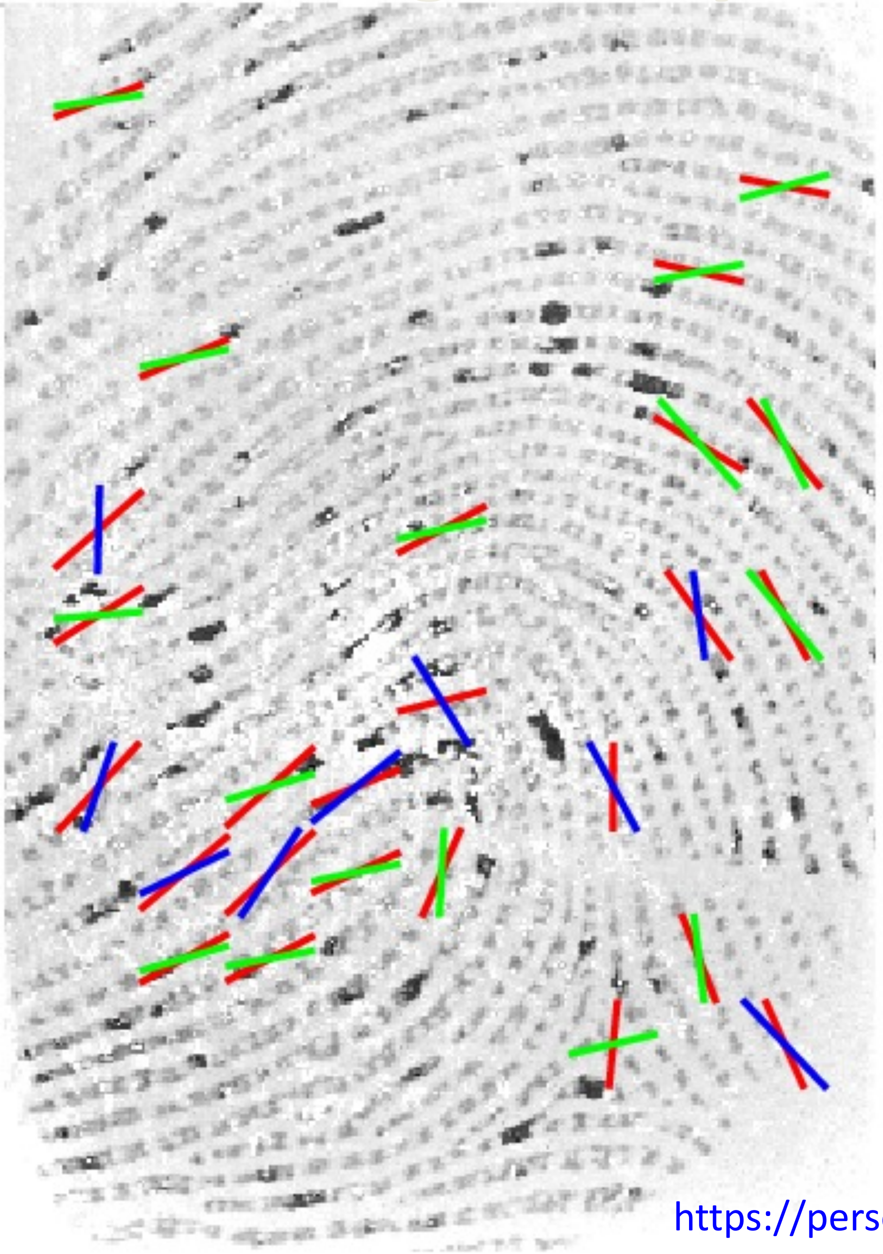
7 Image Analysis–Local Dominant Orientation

- What is the minimum and maximum values of $coh(x,y)$?
- In what cases $coh(x,y)$ reaches to the minimum or maximum value?
- How to choose the appropriate size of the average window $S(x,y)$?
- What factor will affects the selection of the $S(x,y)$?
- What problems of this technique may have?

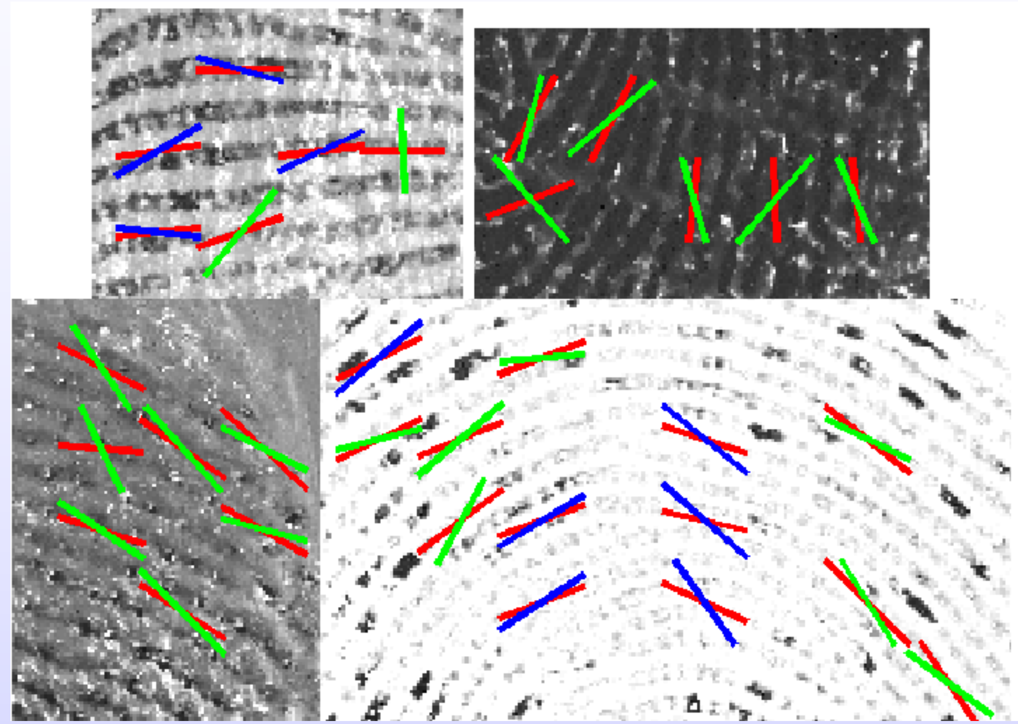
- Rresearch on this topic can be found in the research publication:

X.D. Jiang, "On Orientation and Anisotropy Estimation for Online Fingerprint Authentication," *IEEE Transactions on Signal Processing,* Vol. 53, No. 10, pp. 4038- 4049, October 2005.

7 Image Analysis–Local Dominant Orientation



Examples of different methods

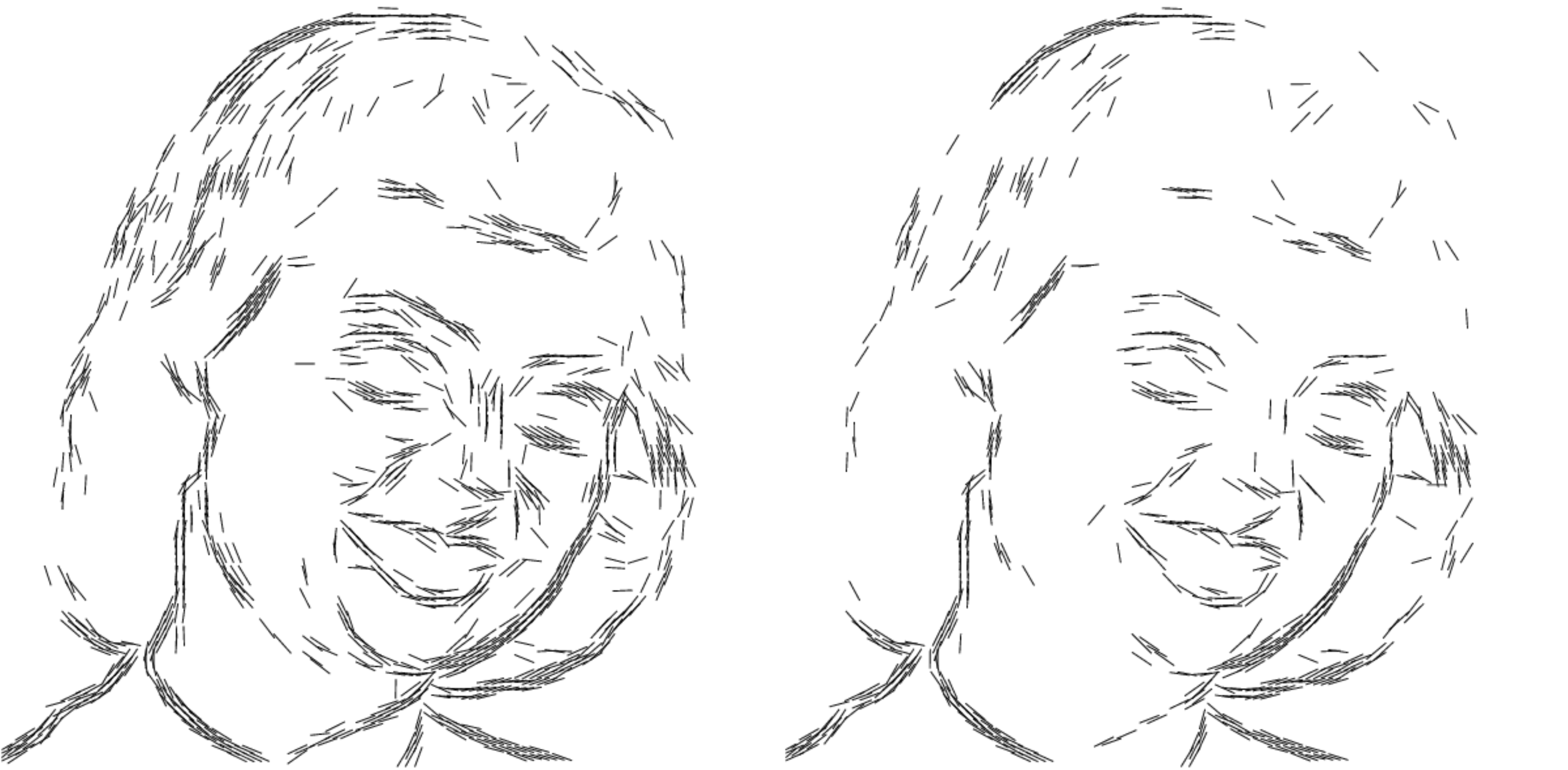


7 Image Analysis–Local Dominant Orientation

A noise free face image and its noising version

7 Image Analysis–Local Dominant Orientation

Extracted significant orientations represented by short lines of the noise free image by two different gradient operators



7 Image Analysis–Local Dominant Orientation

Extracted significant orientations represented by short lines of the noising face image by two different gradient operators