# Calculus - 杂项

硝基苯

# 反三角函数

$$(rcsin x)' = rac{1}{\sqrt{1-x^2}}$$
 $(rccos x)' = -rac{1}{\sqrt{1-x^2}}$ 
 $(rctan x)' = rac{1}{1+x^2}$ 
 $(rccot x)' = -rac{1}{1+x^2}$ 

# 反函数

$$x=f(y)$$
 单调,可导,且  $f'(y) 
eq 0$ 

$$[f^{-1}(x)]' = rac{1}{f'(y)}$$

#### 高阶导数

$$(e^x)^{(n)} = e^x$$
 $(\sin x)^{(n)} = \sin(x + n \cdot \frac{\pi}{2})$ 
 $(\cos x)^{(n)} = \cos(x + n \cdot \frac{\pi}{2})$ 
 $(\ln x)^{(n)} = (-1)^{n-1} \frac{(n-1)!}{x^n}$ 
 $(x^\mu)^{(n)} = \mu(\mu - 1)(\mu - 2) \cdots (\mu - n + 1)x^{\mu - n}$ 

$$egin{split} rac{d^n}{dx^n}f(ax+b) &= a^n \cdot rac{d^n}{d(ax+b)^n}f(ax+b) \ &(u\pm v)^{(n)} &= u^{(n)}\pm v^{(n)} \ &(Cu)^{(n)} &= Cu^{(n)} \ &(uv)^{(n)} &= \sum_{k=0}^n C_n^k u^{(n-k)}v^{(k)} \end{split}$$

### 隐函数

方程两边同时对 x 求导,解出  $\frac{dy}{dx}$ 

对数求导法

$$y = u^v$$

$$y'=u^v(vlnu)'=u^v(v'\ln u+vrac{u'}{u})$$

多因式相乘除 (考虑取绝对值)

### 曲线积分

$$ds = \sqrt{1 + {y'}^2} dx$$
 $K = rac{|y''|}{(1 + {y'}^2)^{3/2}}$ 

# 不定积分公式

$$(rcsin e^x)' = (e^{-2x} - 1)^{-1/2} \ \int rctan x dx = x rctan x - rac{1}{2} \ln(1 + x^2) \ \int rac{dx}{(x^2 + a^2)^{n+1}} = rac{1}{2na^2} rac{x}{(x^2 + a^2)^n} + rac{2n-1}{2na^2} \int rac{dx}{(x^2 + a^2)^n}$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + C$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln |x + \sqrt{x^2 \pm a^2}| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

### 定积分公式

$$\int_0^{rac{\pi}{2}} \sin^n x \, dx = \int_0^{rac{\pi}{2}} \cos^n x \, dx = egin{cases} rac{n-1}{n} \cdot rac{n-3}{n-2} \cdots rac{3}{4} \cdot rac{1}{2} \cdot rac{\pi}{2}, & n ext{ is even} \ rac{n-1}{n} \cdot rac{n-3}{n-2} \cdots rac{4}{5} \cdot rac{2}{3} \cdot 1, & n ext{ is odd} \end{cases}$$

#### 万能公式

$$u= anrac{x}{2}$$
  $\sin x=rac{2u}{1+u^2}$   $\cos x=rac{1-u^2}{1+u^2}$   $dx=rac{2}{1+u^2}du$