Supplemental Materials

February 14, 2026

This document provides supplemental materials for the manuscript entitled "Distributed Coverage Control on Poriferous Surface via Poly-Annulus Conformal Mapping" by Xun Feng, Chao Zhai, and it is composed of two parts. The first part presents some theoretical results, and it includes some key definitions and the derivation process of the mapping. The second part provides simulation results on the Poriferous Surface and relevant discussions, which includes the analysis of computational cost, quantitative metrics, comparisons with baseline approaches, scenarios of multiple obstacles, scalability of the algorithm and the effect of agent failure.

1 Theoretical Results

1.1 Partial Welding Example

To illustrate the mathematical mechanism of partial welding, we present an intuitive example. Reconstruct the topological connectivity of a slit domain $\mathcal{D} = \mathbb{C} \setminus [-i, i]$ by welding its separated boundaries. Let the slit $[-i, i]$ be regarded as two distinct boundary segments: $\gamma^+ = \{iy + \epsilon \mid y \in [-1, 1]\}$, and $\gamma^- = \{iy - \epsilon \mid y \in [-1, 1]\}$, where $\epsilon \to 0^+$. The welding mapping $T : \mathcal{D} \to \Omega$ is defined by the elementary function $w = T(z) = \sqrt{z^2 + 1}$. This mapping transforms the slit domain $\mathcal{D}$ into the $\Omega = \mathbb{C} \setminus (-\infty, 0]$. Crucially, it enforces the boundary identification condition

$$T(iy^+) = T(iy^-) = \sqrt{1 - y^2}, \quad \forall y \in [-1, 1]. \tag{1}$$

This implies that the disjoint banks γ^+ and γ^- are mapped to the exact same locus in Ω, thereby welding the cut back together. In our proposed framework, the optimization of Θ_{uv} (**??**) serves as the numerical inverse operator of such welding. It explicitly identifies the boundary correspondence

$$h_u(\partial S_u) \cong \Theta_{uv}(h_v(\partial S_v)), \tag{2}$$

which effectively restores the continuous topology of poly-annulus S from the decomposed local disks.

1.2 Geometric Computation of Safe Boundary

This appendix details the numerical generation of the partition bar $\hat{\Gamma}_i$ in Ξ, consisting of linear segments and circular arcs. For intersection identification, the i-th nominal partition bar is parameterized as a vector $\hat{\mathbf{x}}(\hat{\xi}) = \hat{\xi}\hat{\mathbf{b}}_i$, where $\hat{\xi} \in [0,1]$. The k-th obstacle is defined by its center $\hat{o}_k$ and radius $\hat{r}_k$. The intersection points are determined by the Euclidean constraint $\|\hat{\mathbf{x}}(\hat{\xi}) - \hat{o}_k\|^2 = \hat{r}_k^2$, which yields the quadratic equation

$$|\hat{\mathbf{b}}_i|^2\hat{\xi}^2 - 2(\hat{\mathbf{b}}_i \cdot \hat{o}_k)\hat{\xi} + (|\hat{o}_k|^2 - \hat{r}_k^2) = 0. \tag{3}$$

Solving for $\hat{\xi}$ gives the entry and exit parameters $\hat{\xi}_{in} < \hat{\xi}_{out}$, with corresponding coordinates $\hat{p}_{in} = \hat{\xi}_{in}\hat{\mathbf{b}}_i$ and $\hat{p}_{out} = \hat{\xi}_{out}\hat{\mathbf{b}}_i$. To bypass the obstacle, a circular arc is generated on the buffer circle with radius $\hat{r}_{k,i} = \hat{r}_k(1 + i\beta)$. The entry and exit angles relative to $\hat{o}_k$ are computed as $\hat{\psi}_{in} = \text{atan}(\hat{p}_{in} - \hat{o}_k)$ and $\hat{\psi}_{out} = \text{atan}(\hat{p}_{out} - \hat{o}_k)$. The safe arc $\hat{A}_{k,i}$ is then discretized via angular interpolation

$$\hat{\mathbf{x}}_{arc}(u) = \hat{o}_k + \hat{r}_{k,i} \begin{bmatrix} \cos(\hat{\psi}_{in} + u\frac{\Delta\tilde{\hat{\psi}}}{U-1}) \\ sin(\hat{\psi}_{in} + u\frac{\Delta\tilde{\hat{\psi}}}{U-1}) \end{bmatrix}, \tag{4}$$

where $u \in \{0, \ldots, U-1\}$. This ensures $\hat{\Gamma}_i$ is piecewise smooth and strictly disjoint from obstacles.

1.3 Spectral Properties of $\mathcal{L}_\mathcal{G}(\hat{\omega})$

$$[\mathcal{L}_\mathcal{G}]_{ij} = \begin{cases} \hat{\omega}_i + \hat{\omega}_{i+1}, & i = j \\ -\hat{\omega}_{\max(i,j)}, & (i,j) \in \mathcal{E} \\ 0. & \text{otherwise} \end{cases} \tag{5}$$

For the weighted Laplacian $\mathcal{L}_\mathcal{G}(\hat{\omega})$ in (5), under $\hat{\omega}_k \geq \underline{\omega} > 0, \forall k \in \mathcal{N}$, the following properties hold

1. $\mathcal{L}_\mathcal{G} = \mathcal{L}_\mathcal{G}^T$ and $\mathcal{L}_\mathcal{G} \succeq 0$.

2. $\ker(\mathcal{L}_\mathcal{G}) = \text{span}(\mathbf{1}_N)$, implying $\sum_{i=1}^N \dot{\hat{m}}_i = 0$.

3. $\lambda_2(\mathcal{L}_\mathcal{G}) \geq \underline{\omega}\lambda_2(\mathcal{L}_S) = 2\underline{\omega}(1 - \cos(2\pi/N)) > 0$.

Proof. Positive Semi-definiteness: The symmetry follows from the undirected adjacency in $\mathcal{G}_N$. For any $x \in \mathbb{R}^N$, the quadratic form is $x^T \mathcal{L}_{\mathcal{G}} x = \sum_{k=1}^{N} \hat{\omega}_k (x_k - x_{k-1})^2 \geq 0$, where $x_0 \equiv x_N$. Then, $\mathcal{L}_{\mathcal{G}} \succeq 0$. Null Space: $x^T \mathcal{L}_{\mathcal{G}} x = 0 \iff x_k = x_{k-1}$ for all k. Hence, $\ker(\mathcal{L}_{\mathcal{G}}) = \mathrm{span}(\mathbf{1}_N)$. Connectivity: Since $\underline{\omega} > 0$, the cycle graph $\mathcal{G}_N$ is connected. By the Courant-Fischer Theorem [**?**, Th. 4.2.2], the second smallest eigenvalue satisfies $\lambda_2(\mathcal{L}_{\mathcal{G}}) \geq \underline{\omega} \lambda_2(\mathcal{L}_S) > 0$, which establishes the exponential stability on the subspace $\mathbf{1}_N^{\perp}$. □

2 Simulations

2.1 Parameters Design

The centroid tracking gain K^* and agent gain k_p are tuned to ensure that agents navigate poly-annlus S toward their respective centroids, while strictly adhering to the Riemannian metric constraints. The partition gain $k_{\hat{\psi}}$ is determined by the spectral properties of the communication topology to guarantee the exponential decay of workload imbalances across multi-agent system. To maintain topological connectivity in the presence of holes, the sequence factor β is introduced, which ensures that the obstacle-avoiding partition bars remain disjoint and strictly separated from the boundary components in Ξ. Finally, the convergence threshold T_ε is calibrated to define the steady-state precision of the coverage mission, so that the terminal configuration satisfies the quasi-optimal workload distribution.

2.2 Algorithm Analysis

This section evaluates the computational efficiency, scalability and robustbness of the proposed diffeomorphic coverage control framework.

2.2.1 Computational Complexity

The framework decouples offline mapping from online distributed execution to ensure real-time performance.

1. Mapping (Algorithm 1): Solving the Laplace equation for the conformal map τ on S with v vertices across M subdomains incurs a complexity of $O((V \cdot M^{-1})^{\frac{3}{2}})$.

2. Partition (Algorithm 2: The linear intersection checks between N partition bars and n obstacles yield a complexity of $O(n)$ per control cycle.

3. Control (Algorithm 3: Each iteration involves $O(D_{\mathcal{G}})$ communication rounds, where $D_{\mathcal{G}}$ is the graph diameter.

The total online complexity is $O(n + K^* \cdot D_{\mathcal{G}})$. As shown in Table II, the average execution time is $t_{avg} \approx 0.02$s per agent, which supports high-frequency deployment.

2.2.2 Scalability Analysis

The poriferous geometry of S imposes a theoretical limit on the agent population N to maintain topological disjointness and input-to-state stability. Given a sequence factor β, the maximal agent capacity N_{max} is determined by

$$N_{max} \leq \min \left\{ \frac{1}{2} \hat{d}_{min} \cdot (\hat{r}_{max}\beta)^{-1}, \gamma\epsilon_{max} \cdot (C_{\hat{\delta}}\beta)^{-1} \right\}, \tag{6}$$

where $\hat{d}_{min}$ is the minimum obstacle separation, $\hat{r}_{max}$ is the maximum obstacle radius, and ϵ_{max} is the allowable workload error. The first term in (6) ensures the non-overlapping of buffer zones ($\hat{r}_k(1 + N\beta) - \hat{r}_k \leq 0.5\hat{d}_{min}$), while the second term guarantees the ultimate workload error $\|\mathbf{e}(\infty)\| \leq C_{\hat{\delta}}\beta \cdot \gamma^{-1}$ remains within ϵ_{max}, where $C_{\hat{\delta}}$ is the geometric sensitivity defined in Lemma IV.1. This bound defines the fundamental trade-off between coverage precision β and the swarm scale N on poly annulus S.

2.2.3 Robustness Analysis

Consider a failure at $t = T_f$ where a subset of agents $N_{\mathcal{F}}$ ceases operation, resulting in an active set $\mathcal{A}$ with size $N_{\mathcal{A}} = N - N_{\mathcal{F}}$. Then, the new equilibrium workload is $\bar{m}_{\mathcal{A}} = M_{total} \cdot N_{\mathcal{A}}^{-1}$. Define the error vector $\mathbf{e} \in \mathbb{R}^{N_{\mathcal{A}}}$ with components $e_i = \hat{m}_i - \bar{m}_{\mathcal{A}}$, satisfying $\mathbf{e} \perp \mathbf{1}_{N_{\mathcal{A}}}$. Under the reduced topology, the error dynamics in Ξ follow the Riemannian Laplacian flow $\dot{\mathbf{e}} = -k_{\hat{\psi}}\mathcal{L}_{\mathcal{G}}(\hat{\omega})\mathbf{e}$, where $\mathcal{L}_{\mathcal{G}}(\hat{\omega})$ is the weighted Laplacian induced by the metric $\hat{\eta}$. For $V = \|\mathbf{e}\|^2/2$, its derivative satisfies

$$\dot{V} = -k_{\hat{\psi}}\mathbf{e}^T\mathcal{L}_{\mathcal{G}}(\hat{\omega})\mathbf{e} \leq -k_{\hat{\psi}} \cdot \underline{\omega} \cdot \lambda_2(\mathcal{L}_{S,N_{\mathcal{A}}}) \cdot \|\mathbf{e}\|^2, \tag{7}$$

where $\underline{\omega} > 0$ is the minimum marginal density. The exponential convergence rate is $\gamma_{\mathcal{A}} = 2k_{\hat{\psi}} \cdot \underline{\omega} \cdot (1 - \cos(2\pi \cdot N_{\mathcal{A}}^{-1}))$. Since $N_{\mathcal{A}} < N$, the spectral property of the cycle graph ensures $\gamma_{\mathcal{A}} > \gamma_{\mathcal{N}}$. The re-balancing speed automatically increases as the swarm size reduces, ensuring that the error $\|\mathbf{e}(t)\| \leq \|\mathbf{e}(T_0)\| \exp(-\gamma_{\mathcal{A}}(t - T_f))$ swiftly decays. This intrinsic self-healing property allows the remaining agents to reach new Riemannian centroids without manual reconfiguration of the map τ or the Riemannian metric $\hat{\eta}$.

References