431 Class 19

Thomas E. Love

2018-11-06

Today's Agenda

- Comparing Rates/Proportions
- The tabyl function in the janitor package
- Analyzing a 2x2 Cross-Tabulation
- Power and Sample Size When Comparing Proportions
- The In-Class Survey (from Class 18)

```
source("Love-boost.R") # helps to load Hmisc explicitly
library(Hmisc); library(pwr); library(broom); library(Epi)
library(magrittr); library(janitor) # new and "exciting"
library(tidyverse) # always load tidyverse last

dm192 <- read.csv("data/dm192.csv") %>% tbl_df()
class18a <- read.csv("data/class18a.csv") %>% tbl_df
class18b <- read.csv("data/class18b.csv") %>% tbl df
```

Comparing Rates/Proportions

Comparing Two Proportions

Quinnipiac U. poll December 16-20, 2015 of 1,140 registered U.S. voters

- Would you support or oppose a law requiring background checks on people buying guns at gun shows or online?
- Do you personally own a gun or does someone else in your household own a gun?

Reported summaries of that poll get me to the following table:

_	Support Law	Oppose Law	Total
No Gun	542	24	566
Gun Household	440	73	513
Total	982	97	1,079

Links to sources: fivethirtyeight and pollingreport

2 x 2 Table of Guns and Support, Prob. Difference

_	Support	Oppose	Total
No Gun in HH	542	24	566
Gun Household	440	73	513
Total	982	97	1,079

- Of those living in a no gun household, 542/566 = 95.8% support universal background checks.
- \bullet Of those living in a gun household, 440/513 =85.8% support universal background checks.
- So the sample shows a difference of 10 percentage points, or a difference of 0.10 in proportions

Can we build a confidence interval for the population difference in those two proportions?

2 x 2 Table of Guns and Support, Relative Risk

_	Support	Oppose	Total
No Gun in HH	542	24	566
Gun Household	440	73	513
Total	982	97	1,079

- $Pr(support \mid no gun in HH) = 542/566 = 0.958$
- $Pr(support \mid gun in HH) = 440/513 = 0.858$
- ullet The ratio of those two probabilities (risks) is .958/.858=1.12

Can we build a confidence interval for the relative risk of support in the population given no gun as compared to gun?

2 x 2 Table of Guns and Support, Odds Ratio

_	Support	Oppose	Total
No Gun in HH	542	24	566
Gun Household	440	73	513
Total	982	97	1,079

- Odds = Probability / (1 Probability)
- Odds of Support if No Gun in HH $= \frac{542/566}{1-(542/566)} = 22.583333$
- Odds of Support if Gun in HH = $\frac{440/513}{1-(440/513)}$ = 6.027397
- Ratio of these two Odds are 3.75

In a 2x2 table, odds ratio = cross-product ratio.

• Here, the cross-product estimate = $\frac{542*73}{440*24} = 3.75$

Can we build a confidence interval for the odds ratio for support in the population given no gun as compared to gun?

2x2 Table Results in R

This twobytwo function is part of the Love-boost. R script we sourced in earlier. Without that, this will throw an error message.

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Full Output

```
2 by 2 table analysis:
```

Outcome : Support

Comparing : No Gun in HH vs. Gun Household

```
Support Oppose P(Support) 95% conf. int.
No Gun in HH 542 24 0.9576 0.9375 0.9714
Gun Household 440 73 0.8577 0.8247 0.8853
```

Exact P-value: 0 Asymptotic P-value: 0

Bayesian Augmentation in a 2x2 Table?

Original command:

Bayesian augmentation approach (add a success and add a failure in each row):

Full Output with Bayesian augmentation

```
2 by 2 table analysis:
```

Outcome : Support

Comparing : No Gun in HH vs. Gun Household

```
Support Oppose P(Support) 95% conf. int.
No Gun in HH 543 25 0.9560 0.9357 0.9701
Gun Household 441 74 0.8563 0.8233 0.8840
```

```
95% conf. interval
Relative Risk: 1.1164 1.0731 1.1614
Sample Odds Ratio: 3.6446 2.2768 5.8342
Conditional MLE Odds Ratio: 3.6405 2.2413 6.0875
Probability difference: 0.0997 0.0655 0.1355
```

Exact P-value: 0 Asymptotic P-value: 0

Using a data frame, rather than a 2x2 table

For example, in the dm192 data, suppose we want to know whether statin prescriptions are more common among male patients than female patients. So, we want a two-way table with "Male", "Statin" in the top left.

```
dm192 %$% table(sex, statin)
```

```
statin
sex 0 1
female 24 74
male 21 73
```

So we want male in the top row and statin yes in the left column...

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female

```
dm192 < -dm192 \%
 mutate(sex f = fct relevel(sex, "male"),
         statin f = fct recode(factor(statin),
                        on statin = "1", no statin = "0"),
         statin f = fct relevel(statin f, "on statin"))
dm192 %$% table(sex f, statin f)
        statin f
sex f on statin no statin
 male
               73
                         21
```

24

74

"Adorning" the taby1

```
dm192 %>% tabyl(sex_f, statin_f) %>%
  adorn_totals() %>%
  adorn_percentages("row") %>%
  adorn_pet_formatting(digits = 1) %>%
  adorn_ns(position = "front") %>%
  adorn_title(row = "Sex", col = "Statin Status") %>%
  knitr::kable(align = "rr", caption = "dm192 statin by sex")
```

Table 5: dm192 statin by sex

	Statin Status	
Sex	on_statin	no_statin
male	73 (77.7%)	21 (22.3%)
female	74 (75.5%)	24 (24.5%)
Total	147 (76.6%)	45 (23.4%)

Running twoby2 against a data set

The twoby2 function from the Epi package can operate with tables (but not, alas, tabyls) generated from data.

```
twoby2(dm192 %$% table(sex_f, statin_f))
```

(edited output on next slide)

With Bayesian Augmentation

```
twoby2(dm192 %$% table(sex_f, statin_f) + 1)
```

(edited output on the slide after that)

twoby2 output on Raw Data (No Augmentation)

```
2 by 2 table analysis:
Outcome: on statin
                       Comparing: male vs. female
     on statin no statin P(on statin) 95% conf. interval
male
          73 21 0.7766 0.6815 0.8496
female
       74 24 0.7551 0.6605 0.8301
                              95% conf. interval
           Relative Risk: 1.0285
                                 0.8795 1.2026
                             0.5775 2.2010
       Sample Odds Ratio: 1.1274
Conditional MLE Odds Ratio: 1.1267 0.5473 2.3330
   Probability difference: 0.0215 -0.0985 0.1399
P-values: Exact: 0.7368 Asymptotic: 0.7253
```

twoby2 WITH Bayesian Augmentation

```
2 by 2 table analysis:
Outcome: on statin
                       Comparing: male vs. female
      on statin no statin P(on statin) 95% conf. interval
                         0.7708 0.6764 0.8441
male
          74 22
female
         75 25 0.7500 0.6561 0.8251
                              95% conf. interval
           Relative Risk: 1.0278
                                0.8783
                                        1,2027
       Sample Odds Ratio: 1.1212 0.5814 2.1624
Conditional MLE Odds Ratio: 1.1206 0.5520 2.2869
   Probability difference: 0.0208 -0.0988 0.1389
P-values: Exact: 0.7414 Asymptotic: 0.7328
```

Power and Sample Size When Comparing Proportions

Relation of α and β to Error Types

Recall the meanings of α and β :

- α is the probability of rejecting H_0 when H_0 is true.
 - So 1α , the confidence level, is the probability of retaining H₀ when that's the right thing to do.
- β is the probability of retaining H_0 when H_A is true.
 - So 1β , the power, is the probability of rejecting H_0 when that's the right thing to do.

_	H _A is True	H ₀ is True
Test Rejects H ₀ Test Retains H ₀	Correct Decision $(1 - \beta)$ Type II Error (β)	Type I Error (α) Correct Decision $(1 - \alpha)$

Tuberculosis Prevalence Among IV Drug Users

Here, we investigate factors affecting tuberculosis prevalence among intravenous drug users.

Among 97 individuals who admit to sharing needles, 24 (24.7%) had a positive tuberculin skin test result; among 161 drug users who deny sharing needles, 28 (17.4%) had a positive test result.

What does the 2x2 table look like?

Tuberculosis Prevalence Among IV Drug Users

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The 2x2 Table is...

- rows describe needle sharing, columns describe TB test result
- row 1 people who share needles: 24 TB+, and 97-24 = 73 TB-
- \bullet row 2 people who don't share: 28 TB+ and 161-28 = 133 TB-

twobytwo (with Bayesian Augmentation)

To start, we'll test the null hypothesis that the population proportions of intravenous drug users who have a positive tuberculin skin test result are identical for those who share needles and those who do not.

```
H_0: \pi_{share} = \pi_{donotshare}
H_A: \pi_{share} \neq \pi_{donotshare}
```

We'll use the Bayesian augmentation.

Two-by-Two Table Result

Outcome : TB test+

Comparing : Sharing vs. Not Sharing

```
TB test+ TB test- P(TB test+) 95% conf. int. Sharing 25 74 0.2525 0.1767 0.3471 Not Sharing 29 134 0.1779 0.1265 0.2443
```

95% conf. interval Relative Risk: 1.4194 0.8844 2.2779

Sample Odds Ratio: 1.5610 0.8520 2.8603

Conditional MLE Odds Ratio: 1.5582 0.8105 2.9844 Probability difference: 0.0746 -0.0254 0.1814

Exact P-value: 0.1588 Asymptotic P-value: 0.1495

What conclusions should we draw?

Designing a New TB Study

PI:

- OK. That's a nice pilot.
- We saw $p_{nonshare} = 0.18$ and $p_{share} = 0.25$ after your augmentation.
- Help me design a new study.
 - This time, let's have as many needle-sharers as non-sharers.
 - We should have 90% power to detect a difference as large as what we saw in the pilot, or larger, so a difference of 7 percentage points.
 - We'll use a two-sided test, and $\alpha = 0.05$, of course.

What sample size would be required to accomplish these aims?

How power.prop.test works

power.prop.test works much like the power.t.test we saw for means.

Again, we specify 4 of the following 5 elements of the comparison, and R calculates the fifth.

- The sample size (interpreted as the # in each group, so half the total sample size)
- The true probability in group 1
- The true probability in group 2
- The significance level (α)
- The power (1β)

The big weakness with the power.prop.test tool is that it doesn't allow you to work with unbalanced designs.

Using power.prop.test for Balanced Designs

To find the sample size for a two-sample comparison of proportions using a balanced design:

- we will use a two-sided test, with $\alpha = .05$, and power = .90,
- we estimate that non-sharers have probability .18 of positive tests,
- and we will try to detect a difference between this group and the needle sharers, who we estimate will have a probability of .25

R Command to find the required sample size

Two-sample comparison of proportions power calculation n=721.7534 p1=0.18, p2=0.25 sig.level = 0.05, power = 0.9, alternative = two.sided NOTE: n is number in *each* group

So, we'd need at least 722 non-sharing subjects, and 722 more who share needles to accomplish the aims of the study, or a total of 1444 subjects.

Suppose we can get 400 sharing and 400 non-sharing subjects. How much power would we have to detect a difference in the proportion of positive skin test results between the two groups that was identical to the data above or larger, using a *one-sided* test, with $\alpha=.10$?

```
Two-sample comparison of proportions power calculation n=400, p1=0.18, p2=0.25 sig.level = 0.1, power = 0.8712338 alternative = one.sided NOTE: n is number in *each* group
```

We would have just over 87% power to detect such an effect.

Using the pwr package to assess sample size for Unbalanced Designs

The pwr.2p2n.test function in the pwr package can help assess the power of a test to determine a particular effect size using an unbalanced design, where n_1 is not equal to n_2 .

As before, we specify four of the following five elements of the comparison, and R calculates the fifth.

- n1 = The sample size in group 1
- n2 = The sample size in group 2
- sig.level = The significance level (α)
- power = The power (1β)
- h =the effect size h, which can be calculated separately in R based on the two proportions being compared: p_1 and p_2 .

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Calculating the Effect Size h

To calculate the effect size for a given set of proportions, use ES.h(p1, p2) which is available in the pwr package.

For instance, in our comparison, we have the following effect size.

$$ES.h(p1 = .18, p2 = .25)$$

[1] -0.1708995

Using pwr.2p2n.test in R

Suppose we can have 700 samples in group 1 (the not sharing group) but only 400 in group 2 (the group of users who share needles).

How much power would we have to detect this same difference (p1 = .18, p2 = .25) with a 5% significance level in a two-sided test?

R Command to find the resulting power

```
pwr.2p2n.test(h = ES.h(p1 = .18, p2 = .25),

n1 = 700, n2 = 400, sig.level = 0.05)
```

difference of proportion power calculation for binomial distribution (arcsine transformation)

```
h = 0.1708995, n1 = 700, n2 = 400
sig.level = 0.05, power = 0.7783562
alternative = two.sided
NOTE: different sample sizes
```

We will have about 78% power under these circumstances.

Comparison to Balanced Design

How does this compare to the results with a balanced design using 1100 drug users in total, i.e. with 550 patients in each group?

which yields a power estimate of 0.809. Or we could instead have used...

which yields an estimated power of 0.808.

Each approach uses approximations, and slightly different ones, so it's not surprising that the answers are similar, but not identical.

Exploring the In-Class Survey from Class 18

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In-Class Survey

We chose (using a computer) a random number between 0 and 100.

Your number is X = 10 (or 65).

- Do you think the percentage of countries which are in Africa, among all those in the United Nations, is higher or lower than X?
- Question of the percentage of countries which are in Africa, among all those in the United Nations.

The facts

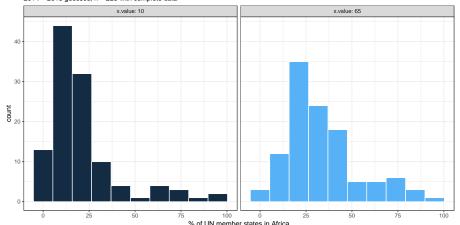
- There are 193 sovereign states that are members of the UN.
- The African regional group has 54 member states, so that's 28%.
- UN regions for countries found at this Wikipedia link.

A troubling situation

We chose (using a computer) a random number	between o and 100. Yo	ur number is $X = 65$.	
Do you think the percentage of countries white United Nations, is higher or lower than X? Circle your answer: HIGHER than X	ch are in Africa, among LOWER than X	all those in the	
2. Give your best estimate of the percentage of those in the United Nations.	countries which are in		
	My Answer:	20 percen	it.

class18a Africa percentage guess by X = 10 or 65

% of UN in Africa Guess, by Prompting X value 2014 – 2018 guesses, n = 226 with complete data

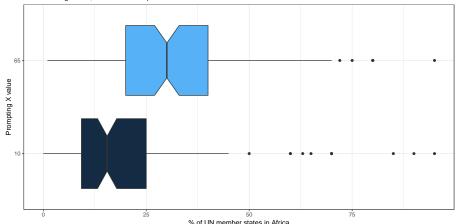


class18a Analysis, Step-by-Step

- What is the outcome under study?
- What are the (in this case, two) treatment/exposure groups?
- Were the data collected using matched / paired samples or independent samples?
- Are the data a random sample from the population(s) of interest? Or is there at least a reasonable argument for generalizing from the sample to the population(s)?
- What is the significance level (or, the confidence level) we require here?
- Are we doing one-sided or two-sided testing/confidence interval generation?
- If we have paired samples, did pairing help reduce nuisance variation?
- If we have paired samples, what does the distribution of sample paired differences tell us about which inferential procedure to use?
- If we have independent samples, what does the distribution of each individual sample tell us about which inferential procedure to use?

class18a Africa percentage guess by X = 10 or 65

% of UN in Africa Guess, by Prompting X value 2014 – 2018 guesses, n = 226 with complete data



10 114 22 18.8 15.5 65 112 33.6 19.0 30

```
class18a %>%
 filter(complete.cases(africa.pct)) %>%
 group_by(x.value) %>%
 summarise(n = n(),
            mean = round(mean(africa.pct),2),
            sd = round(sd(africa.pct),2),
            median = median(africa.pct))
# A tibble: 2 \times 5
 x.value
                     sd median
             n mean
    <int> <int> <dbl> <dbl> <dbl>
```

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class18a comparisons (results: next slide)

class18a Comparing Two Populations

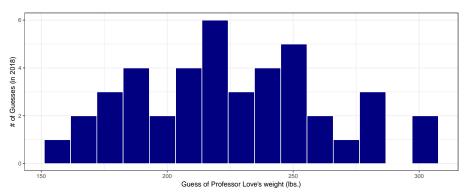
$$\Delta = \mu_{65} - \mu_{10}$$

Procedure	Est. Δ	95% CI for Δ	р
Welch t	11.6	(6.7, 16.6)	6.5e-06
Pooled t	11.6	(6.7, 16.6)	6.5e-06
Rank Sum	12.0	(8.0, 15.0)	2.9e-09
Bootstrap	11.6	(6.5, 16.4)	< .05

Conclusions?

In-Class Survey (class18b data)

Provide a point estimate for Dr. Love's current weight (in pounds.)



- 2018 Weight Guesses: n = 42, $\bar{x} = 225.6$ lbs., s = 36.6 lbs.
- Five Number Summary: 154 200 220 250 300

Now estimate one interval, which you believe has a 50% chance of including Dr. Love's current weight (again, in pounds.) Then do the same for a 90% interval.

We have n=42 independent guesses, with $\bar{x}=225.6$ lbs., s=36.6 lbs. Let's first obtain quantiles, and use the crowd's wisdom.

```
quantile(class18b$love.lbs,
  probs = c(0.05, 0.10, 0.25, 0.50, 0.75, 0.90, 0.95))
```

5% 10% 25% 50% 75% 90% 95% 170.25 177.30 200.00 220.00 250.00 279.00 280.00

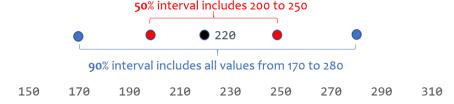
- What's a rational 50% interval for estimating my weight?
- How about a 90% interval?

One Possible, Rational, Set of Intervals

Suppose my estimate is 220 pounds.

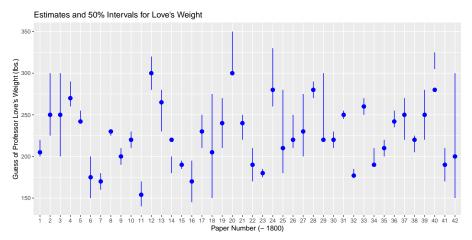
- Then suppose I assign probability 0.50 to the interval (200, 250)
- And suppose I assign probability 0.90 to the interval (170, 280)

Estimated Intervals from Distribution of 2018 Weight Guesses (in lbs.)



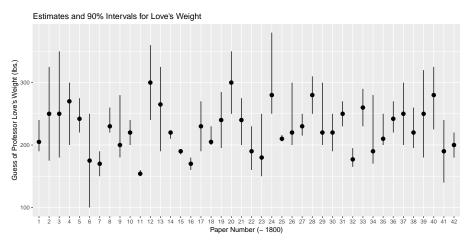
In-Class Survey (class18b data)

4a. Now estimate one interval, which you believe has a 50% chance of including Dr. Love's current weight (again, in pounds.)



In-Class Survey (class18b data)

4b. Now do the same, but for a 90% interval...



3. Provide a point estimate for Dr. Love's current weight (in pounds.) If you think in kilograms, multiply kg by 2.2 to get pounds.
My Answer: 260 pounds.
4. Now estimate one interval, which you believe has a 50% chance of including Dr. Love's current weight (again, in pounds.) Then do the same for a 90% interval.
50% interval: Dr. Love's weight is (150, 300) pounds.
90% interval: Dr. Love's weight is (220, 280) pounds.

• Why does this set of intervals not make sense?

3. Provide a point estimate for Dr. Love's current weight (in pounds.) If you think in kilograms, multiply kg by 2.2 to get pounds.	
My Answer: 260 pounds.	
4. Now estimate one interval, which you believe has a 50% chance of including Dr. Love's current weight (again, in pounds.) Then do the same for a 90% interval.	
50% interval: Dr. Love's weight is (150, 300) pounds.	
90% interval: Dr. Love's weight is (220, 280) pounds.	

- Why does this set of intervals not make sense?
- There were **8** students (out of 44) who had a wider 50% interval than 90% interval.

ght (in pounds.) If	you think in	
	~	
100,) pounds.	
200,	260) pounds.	
	My Answer:	my Answer: 240 pounds. My Answer: 240 pounds. 50% chance of including Dr. Love's or a 90% interval. [00] [60] pounds. 260] pounds.

• It wasn't clear enough that the interval estimate was meant to surround the point estimate.

nt (in pounds.) If	you think in
	240 pounds.
	cluding Dr. Love's
100,	160) pounds.
200,_	260) pounds.
	0% chance of in

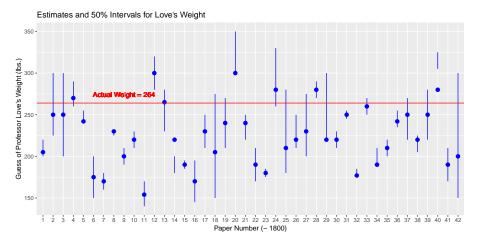
- It wasn't clear enough that the interval estimate was meant to surround the point estimate.
- There were **2** students out of 42 with this problem in their 50% interval, **0** in their 90% interval.

3. Provide a point estimate for Dr. Love's current wei	ght (in pounds.) If you think in
kilograms, multiply kg by 2.2 to get pounds.	My Answer: 240 pounds.
4. Now estimate one interval, which you believe has a	
current weight (again, in pounds.) Then do the same t	
50% interval: Dr. Love's weight is ([00, [60) pounds.
90% interval: Dr. Love's weight is (_	200, 260) pounds.
50% interval: Dr. Love's weight is (_	(00), (60) pounds.

- It wasn't clear enough that the interval estimate was meant to surround the point estimate.
- There were 2 students out of 42 with this problem in their 50% interval, 0 in their 90% interval.
- For **8** students, the 90% interval did not contain the 50% interval.

The facts (with 50% intervals)

On 2018-11-01, Dr. Love actually weighed 264 lbs. or 119.7 kg or 18 stone and 12 pounds, dressed but without shoes.

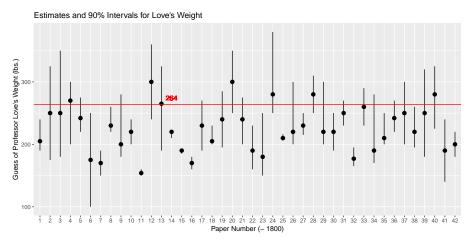


ullet 14 of the 42 50% intervals estimated by students included 264 lbs.

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The facts (with 90% intervals)

On 2018-11-01, Dr. Love actually weighed 264 lbs.



• 22 of the 42 90% intervals (or 52.3% of them) included 264 lbs.

Coming Up ...

- Larger Cross-Tabulations
- Three-Way Tables
- Comparing Three or More Population Means with ANOVA

Please don't forget to do the Minute Paper after Class 19.