

- ①
1.  $M_i$  for the possibility get  $i$  side for  $i=1, 2, 3, 4, 5$   
 $M_6 = 1 - \sum_{i=1}^5 M_i$
  2.  $M_i = \frac{1}{6}$  for  $i=1, 2, 3, 4, 5, 6$
  3.  $M_2 = 1$   $M_1 = M_3 = M_4 = M_5 = M_6 = 0$
  4.  $0 \leq M_i \leq 1$   $\sum_{i=1}^6 M_i = 1$   $i=1, 2, 3, 4, 5, 6$

②  $E(w) = \frac{1}{2} \sum_{n=1}^N \alpha_n (t_n - w^T \phi(x_n))^2$  want  $\nabla E = 0^T$

$$\frac{\partial}{\partial w} E(w) = \sum_{n=1}^N \alpha_n (t_n - w^T \phi(x_n)) \phi(x_n)$$

$$0 = \sum_{n=1}^N \alpha_n t_n \phi(x_n)^T - \sum_{n=1}^N \alpha_n w^T \phi(x_n) \phi(x_n)^T$$

$$w^T \sum_{n=1}^N \alpha_n \phi(x_n) \phi(x_n)^T = \sum_{n=1}^N \alpha_n t_n \phi(x_n)^T$$

$$w^T \alpha X^T X = \alpha^T X^T T$$

$$w = (X^T \alpha X)^{-1} X^T \alpha T$$

- ③ 1, NO, one counter example is if validation data is same as train data, the error will be same, not higher

2 Yes, for unregularized regression, higher degree can make line more close to data, which means lower or same error

3 No, one counter example is for  $\lambda = \infty$  in regularized regression. the  $w$  will be real small compare to value  $w$  should be. So the regularized error will be higher in this case.

$$(F) E(w) = \sum_{n=1}^N (t_n - w^T \phi(x_n)) + \sum_{i=1}^J \lambda_i w^j$$

$\lambda$  and  $j$  is hyper parameter input  $j = \{1, 2\}$   $\lambda$  is weight

$$\nabla E(w) = - \sum_{n=1}^N (t_n - w^T \phi(x_n)) \phi(x_n)^T + j \sum_{i=1}^J \lambda_i w^{j-1}$$

$$= - \sum_{n=1}^N (t_n - w^T \phi(x_n)) \phi(x_n)^T + 2 \sum_{i=1}^{J_1} \lambda_{J_1} w + \sum_{i=1}^{J_2} \lambda_{J_2}$$

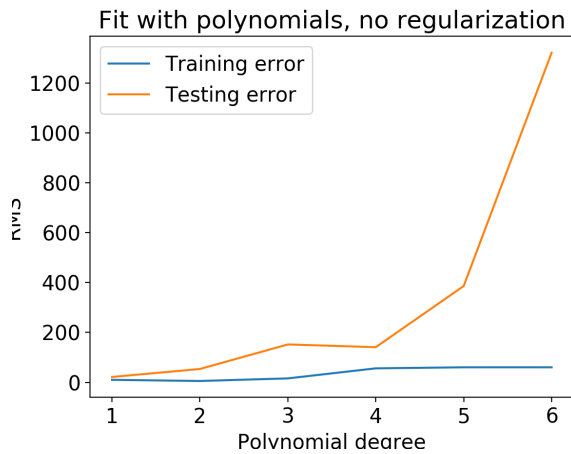
## Problem 5

### 5.1

1. Niger - 313.7
2. Sierra Leone - 185.3
3. Replace the missing value with mean in column.

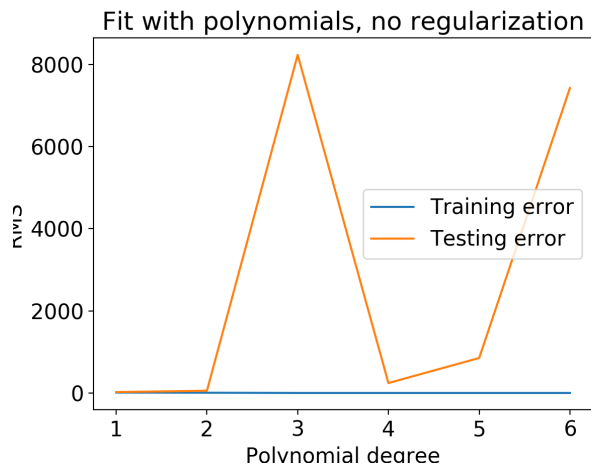
### 5.2

1.



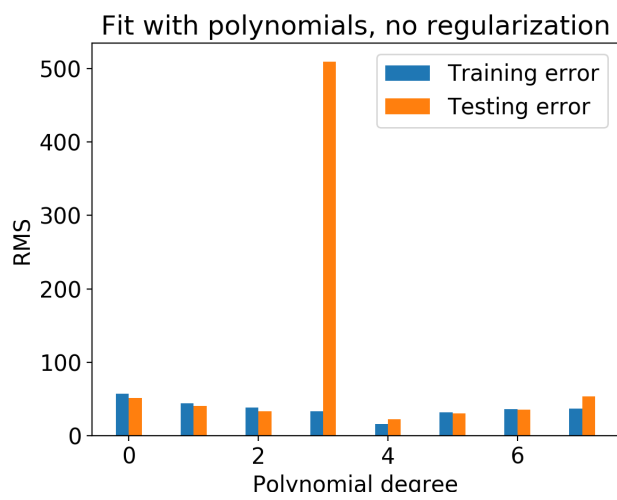
Before normalization

With the increase of polynomial degree, the testing error goes up.

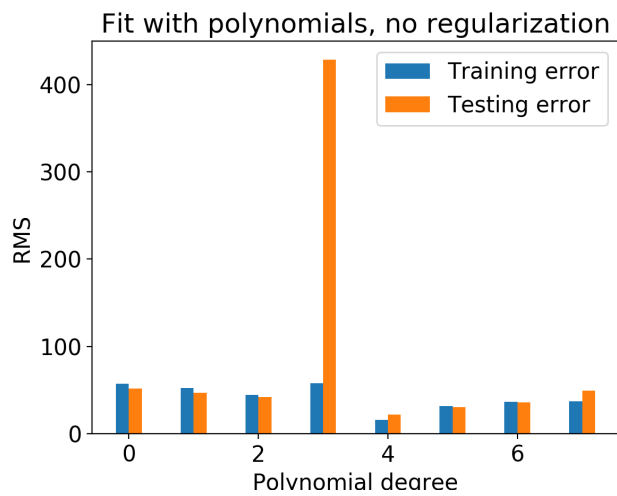


After normalization

### 5.2.2

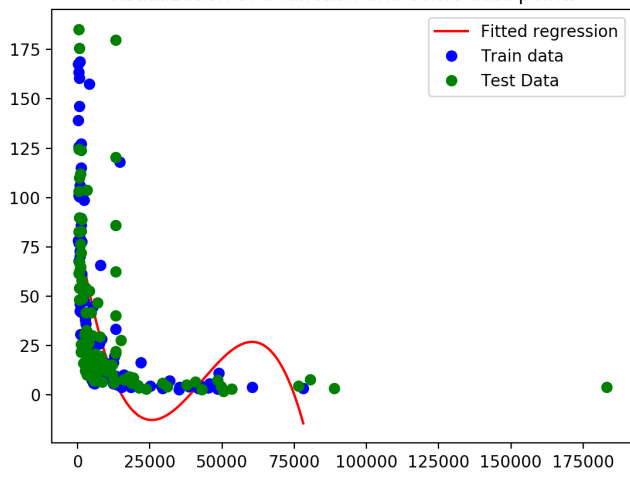


With bias



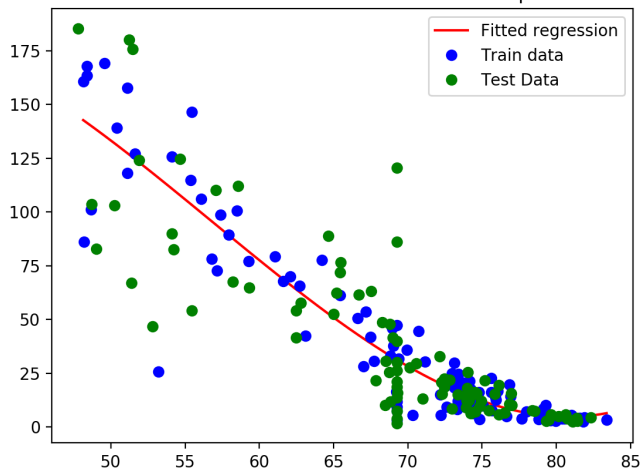
Without bias

Visualization of a function and some data points



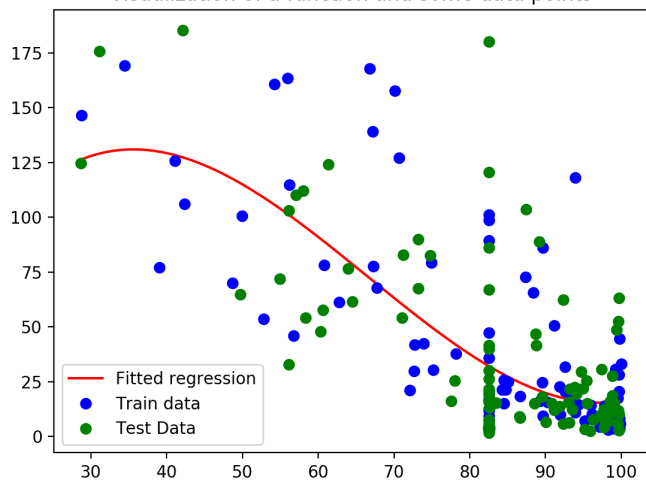
GNI

Visualization of a function and some data points



Life expectancy

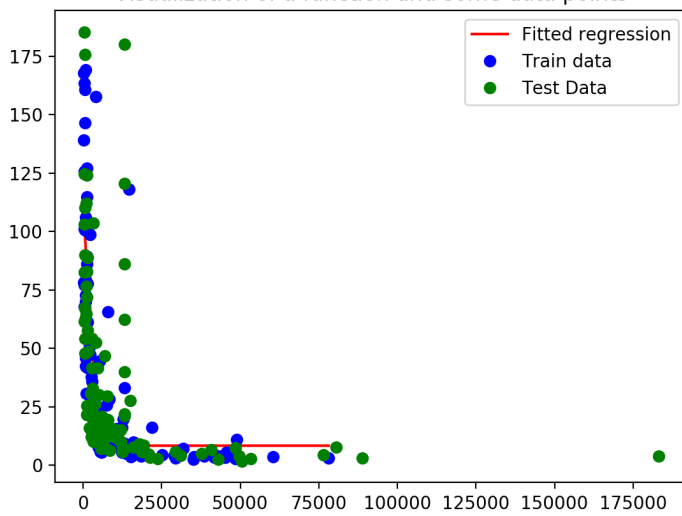
Visualization of a function and some data points



Literacy

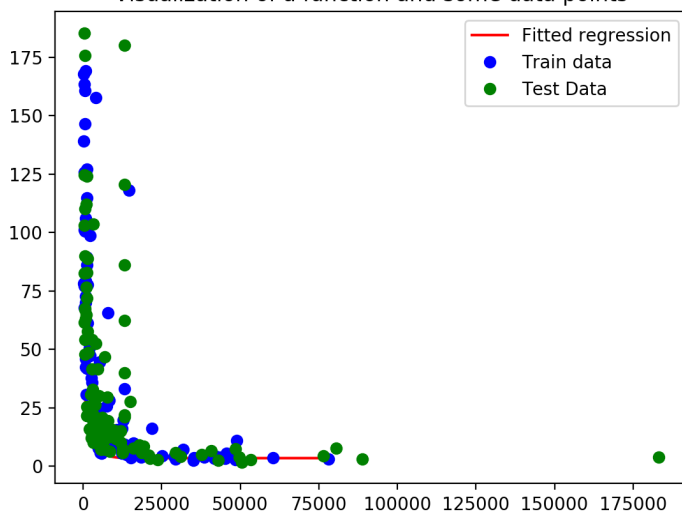
### 5.3

Visualization of a function and some data points



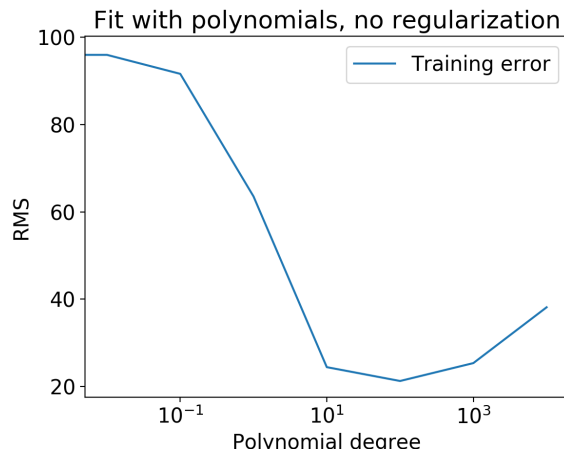
$\mu = 100$   
 Train error = 28.555023448223448  
 Test error = 34.04101841026058

Visualization of a function and some data points



$\mu = 10000$   
 Train error = 38.408782047940605  
 Test error = 39.187056804369604

## 5.4



Lambda = 100 has the smallest error.

0	134.0872480012051,
0.01	89.93358664559737,
0.1	49.182403187023496,
1	54.03122827800846,
10	38.11164190068872,
$10^2$	24.056916620748765,
$10^3$	27.615413925888504,
$10^4$	44.260654394116614

The 0 and  $10^4$  don't show up in graph, but in chart