$$X \sim U(0,1),$$

$$Y = \begin{cases} 1, & X \leq \dfrac{1}{3}, \\ 2, & X > \dfrac{1}{3}. \end{cases}$$

1 $p_Y(y)$

Вероятность события $X \leq \frac{1}{3}$:

$$P\left(X \leq \frac{1}{3}\right) = \int_0^{1/3} f_X(x)\, dx = \int_0^{1/3} 1\, dx = \frac{1}{3}.$$

$$P\left(X > \frac{1}{3}\right) = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$P(Y = 1) = P\left(X \leq \frac{1}{3}\right) = \frac{1}{3},$$

$$P(Y = 2) = P\left(X > \frac{1}{3}\right) = \frac{2}{3}.$$

$$p_Y(y) = \begin{cases} \dfrac{1}{3}, & y = 1, \\ \dfrac{2}{3}, & y = 2, \\ 0, & \text{в др случаях.} \end{cases}$$

2 $E(Y)$

2.1 через $p_Y(y)$

$$E(Y) = \sum_y y \cdot p_Y(y) = 1 \cdot \frac{1}{3} + 2 \cdot \frac{2}{3} = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}.$$

2.2 через $f_X(x)$

$$E(Y) = E[g(X)], \quad g(x) = \begin{cases} 1, & x \leq \dfrac{1}{3}, \\ 2, & x > \dfrac{1}{3}. \end{cases}$$

$$E(Y) = \int_{-\infty}^{+\infty} g(x) f_X(x)\, dx = \int_0^{1/3} 1 \cdot 1\, dx + \int_{1/3}^1 2 \cdot 1\, dx = [x]_0^{1/3} + [2x]_{1/3}^1.$$

$$\int_0^{1/3} 1\, dx = \frac{1}{3},$$

$$\int_{1/3}^{1} 2\,dx = 2 \cdot \left(1 - \frac{1}{3}\right) = 2 \cdot \frac{2}{3} = \frac{4}{3}.$$

$$E(Y) = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}.$$

3 Ответ

1. Распределение y:

$$p_Y(1) = \frac{1}{3}, \quad p_Y(2) = \frac{2}{3}.$$

2. Математ ожидание:

$$E(Y) = \frac{5}{3}.$$