Задача 10

$H_0: \xi \sim p_0(x) = \begin{cases} 1, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$ $\qquad$ $H_1: \xi \sim p_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$

а) $n=1$ α

$l = \frac{p_1}{p_0} = \frac{e}{e-1} e^{-x} \geq C$ $\qquad$ $e^{-x} \geq B$

$G: x \leq A$

$P(x \leq A \mid H_0) = \alpha$

$\int_0^A 1 dx = \alpha \qquad A = \alpha$

$G: x \leq \alpha$

$\alpha_1 = P(H_1 \mid H_0) = \alpha$

$W = P(x \leq A \mid H_1) = \int_0^\alpha \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1} e^{-x} \Big|_\alpha^0 = \frac{e}{e-1}(1 - e^{-\alpha})$

$\alpha_2 = 1 - W = 1 - \frac{e}{e-1}(1 - e^{-\alpha}) = \frac{e}{e-1}\left(\frac{e-1}{e} - 1 + e^{-\alpha}\right) =$

$= \frac{e}{e-1}\left(\frac{e^{1-\alpha} - 1}{e}\right) = \frac{e^{1-\alpha} - 1}{e-1}$

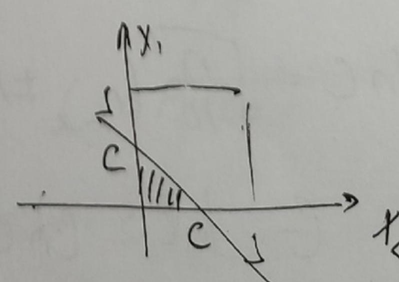
б) $n=2$

$l = \frac{L_1}{L_0} = \frac{\left(\frac{e}{e-1}\right)^2 e^{-x_1} e^{-x_2}}{1 \cdot 1} \geq C$ $\qquad$ $e^{-x_1 - x_2} \geq B$

$G: x_1 + x_2 \leq C$

$\alpha_1 = P(X \leq C \mid H_0) = \alpha$

$P(x_1 + x_2 \leq C \mid H_0) = \frac{C^2}{2} = \alpha$



$C = \sqrt{2\alpha}$ $\qquad$ $G: x_1 + x_2 \leq \sqrt{2\alpha}$

$W = P(x_1 + x_2 \le A \mid H_1) = \int_0^A dx_1 \int_0^{A-x_1} \left(\frac{e}{e-1}\right)^2 e^{-x_1-x_2} dx_2 = \left(\frac{e}{e-1}\right)^2 \left(1 - e^{-\sqrt{2\alpha}} - e^{-\sqrt{2\alpha}}\sqrt{2\alpha}\right)$

$\cancel{\alpha_2 = 1 - W = 1 - \left(\frac{e}{e-1}\right)^2 \left(1 - e^{-\sqrt{2\alpha}} - e^{-\sqrt{2\alpha}}\sqrt{2\alpha}\right)}$

$= \left(\frac{e}{e-1}\right)^2 e^{-A}\left(-A - 1 + e^{A}\right) = \left(\frac{e}{e-1}\right)^2 \left(1 - (\sqrt{2\alpha}+1) e^{-\sqrt{2\alpha}}\right)$

$\alpha_2 = 1 - W$

c) $H_0: p(x) = 1 \ (0,1)$

$H_1: p(x) = \frac{e}{e-1} e^{-x} \ (0,1)$

$l = \frac{l_1}{l_0} = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \ge c$

$\alpha_1 = P(l \ge c \mid H_0) = \alpha$

$\ln l = \sum_{i=1}^{n} \ln \frac{p_1(x_i)}{p_0(x_i)} = \sum \eta_i$

$\frac{p_1(x_i)}{p_0(x_i)}$; $p_0(x_i) = 1$

Т. Фишера

$\frac{\sum \eta_i - nM[\eta_i]}{\sqrt{nD[\eta_i]}} \sim N(0,1)$

H_0: $M[\eta_i] = M\left[\ln \frac{e}{e-1} e^{-x_i}\right] = M\left[\ln\frac{e}{e-1} - x_i\right] = \ln\frac{e}{e-1} - \frac{1}{2}$

$D[\eta_i] = D\left[\ln\frac{e}{e-1} e^{-x_i}\right] = D\left[\ln\frac{e}{e-1} - x_i\right] = D[x_i] = \frac{1}{12}$

$\alpha_1 = P(\ln l \ge \ln c \mid H_0) = P\left(\frac{\ln l - n\left(\ln\frac{e}{e-1} - \frac{1}{2}\right)}{\sqrt{n \cdot \frac{1}{12}}} \ge \frac{\ln c - n\left(\ln\frac{e}{e-1} - \frac{1}{2}\right)}{\sqrt{n/12}}\right) = \alpha$

$A = u_{1-\alpha}$

$\ln c = \sqrt{\frac{n}{12}}\, u_{1-\alpha} + n \ln\frac{e}{e-1} - \frac{n}{2}$

$G: \ln l \ge \ln c$

$$\ln \ell = \sum_{i=1}^{n} l_i = \sum_{i=1}^{n} \ln \frac{e}{e-1} e^{-x_i} = \sum_{i=1}^{n}\left(\ln \frac{e}{e-1} - x_i\right) = n\ln\frac{e}{e-1} - n\bar{x} \geq \sqrt{n\frac{1}{12}}\,u_{1-\alpha} + n\ln\frac{e}{e-1} + -\frac{n}{2}$$

$$G: \bar{x} \leq \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}$$

$\alpha_1 = \alpha$

$$W = P\left(\bar{x} \leq \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}} \,\Big|\, H_1\right) \qquad \frac{\bar{x} - M[\xi]}{\sqrt{D\xi}}\sqrt{n} \sim N(0,1)$$

$$H_1: M[\xi] = \int_0^1 \frac{e}{e-1} x e^{-x}\,dx = \frac{e-2}{e-1}$$

$$M\xi^2 = \int_0^1 \frac{e}{e-1} x^2 e^{-x}\,dx = \frac{2e-5}{e-1}$$

$$D\xi = \frac{e^2-3e+1}{(e-1)^2}$$

$$W = P\left(\frac{\bar{y}-\alpha_1}{\sqrt{\mu_2}}\sqrt{n} \leq \overbrace{\frac{\left(\frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}\right) - \alpha_1}{\sqrt{\mu_2}}\sqrt{n}}^{B}\right) = \int_{-\infty}^{B} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\,dx$$

$\alpha_2 = 1 - W$

d) $\alpha_1 = P(X_{min} < c \mid H_0) = \alpha = F_{min}(c)$

$$F_{min} = 1 - (1-F(x))^n$$

$$\alpha = 1 - (1 - F_0(c))^n \qquad F_0(c) = 1 - \sqrt[n]{1-\alpha}$$

$$c = 1 - \sqrt[n]{1-\alpha}$$

$$G: X_{min} < \left(1 - \sqrt[n]{1-\alpha}\right)$$

$$W = P(X_{min} < c \mid H_1) = F_{min}(c) = 1 - (1 - F_1(c))^n =$$

$$= 1 - \left(1 - \frac{e}{e-1}\left(1 - e^{\sqrt[n]{1-\alpha}-1}\right)\right)^n = 1 - \left(1 - \frac{e}{e-1} + \frac{e^{\sqrt[n]{1-\alpha}}}{e-1}\right)^n$$

$H_0:\ q \sim p(x) = \frac{1}{4}\delta(x-1) + \frac{1}{4}\delta(x-2) + \frac{1}{6}\delta(x-3) + \frac{1}{3}\delta(x-4)$

$\alpha_1 = \alpha$

$$\alpha_2 = 1 - \omega\left(\frac{e^{\sqrt[n]{1-\alpha}}}{e} - \frac{1}{e-1}\right)^n$$