HW3

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1.1

```
source("test-data.R")
library("leaps")
truncated.power.design.matrix <- function(x) {
  result <- matrix(0,nrow=length(x),ncol=length(x))
  for (i in 1:length(x)) {
    for (j in 1:length(x)) {
      result[i,j] <- max(0,x[i]-x[j])
    }
  }
  for (j in 1:length(x)) {
    result[j,length(x)] <- 1
  }
  return(result)
}
```

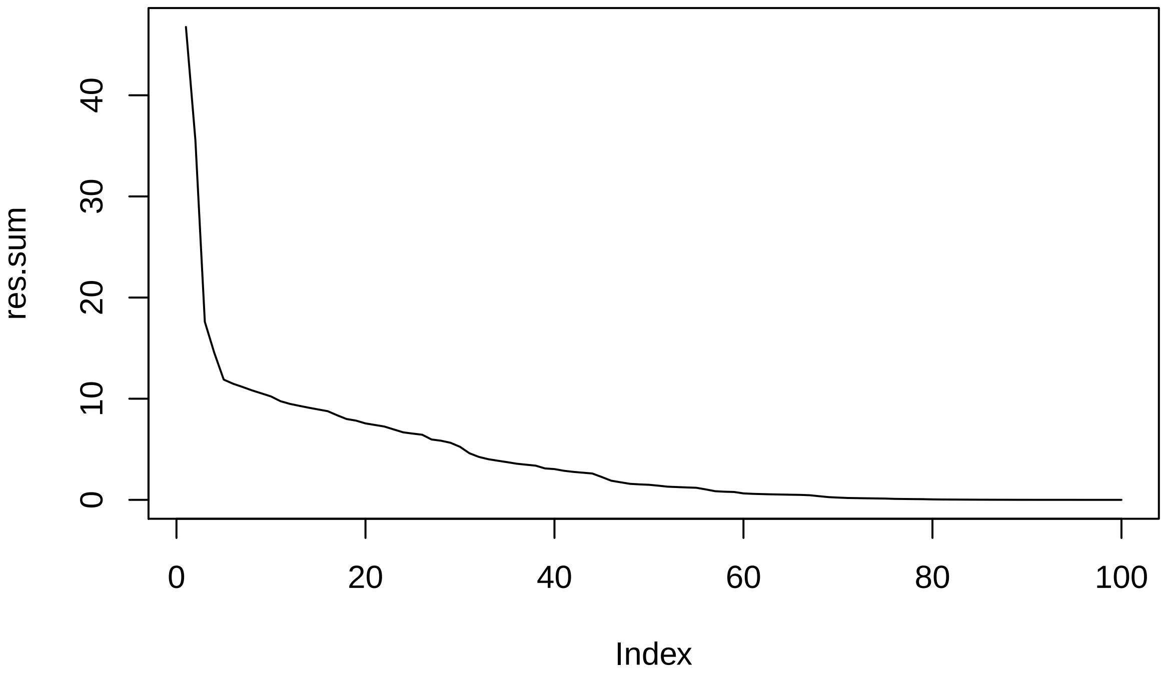
1.2

```
regsubsets.fitted.values <- function(X, regsubsets.out, nterm) {
  out.summary <- summary(regsubsets.out)
  vorder <- regsubsets.out$vorder[1:nterm][order(regsubsets.out$vorder[1:nterm])]
  coeff <- coef(regsubsets.out,nterm)
  coefficient <- rep(0,100)
  for (i in 1:nterm) {
    coefficient[vorder[i]] <- coeff[i]
  }
  return(X%*%coefficient)
}
```

1.3

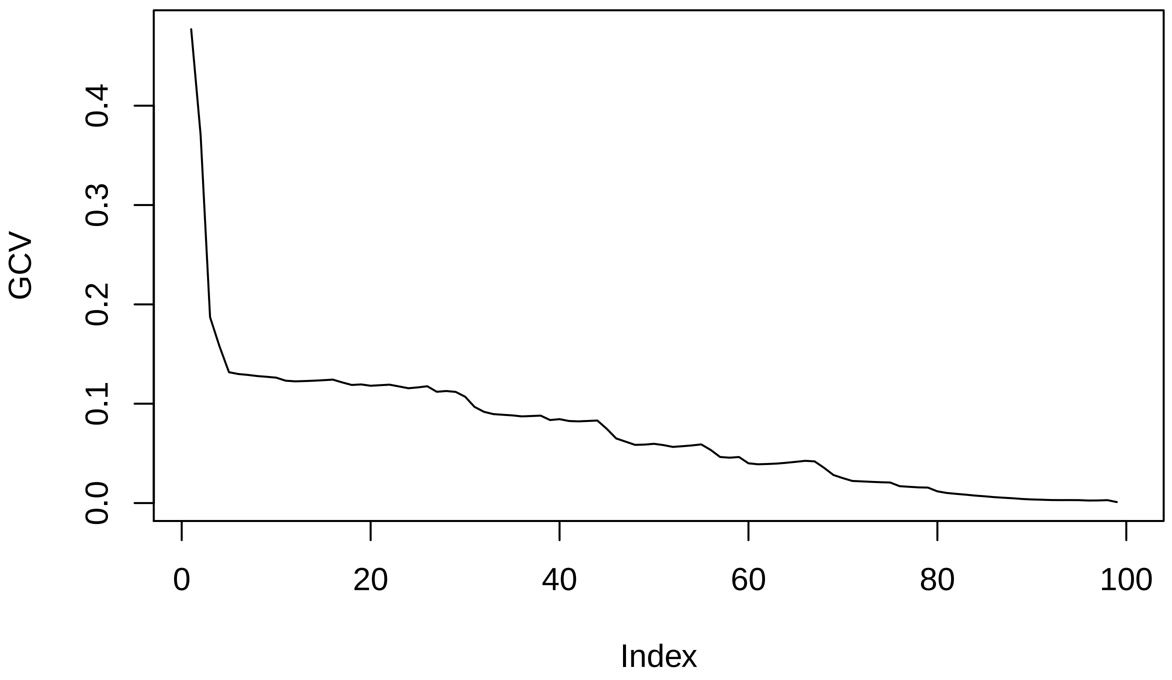
```
truncated <- truncated.power.design.matrix(x)
regsubsets.out <- regsubsets(truncated,y,method="forward",nvmax=100,intercept = FALSE)
out.summary <- summary(regsubsets.out)
res.sum <- rep(0,100)
for(k in 1:100) {
  fit.values <- regsubsets.fitted.values(truncated,regsubsets.out,k)
```

```r
  res.sum[k] <- sum((y-fit.values)^2)
}
plot(res.sum,type='l')
```



1.4

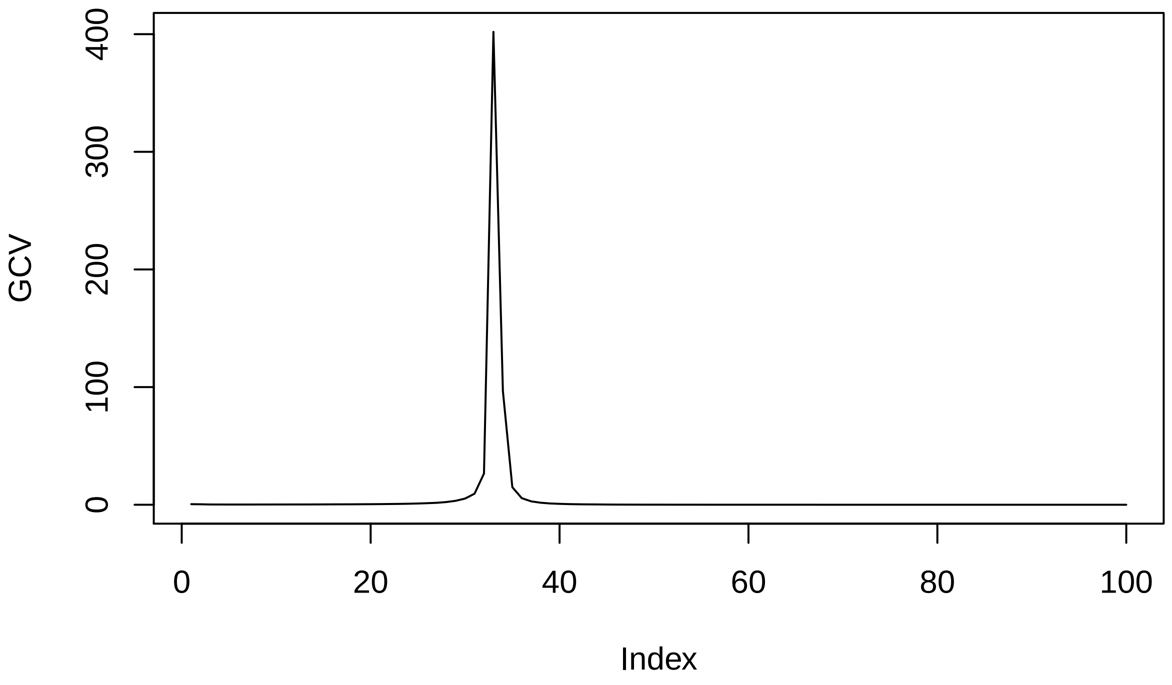
```r
GCV <- rep(0,100)
for(i in 1:100) {
  fit.values <- regsubsets.fitted.values(truncated,regsubsets.out,i)
  GCV[i] <- mean((y-fit.values)^2/(1-i/100)^2)
}
plot(GCV,type='l')
```



Yes, because GCV score is high with small k, because GCV is basically residual sum, $1 - \frac{k}{n}$ is decreasing

1.5

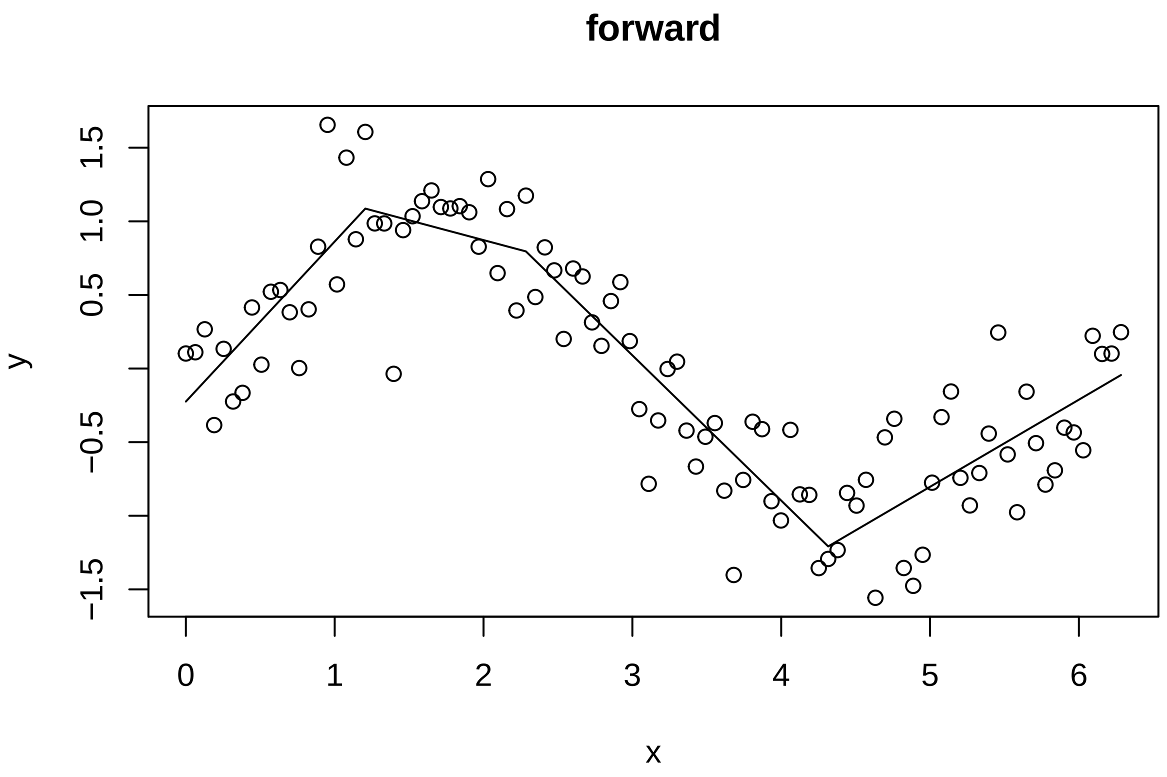
```r
GCV <- rep(0,100)
for(i in 1:100) {
  fit.values <- regsubsets.fitted.values(truncated,regsubsets.out,i)
  GCV[i] <- mean((y-fit.values)^2/(1-3*i/100)^2)
}
plot(GCV,type='l')
```



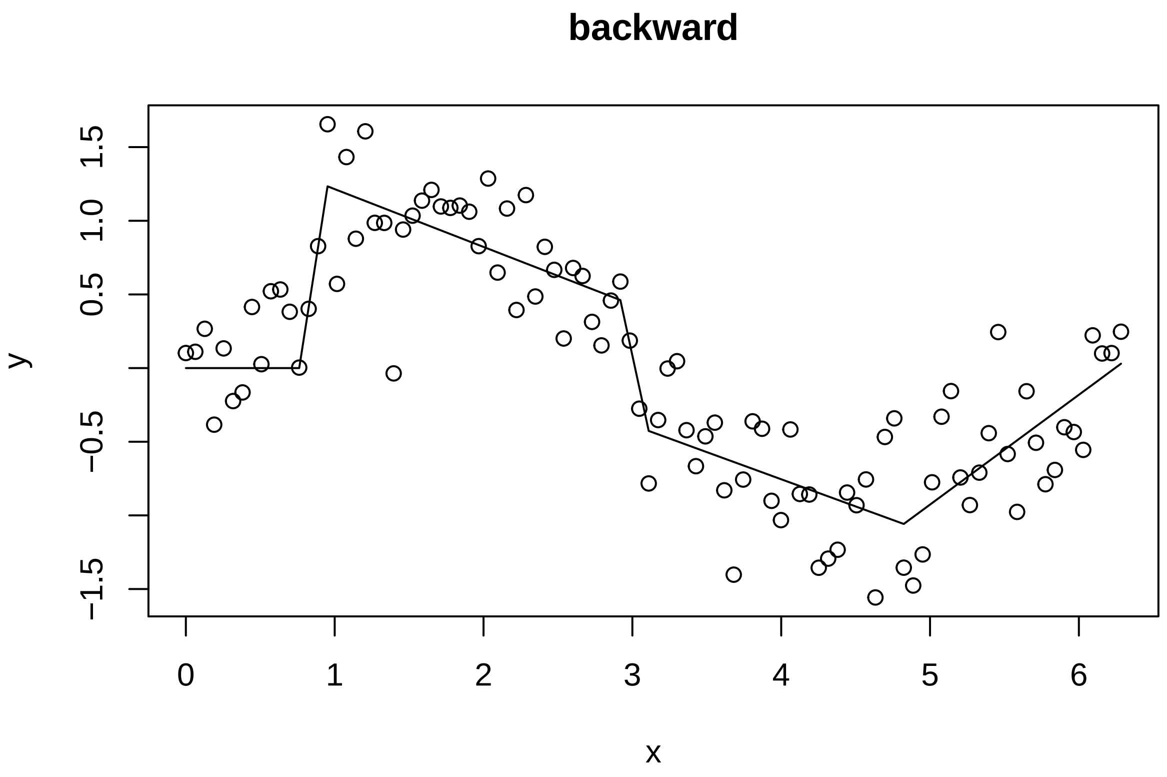
No, 3*k close to 100 when k is 33 so it gives highest score. With different degree of freedom, it gives different highest score

1.6

```r
k <- which(GCV==min(GCV[1:30]))
fit.values <- regsubsets.fitted.values(truncated,regsubsets.out,k)
plot(x,y,main='forward')
lines(x,fit.values)
```



```
regsubsets.out <- regsubsets(truncated,y,method="backward",nvmax=100,intercept = FALSE)
fit.values <- regsubsets.fitted.values(truncated,regsubsets.out,k)
plot(x,y,main='backward')
lines(x,fit.values)
```



2.1

$\hat{a} = argmin||y - Xa||^2 + \lambda a^T \Omega a$ $\frac{d}{da}||y - Xa||^2 + \lambda a^T \Omega a = 0 = \frac{d}{da} a^T X^T a - a^T X^T y - y^T X a + y^T y + \lambda a^T \Omega a$ $= 2X^T X a - 2X^T y + 2\lambda \Omega a = X^T X a - X^T y + \lambda \Omega a$ $a = (X^T X + \lambda \Omega)^{-1} X^T y$

so

$\hat{y} = X(X^T X + \lambda \Omega)^{-1} X^T y = S_\lambda y$

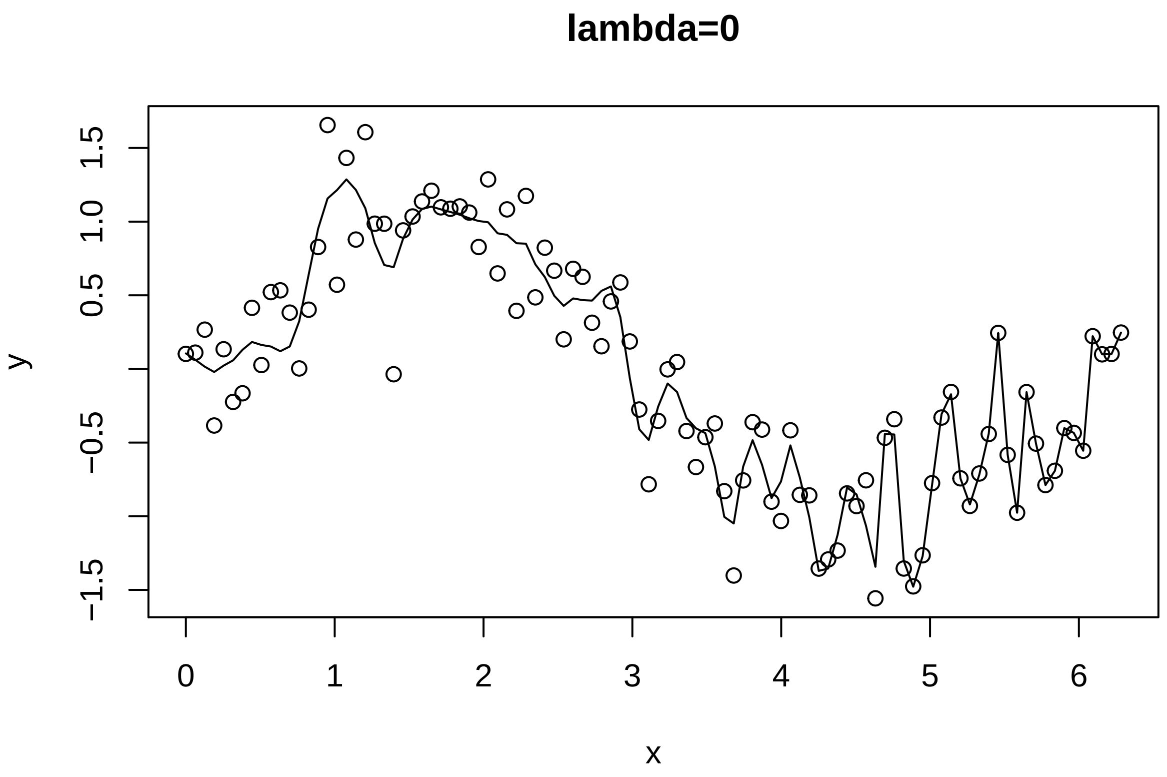
2.2

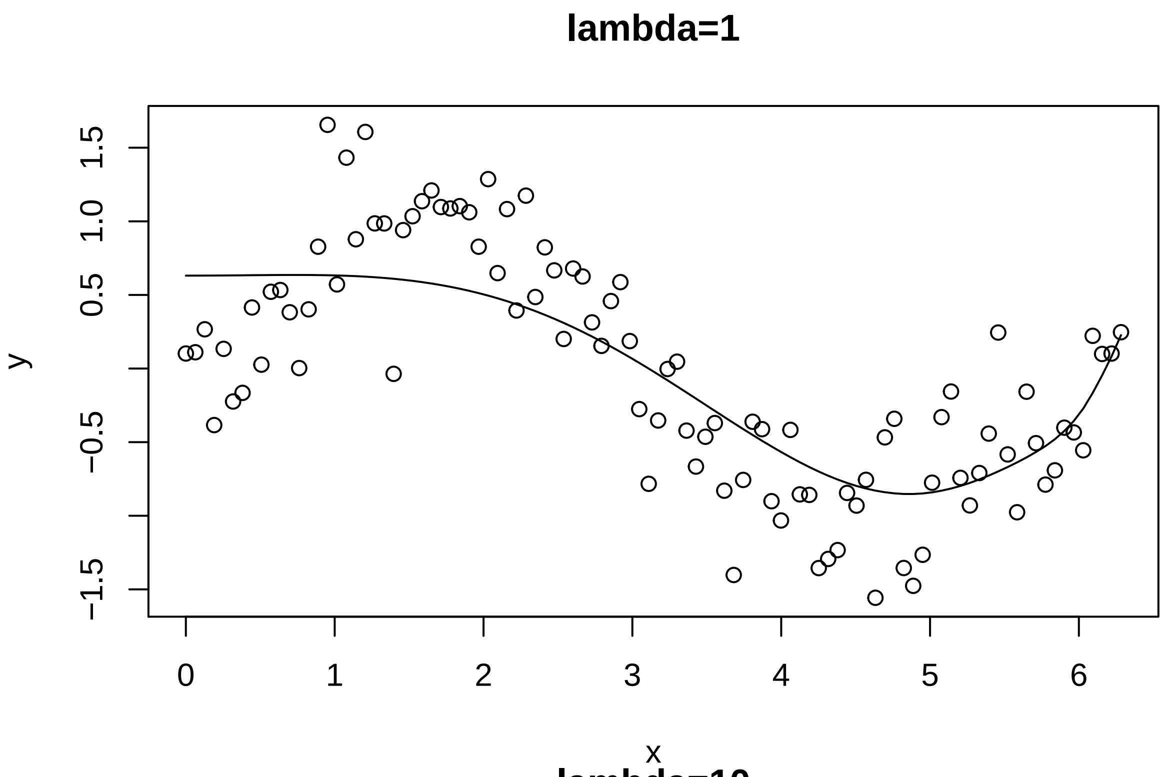
```r
library('glmnet')
```

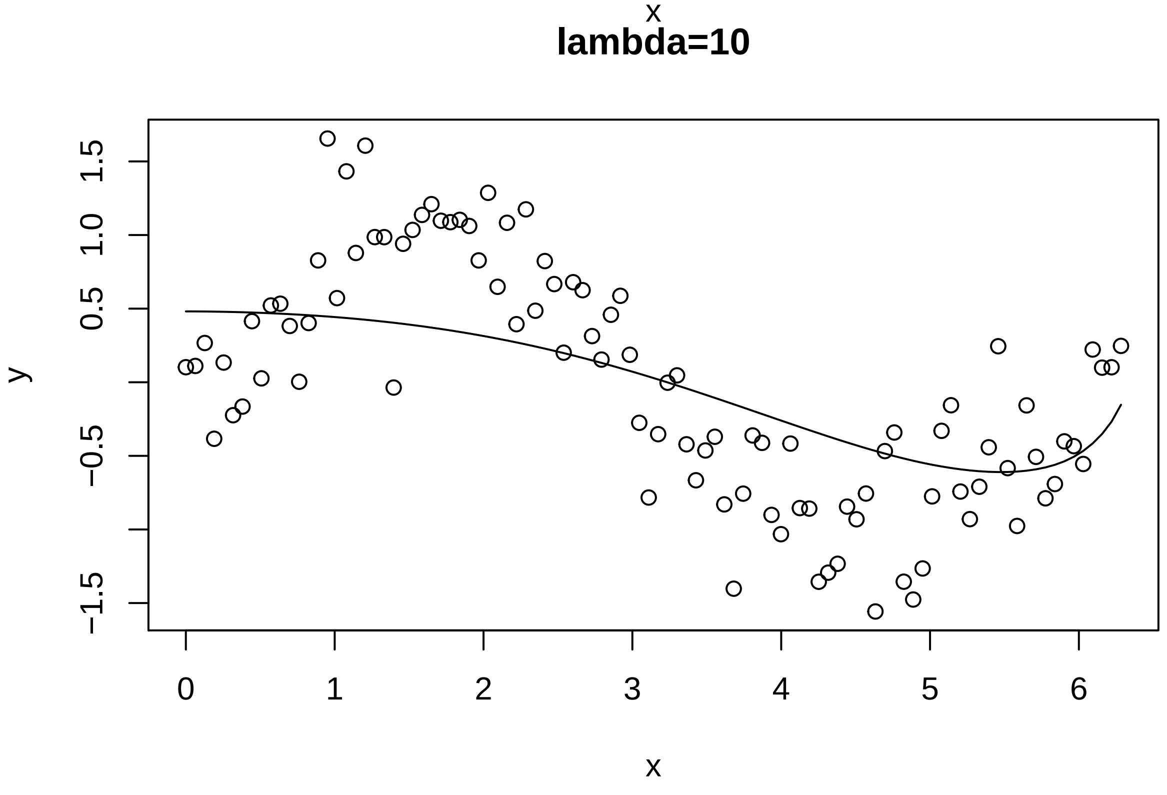
```
## Loading required package: Matrix
```

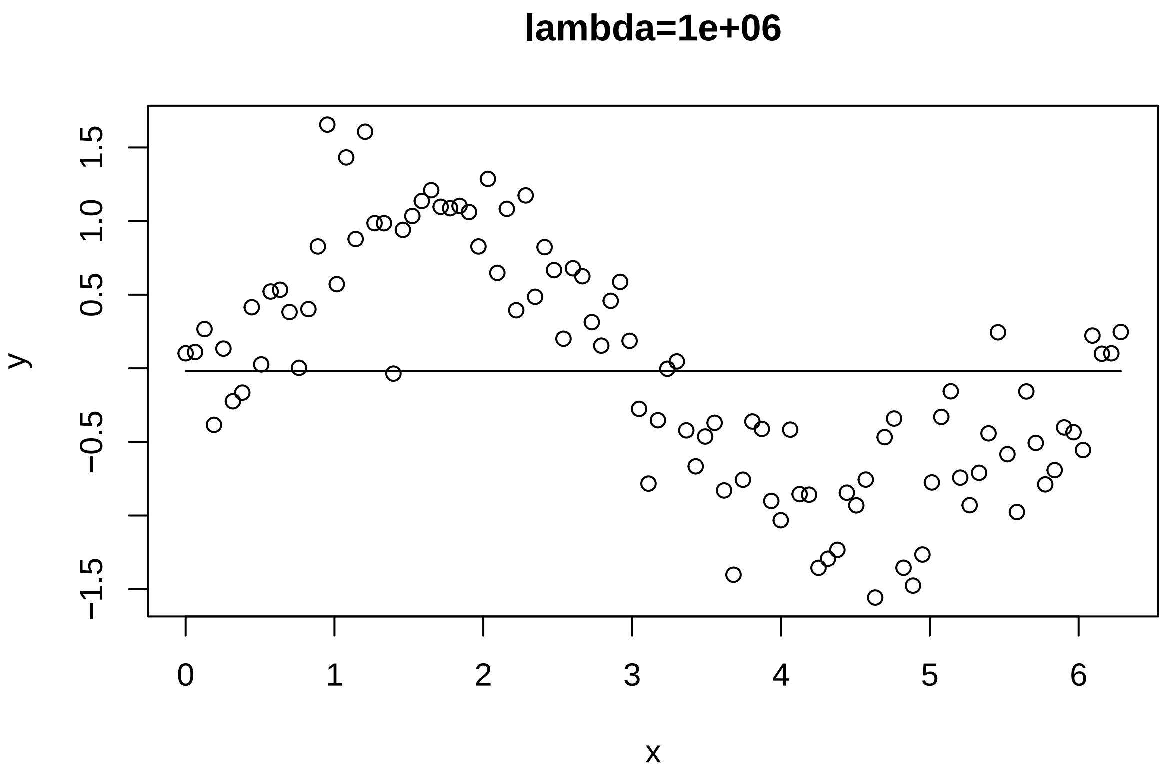
```
## Loaded glmnet 3.0-2
```

```r
lambda <-  c(0,1,10,1000000)
ridge.mod <- glmnet(truncated,y,alpha=0,lambda = c(0,1,10,1000000))
coeff <- coef(ridge.mod)
coeff[101,]=coeff[1,]
for (i in 1:4) {
  plot(x,y,main = paste0("lambda=",lambda[i]))
  lines(x,truncated%*%coeff[,5-i][2:101])
}
```



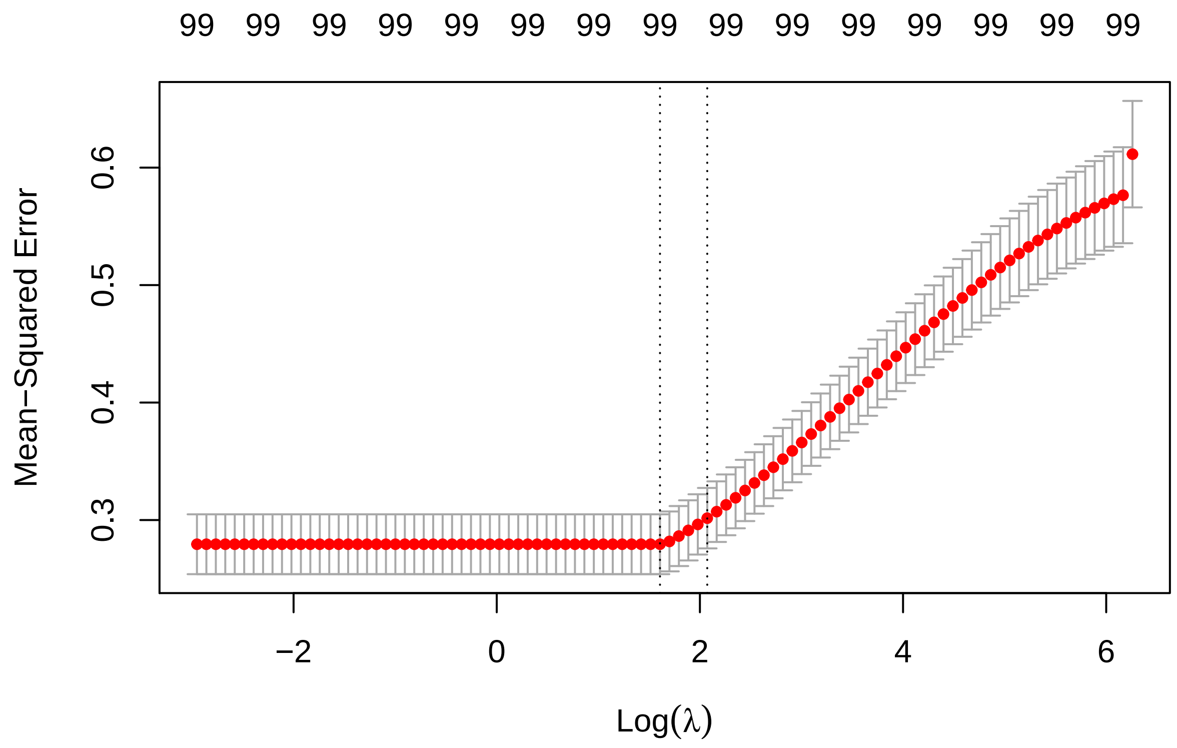

lambda=1
y
x
−1.5
−0.5
0.5
1.0
1.5
0
1
2
3
4
5
6


lambda=10
y
x
−1.5
−0.5
0.5
1.0
1.5
0
1
2
3
4
5
6

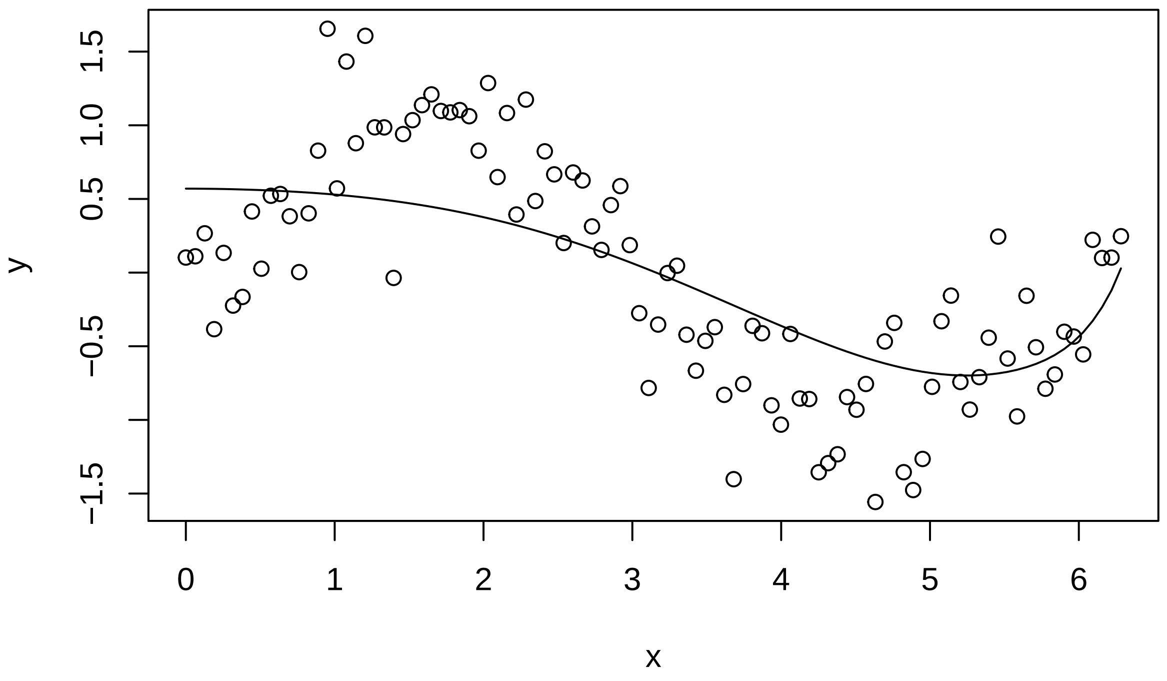


2.3

```
cv.out <- cv.glmnet(truncated,y,alpha=0)
plot(cv.out)
```



```r
optlam <- cv.out$lambda.min
ridge.mod <- glmnet(truncated,y,alpha=0,lambda = optlam)
coeff <- coef(ridge.mod)
coeff[101,]=coeff[1,]
y_hat <- truncated%*%coeff[2:101]
plot(x,y)
lines(x,y_hat)
```



3

a

Best subset gives the smallest training RSS, because it chooses the best k among all predictors as backward and forward both choose k from top to bottom or vice versa.

b

Also best subset, because we don't have control on test set, in general it gives the best prediction.

c

1

True, because forward choose k from beginnig to the end, so a k+1 model must include.

2

True, same reason as last question.

3

False, forward and backward choose from different direction.

4

False, same as last question.

5

False, the additional variable could take one of the predictor's place.

4

a

Increase, as λ increase, the model is more off, because of the $-\lambda \sum \beta^2$ part.

b

Decrease then increase, first, increasing λ can make the model close to the true model, but it will eventually off the model.

c

Decrease, it lessen the variance of x_{ij}

d

Increase, it makes the model off the predict model.

e

Not change, irreducible error does not depend on the model