CS 161
Due: 11/6/18

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1. $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$

$\neg P \vee \neg Q = \neg Q \vee \neg P$

Q	P	$\neg Q$	$\neg P$	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

match, so same

(moreover, in CNF, they are $\neg P \vee \neg Q$ and $\neg Q \vee \neg P$ which are equivalent statements)

$P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

Q	P	$\neg Q$	$P \wedge \neg Q$ (1)	$\neg P$	$\neg P \wedge Q$ (2)	(1) $\vee$ (2)	$P \Leftrightarrow \neg Q$
0	0	1	0	1	0	0	0
0	1	1	1	0	0	1	1
1	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0

match, so same

2. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

let Smoke be denoted by S

Fire be denoted by F

S	F	$\neg S$	$\neg F$	① $S \Rightarrow F$	② $\neg S \Rightarrow \neg F$	① $\Rightarrow$ ②
0	0	1	1	1	1	1
0	1	1	0	1	0	0
1	0	0	1	0	1	1
1	1	0	0	1	1	1

Because not all cases are true, the sentence is $\boxed{\text{neither}}$ valid nor unsatisfiable.
It's satisfiable in 3/4 worlds, unsatisfiable in 1/4.

$(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$

let smoke be S

Fire be F

Heat be H

S	F	H	① $S \Rightarrow F$	② $S \vee H$	③ ② $\Rightarrow F$	① $\Rightarrow$ ③
0	0	0	1	0	1	1
0	0	1	1	1	0	0
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	0	1	0	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Because not all cases are true, the sentence is $\boxed{\text{neither}}$ valid nor unsatisfiable. It's satisfiable in 7/8 worlds, unsatisfiable in 1/8.

$((\text{Smoke} \wedge \text{Heat}) \Rightarrow \text{fire}) \Leftrightarrow ((\text{Smoke} \Rightarrow \text{fire}) \vee (\text{Heat} \Rightarrow \text{fire}))$

let Smoke be S
Heat be H
Fire be F

S	H	F	① $S \wedge H$	④ ① $\Rightarrow F$	② $S \Rightarrow F$	③ $H \Rightarrow F$	⑤ ② $\vee$ ③	④ $\Leftrightarrow$ ⑤
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	1	1	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

Because all cases are true the sentence is both valid and satisfiable.

3a. $\text{mythical} \Rightarrow \text{immortal}$
$\neg\text{mythical} \Rightarrow (\neg\text{immortal} \wedge \text{mammal})$
$(\text{immortal} \vee \text{mammal}) \Rightarrow \text{horned}$
$\text{horned} \Rightarrow \text{magical}$

3b converting the Knowledge Base to CNF in same order as above:

$(\neg\text{mythical} \vee \text{immortal})$
$\wedge\ (\text{mythical} \vee (\neg\text{immortal} \wedge \text{mammal}))$
$\wedge\ ((\neg\text{immortal} \wedge \neg\text{mammal}) \vee \text{horned})$
$\wedge\ (\neg\text{horned} \vee \text{magical})$ $\Rightarrow$

$= (\neg\text{mythical} \vee \text{immortal}) \wedge (\text{mythical} \vee \text{mortal}) \wedge$
$(\text{mythical} \vee \text{mammal}) \wedge (\neg\text{immortal} \vee \text{horned}) \wedge (\neg\text{mammal} \vee \text{horned})$
$\wedge\ (\neg\text{horned} \vee \text{magical})$

3c is unicorn mythical? magical? horned?

KB

1. $\neg$mythical $\vee$ immortal
2. mythical $\vee$ $\neg$immortal
3. mythical $\vee$ mammal
4. $\neg$immortal $\vee$ horned
5. $\neg$mammal $\vee$ horned
6. $\neg$horned $\vee$ magical

7. $\neg$ mythical ← assumption
8. $\neg$immortal (2,7)
9. mammal (3,7)
10. horned (5,9)
11. magical (6,10)
12. $\neg$mythical $\vee$ horned (1,4)

no contradictions found. can't conclude that unicorn is mythical.

7. $\neg$magical ← assumption
8. $\neg$horned (6,7)
9. $\neg$immortal (4,8)
10. $\neg$mythical (1,9)
11. mammal (3,10)
12. horned (5,11)

(8,12) are contradictions, thus proven that unicorn is magical.

7. $\neg$horned ← assumption
8. $\neg$immortal (4,7)
9. $\neg$mythical (1,8)
10. mammal (3,9)
11. $\neg$mammal (5,7)

(10, 11) are contradictions, thus proven that $\boxed{\text{unicorn is horned}}$

4. **Figure 1**: $\boxed{\text{yes, decomposable}}$ because none of the variables from the "and" gates overlap from its subcircuits

$\boxed{\text{not deterministic}}$ because the variables from the subcircuits of the "or" gates are not mutually exclusive ($\neg\neg C$ present in both subcircuits of topmost "or" gate)

$\boxed{\text{not smooth}}$ because not all subcircuits of the "or" gates have the same variables (3rd level, second from left "or" has one subcircuit of C, and the other of C and D, even though D not mentioned in other subcircuit.

Figure 2: yes, decomposable because none of the variables from the "and" gates overlap from its subscripts.

not deterministic because the variables from the subcircuits of the "or" gates are not mutually exclusive ($\neg A$ and B present in both subcircuits of 3rd level, 3rd from left "or" gate).

yes, smooth, because all subcircuits of the "or" gates have the same variables

5a. $(\neg A \wedge B) \vee (\neg B \wedge A)$

A	B	$\neg A$	$\neg B$	(1) $\neg A \wedge B$	(2) $\neg B \wedge A$	(1) $\vee$ (2)	
0	0	1	1	0	0	0	
0	1	1	0	1	0	1	← model 1
1	0	0	1	0	1	1	← model 2
1	1	0	0	0	0	0	

weight of model 1

$w(B) \cdot w(\neg A) = 0.3 * 0.9 = 0.27$

weight of model 2

$w(A) \cdot w(\neg B) = 0.1 * 0.7 = 0.07$

WMC $= 0.27 + 0.07 = \boxed{0.34}$

5b. in CNF: $(\neg A \land B) \lor (\neg B \land A)$

A	B	$\neg A$	$\neg B$	① $\neg A \land B$	② $\neg B \land A$	① $\lor$ ②	
0	0	1	1	0	0	0	
0	1	1	0	1	0	1	← model 1
1	0	0	1	0	1	1	← model 2
1	1	0	0	0	0	0	

<u>weight of model 1</u>

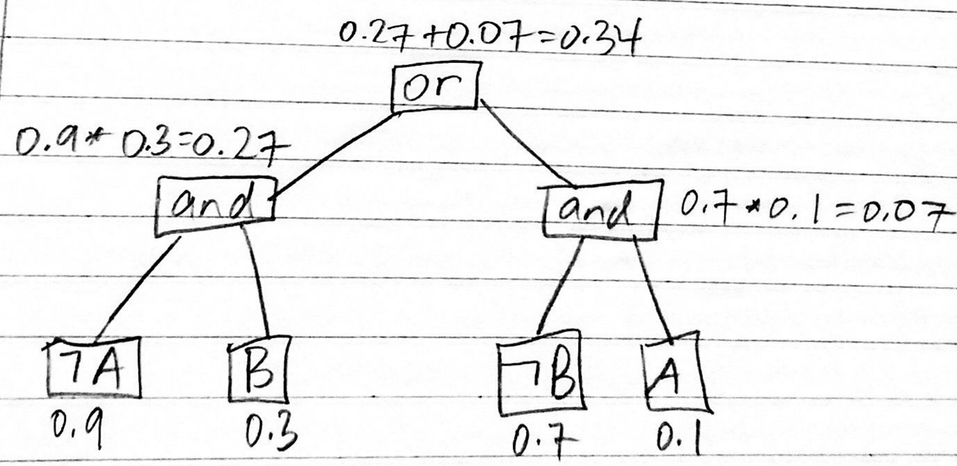
$w(B) \cdot w(\neg A) = 0.3 * 0.9 = 0.27$

<u>weight of model 2</u>

$w(A) \cdot w(\neg B) = 0.1 * 0.7 = 0.07$

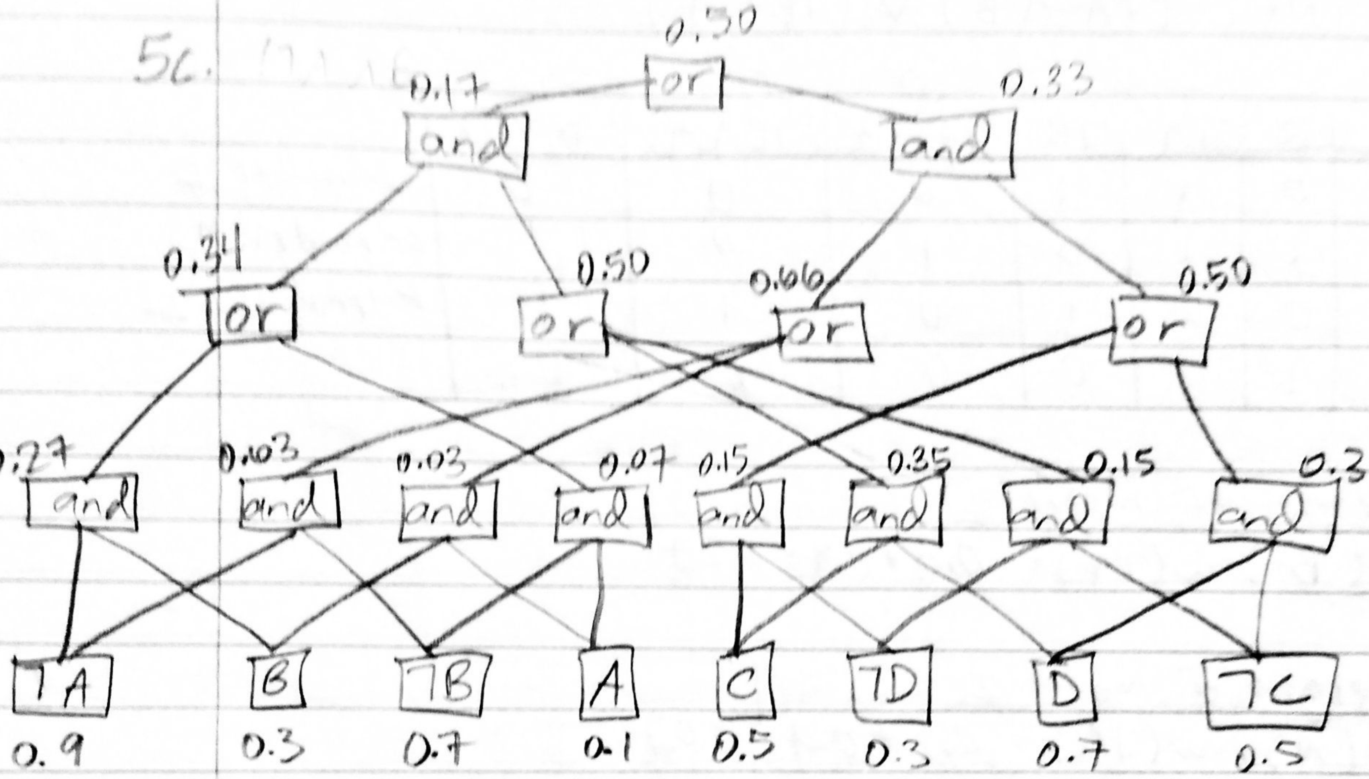
<u>WMC</u> $= 0.27 + 0.07 = 0.34$

(note that this NNF circuit is the same as the formula from 5a)



This is so incredibly fascinating!! The values are the same between WMC and the count on the root. It seems that each "and" gate represents a model, and the "or" sums up the values.

56.



the weighted model count is 0.50.
quite thankful I didn't have to do this using truth tables!