Directional Forecasts with Risk-Neutral Probabilities and Skewness-Based Trading Strategy

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Abstract

This report investigates whether market-implied probabilities derived from options prices predict S&P500 returns, reversal, and volatility. A trading strategy is developed using distribution statistics skewness to generate signals. Option prices from 2011-2021 are used to estimate daily risk-neutral distributions and related metrics. Preliminary analysis shows large probability spikes capture market volatility over 1 year. To predict reversals, a SARIMAX model indicates the upside probability significantly predicts reversals. For the trading strategy, an optimized grid search identifies skewness thresholds that maximize returns while minimizing risk. Despite limited data, incorporating a decision tree boosts performance by refining signals. The tree outperforms with features like skewness, probability differences, and momentum indicators. Future work should acquire more data for validation and explore sophisticated models like random forests and RNNs.

1 Introduction

The ability to predict future asset price movements is invaluable for investors seeking to maximize returns. One approach is using market-implied distributions based on option prices, as proposed by Breeden and Litzenberger [1]. The Minneapolis Fed has computed such risk-neutral probabilities for various assets, including the S&P500 index. This report investigates whether these probabilities contain useful information for predicting S&P500 returns and volatility.

Risk-neutral probabilities represent the market's estimate of the probability distribution of future asset prices, derived from current option prices using a risk-neutral valuation framework. The Minneapolis Fed provides biweekly estimates of these probabilities for market moves of various magnitudes over 6 and 12 month horizons.

The analysis in the report tests whether various summary statistics and the risk-neutral distributions have predictive power for future S&P500 returns. Specifically, it examines the usefulness of the market implied distribution through its means, medians, standard deviations, skewness and kurtosis. A trading strategy is developed and backtested based on the statistics of the distribution. The strategy generates buy and sell signals based on threshold rules for the skewness and kurtosis. The out-of-sample performance is evaluated based on risk-adjusted returns, drawdowns, and other metrics. Combining the risk-neutral probabilities with other signals is also explored.

The report provides new insight into the informational content of market-implied distributions for predicting market movements. The results have important implications for investors, regulators, and academics. The introduction motivates the analysis and outlines the structure of the report. Additional details on data and methodology are provided in later sections.

2 Preliminary Analysis

The Minneapolis Fed's market-based probability estimates for the S&P 500 index, derived from traded options, offer a unique perspective on market expectations. These estimates provide a window into the risk-neutral probabilities of future asset price movements, incorporating the collective wisdom of market participants. In our preliminary analysis, we examined the Fed's biweekly data for the probabilities of the S&P 500 index moving up or down by 20% over a twelve-month horizon.

To contextualize these probability estimates, we matched each biweekly interval with the corresponding realized average level of the S&P 500 index exactly one year into the future. This approach allowed us to juxtapose the market's ex-ante expectations with the ex-post realizations, providing a visual representation of the predictive power of these estimates.

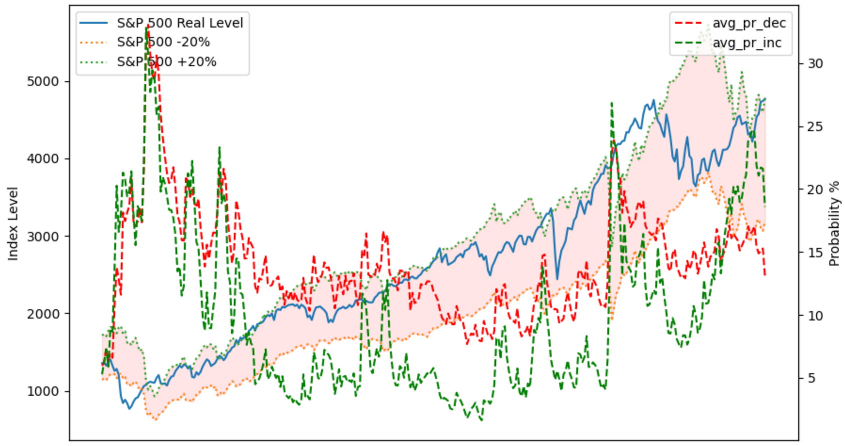

Figure 1: Predictive Power of Estimates

This plot revealed that the provided market-based probability estimates exhibited a notable ability to capture broad market volatility within the examined time frame. Whenever the probabilities of a 20% upward or downward movement in the S&P 500 index experienced a sharp rise, the ex-post realization of the index level one year into the future was likely to display a substantial change. Meanwhile, a limitation emerged as the upward and downward probability estimates often moved in tandem, especially during periods of high volatility. This makes it challenging to infer the precise direction of the anticipated large change in the index level.

This observation underscored the need for more fine-grained research methodologies and robust data support to interpret the directional implications of the probability movements. However, the biweekly frequency, coupled with the relatively short time frame (the estimates data are only available from 2007 onwards) of the data posed limitations for conducting more granular research and developing robust trading strategies.

To address this limitation, we recognized the need to recreate the process of calculating probability estimates at a higher frequency. Drawing inspirations from methods referenced by the

Minniapolis Fed for their estimates calculation, we recreated the process and used it to calculate the daily market based probability data over the span of around 10 years from 2011 to 2021.

3 Data and Methodology

The dataset for the research is based on historical option trading records spanning from 2011 to 2021 from IvyDB (a widely recognized database for historical options prices, trading volumes, and implied volatility information, developed and maintained by OptionMetrics). The analysis in this report specifically targets options related to the S&P 500 index, with a particular focus on options expiring approximately five to seven months from the current date.

For each year, all options, that match the target symbol with the specific expiration date, are ordered in descending order by trading dates. From this ordered list, the top 5 unique trading dates were selected to capture the most significant and recent trading activities. Among these filtered dates, the earliest (minimum) date is identified to define the start of analysis period, ensuring the analysis is on the most pertinent trading activity leading up to the expiration date.

After selecting all relevant period option data, important data attributes are extracted for analysis, including trading date, option symbol, strike price, expiration date, best bid, best offer, trading volume, option ID, and option type (call or put). The strike price was normalized by dividing by 1000 for consistency and ease of analysis. Dates were formatted to ensure uniformity across the dataset.

After data preparation, the analysis begins by constructing a Pandas `dataframe` with date as row index and all possible strike prices as column indices. Key processing steps are taken to convert the date and expiration columns into Pandas `datetime` formats. Additional market data for the S&P 500 index and risk-free rate are downloaded from Yahoo Finance and merged.

Specifically, The `date_strike` function maps the option prices from the CSV file into a main dataframe that has `datetime` indexes and strike prices as columns. If both the call and put prices exist for the same strike price, the option price that is out of the money is chosen. If the trading volume is zero, indicating that no valid price exists for the particular trading day, the latest valid trading price with trading volume larger than 0 in the past four days is used.

Implied volatilities are then calculated from the option prices using the `get_iv` function. This applies the Newton-Raphson root finding method to invert the Black-Scholes formula. NaN values are handled by re-adjusting the step size in Newton's method. The final output is then a dataframe with indexes of `datetime` and columns of strikes containing the implied volatilities.

The core modeling section implements a polynomial function (with degree of two) [2] fit and numerical integration to estimate probability masses above and below threshold index levels. It is critical to remove the impact of outliers in order to get a smooth curve of implied volatilities. Therefore, implied volatilities that are below zero or above 200% are removed to avoid fitting bias. Moreover, the polynomial function is fitted by minimizing the sum of absolute errors, considering the relative robustness of absolute error to outliers.

Since it is likely that only the near-the-money options are traded and have valid prices, the fitted curve only includes implied volatilities with strike prices that are near the current index price. Thus, the x-axis of the curve is extended to -/+50% of the current index price.

Based on the smoothed IV curve, call prices with all different strikes are generated using BSM formula. For later use, call prices are generated for every increase of 1 in strike price so that a call price function with respect to strike prices can be numerically approximated.

From Breeden-Litzenberger [1],

$$f(K) = e^{-rT} \frac{\sigma^2 C}{\sigma K^2}$$

The probability distribution of the underlying index in 6 months is then calculated based on the call price function. By integration, the downside probability of index decreasing by 10% and the upside probability of index increasing by 10% is also calculated. Additional distribution metrics like skewness, kurtosis, mean, variance, 10 percentile and 90 percentile are computed.

To summarize, the key outputs are time series tracking the upside/downside probabilities, and related descriptive statistics. Further analysis and trading strategies could then build on these risk-neutral probabilities.

4 Trading Strategy

4.1 MPD Estimates Reversal Prediction ability

This section analyzes the ability of features from Market Probability Density (MPD) to forecast daily stock return reversals. The study refines SARIMAX informed by Assaf's 2004 research on MENA stock prices[3]. By incorporating ARFIMA processes, as assessed by Liu, Chen, and Zhang[4], the model better handles financial time series' long-term dependencies, improving market prediction and trading strategy analysis.

The modified SARIMAX model, which is essentially a Fractionally Integrated ARFIMA (p,d,q) model, allows for a non-integer order of differencing, denoted by 'd', enabling the model to account for the persistence in the autocorrelation structure of the time series data. The ARFIMA model is expressed as:

$$\Phi(B)(1-B)^d X_t = \Theta(B)\varepsilon_t \quad (10) \tag{1}$$

where $\varepsilon_t \sim \text{iid}(0, \sigma^2)$,

Here, B represents the backward shift operator. The parameters $\Phi(B)$ and $\Theta(B)$ are polynomials in B of orders p and q, respectively, capturing the autoregressive and moving average dynamics.

The empirical study started by reducing features to address multicollinearity, excluding those with correlations above 0.7 to bolster model accuracy and effectiveness. The refined feature set was then subjected to time series cross-validation within the training dataset to determine optimized parameters for the SARIMAX model, namely the orders of the autoregressive (AR), differencing (I), and moving average (MA) components. This cross-validation process, utilizing the TimeSeriesSplit from scikit-learn, ensured that the model's hyperparameters were tuned to capture the temporal structure of the data effectively. Upon obtaining the optimized parameters, the SARIMAX model was applied for out-of-sample testing. The model, enhanced with memory-preserving properties via fractional differencing, was tasked with forecasting stock price reversals. The exogenous features were standardized to maintain consistency in the scale and to facilitate the model's interpretability.

The results in Table 1 present significant insights into the influence of various factors on daily stock price reversals. The up_prob variable shows a negative coefficient of -0.0014 with a p-value of 0.000, strongly suggesting a negative correlation with the likelihood of stock price reversals, consistent with the idea that a higher chance of upward movement might signal a potential reversal. Meanwhile, skewness is also statistically significant, with a p-value of 0.032, indicating a more modest effect on reversal forecasts, implying that while return distribution asymmetry matters, its influence on market behavior is less direct. The mean and variance variables, with p-values of 0.100 and 0.723 respectively, do not show significant predictive power, indicating that these factors on their own may not be effective predictors of market reversals.

Table 1: SARIMAX Results

	coef	std err	z	P>|z|	[0.025	0.975]
up_prob	-0.0014	0.000	-3.750	0.000	-0.002	-0.001
skewness	-0.0006	0.000	-2.150	0.032	-0.001	-5.13e-05
mean	-0.0006	0.000	-1.646	0.100	-0.001	0.000
variance	0.0002	0.001	0.354	0.723	-0.001	0.002
ma.L1	-0.0398	0.014	-2.748	0.006	-0.068	-0.011
sigma2	7.884e-05	1.45e-06	54.289	0.000	7.6e-05	8.17e-05

In conclusion, the application of fractional differencing in conjunction with ARIMA modeling techniques has been instrumental in developing a strategy for predicting stock price reversals. The results manifest the potential of up_prob and skewness features to anticipate directional changes.

4.2 Skewness-Based Trading Strategy

The report utilized a sample of option prices to estimate the higher moments of the return distribution for the underlying index. It is observed that positively skewed returns correlate with subsequent elevated returns. The trading strategy devised from this insight aims to exploit the predictive nature of positive skewness in forecasting upward trends in returns.

The concept that investors take into account higher moments in returns, including skewness, is well-established in financial literature. Theoretical models proposed by scholars like Kraus and Litzenberger (1983) integrate skewness into frameworks predicting expected returns[5]. Empirical studies have further confirmed the significance of higher moments in the valuation of securities. The research by Barberis and Huang (2009), along with the findings of Mitton and Vorkink (2007), suggest that the skewness of individual securities' returns may influence portfolio choices made by investors[6][7].

This report introduces a method to assess the influence of higher moments, particularly skewness, on asset returns. Utilizing option market data, it infers the higher moments of the return distribution for the underlying assets, positing that option prices reflect market consensus on future expectations. Analysis of option prices reveals a positive correlation between skewness and future S&P 500 returns. The derived trading strategy employs skewness as a critical indicator, advocating for buying when the skewness level of the S&P 500 index surpasses a predetermined threshold, and selling when it falls below another threshold.

Given the trading strategy, a comprehensive optimization process is employed to determine the most effective skewness buy and sell thresholds. Initially, the dataset is partitioned into training, validation, and test segments. The training set is used to initialize skewness thresholds using a return optimization approach, which sets the thresholds around the mean skewness, adjusted by a multiple of the standard deviation, which will be refined subsequently.

The validation set is utilized for fine-tuning thresholds through a grid search, guided by a custom objective function. The control variate method is implemented to ascertain optimal parameters within the grid search, and batch processing is employed to account for market fluctuations, ensuring the robustness and adaptability of the threshold estimations.

The objective function for the grid search is formulated to balance expected profits against empirical risk. The definition of empirical risk is adapted from Spyros Skouras [8]:

$$(\hat{b}_N, \hat{c}_N) = \arg\min_{(b,c)\in B} -\sum_{n=1}^{N} R_{n+1} \cdot I[c_0 + c_1 R_n] \tag{2}$$

This approach is chosen due to the simplicity of the forecasting model which is only based on skewness trading thresholds and the limitations inherent in market data, where minimizing empirical risk provides a more accurate parameter estimation than traditional maximum likelihood methods proved by Spyros Skouras[8]. Furthermore, The skewness measure is derived from a risk-neutral probability distribution, implying an assumption of risk-neutral market behavior that prioritizes the prediction of stock price direction. This indicator is apt for a risk-neutral strategy as it reflects directional forecasts.

Based on defined trading strategy, below is the revised empirical risk function:

$$(\hat{b}_N, \hat{s}_N) = \arg\min_{(b,s)\in B} -\sum_{n=1}^{N} R_{n+1} \cdot I[skew_n > b, skew_n < s] \tag{3}$$

In the revised empirical risk aspect of the objective function, the AR(1) model gives way to a skewness threshold as the forecast indicator, where `R_n` denotes the S&P 500's daily returns, and b and s represent the skewness buy and sell thresholds, respectively, with `skew_n` as the observed skewness. The function uses an indicator I, triggering trades only when the current day's skewness surpasses a set threshold, thereby optimizing the skewness thresholds to minimize the gap between predicted risk and observed returns, accounting for return asymmetry.

To make strategy capturing higher profit along with minimizing empirical risk, we make creation to also combine optimized function the expected profit defined by Spyros Skouras[8]:

$$X|Y_1, Y_2 = \begin{cases} \mu_1 + \sigma_1 Y_1 & \text{if } Y_2 > \frac{-\mu_2}{\sigma_2}, \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

$$\begin{aligned} E(X) &= \mu_1 \Phi\left(\frac{\mu_2}{\sigma_2}\right) + \frac{\sigma_1}{\sqrt{2\pi}} \rho \exp\left(-\frac{1}{2}\left(\frac{\mu_2}{\sigma_2}\right)^2\right) \\ &= \mu_1 \Phi\left(\frac{\mu_2}{\sigma_2}\right) + \frac{\sigma_{12}}{\sigma_2\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\mu_2}{\sigma_2}\right)^2\right) \end{aligned} \tag{5}$$

The trading strategy employed analyzes S&P 500 daily returns `Y1` against their predicted cumulative returns `Y2` derived from trading signals. The focus is on expected profits condition on positive cumulative returns over a chosen holding period. To ensure the model's robustness and adaptability, parameters are adjusted in batches, allowing the expected return to adapt to changing data characteristics like mean and variance. This method dynamically tunes the strategy to balance risk and return, adjusting to market shifts.

The objective function employs Grid Search to optimize skewness buy and sell thresholds, aiming for maximum expected profit and minimum empirical risk. Efficiency is enhanced via an early stopping mechanism, ceasing optimization when returns do not improve significantly over a defined iteration span, termed "patience." Parameters explored include skewness thresholds, learning rate, deviation range around optimized thresholds based on standard deviation, and the patience threshold.

The control variate method focuses on iterative optimization, adjusting parameters based on the objective function. Initially, the learning rate was varied within a specific range, with other parameters fixed, to conduct grid searches for its optimal value. The optimal learning rate was determined by the highest score of expected profit minus empirical risk. Subsequently, with the learning rate set, the standard deviation parameter was adjusted to identify the value providing the best score. This iterative approach continued for each parameter, ensuring a holistic optimization strategy that effectively balances risk and profitability.

Upon identifying optimal parameters, these are fixed in the grid search and trading strategy to determine effective skewness thresholds. These optimized parameters and thresholds are then applied in a trading strategy framework on a test dataset to produce trading signals. Signal performance is evaluated through backtesting, focusing on cumulative returns, Sharpe ratio, maximum drawdown, and other metrics to assess the strategy's efficacy and risk-adjusted performance.

4.3 Results

For backtesting trading strategies, a benchmark was defined that capitalizes on the discrepancies between the probabilities of market increase `up_prob` and decrease `down_prob` as determined from options data. This method automatically triggers a 'buy' action when `up_prob` exceeds `down_prob`, and a 'sell' action in anticipation of a market downturn. We also evaluate the differences in trading strategies when incorporating both skewness and kurtosis, as opposed to considering skewness alone.

In evaluating the trading strategies, the benchmark strategy outperformed others with a 53.75% return, albeit with a substantial maximum drawdown of 31.63% and negative risk-adjusted metrics, Sharpe of -2.51 and Sortino of -2.69. This strategy's win rate was marginally over 50%, suggesting moderate trading success.

The skewness-only strategy underperformed compared to the benchmark, achieving a 33.86% return. With lower volatility, its negative Sharpe ratio of -24.35 and Sortino ratio of -22.85, along with a win rate of 33.33%, suggest it was less proficient in managing risk versus reward. Although it outperformed the benchmark towards the end of 2018 and the beginning of 2019, the performance gap widened unfavorably after the COVID period trough.

Adding kurtosis to the strategy worsened the outcome, reducing the return to 2.35% and further degrading the Sharpe -59.92 and Sortino -52.19 ratios. The win rate saw a negligible improvement to 35.22%, but the strategy's maximum drawdown increased to 34.33%, the worst of all strategies tested. This illustrates that the effort to mitigate extreme risks by kurtosis can inadvertently limit the strategy's ability to leverage asymmetric opportunities skewness, reducing overall profitability.

Critically, the negative Sharpe and Sortino ratios across the board indicate that all strategies failed to surpass the risk-free rate after accounting for volatility and downside risk, highlighting inadequate risk compensation. Especially, during the COVID period, the risk-free rate became more attractive due to government incentives in bonds.

In summary, although the Skewness Strategy attains lower daily and annual volatility compared to the benchmark, its overall efficacy is compromised, suggesting that skewness can be a bonus but alone may not suffice for a profitable trading approach. Integrating kurtosis into the optimization of the skewness strategy has led to a significant underachievement relative to the benchmark. The use of kurtosis, which measures the tail heaviness of the distribution, marginally reduces volatility but at the expense of potential gains from tail events, resulting in inferior returns when factored into the strategy.

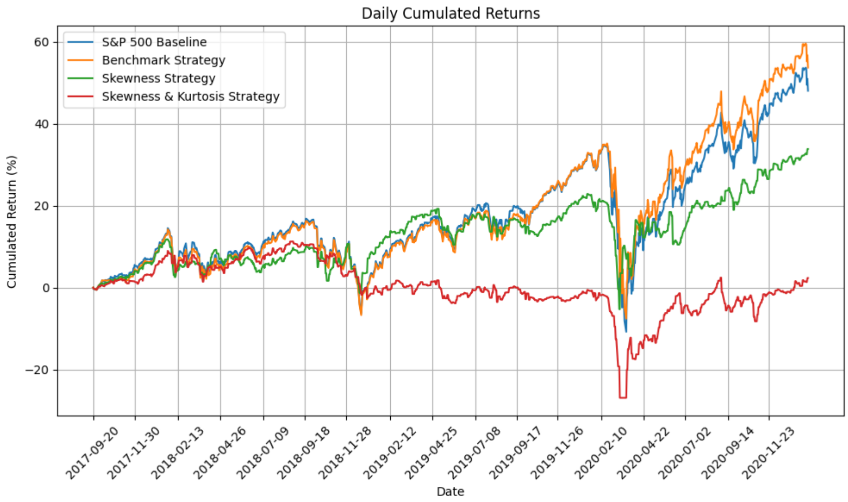

Figure 2: Daily Cumulated Returns

Interestingly, the visual analysis of volatility in Figure 3 demonstrates that the benchmark and skewness-based trading strategies are effective at forecasting the S&P 500's volatility. The benchmark, using `up_prob` and `down_prob`, closely matches the actual rolling volatility of the S&P 500, evidenced by an MSE of 0.0002 and an MAE of 0.0067. Although the skewness strategy slightly underestimates volatility compared to the benchmark, it remains effective, with an MSE of 0.006 and an MAE of 0.0477.

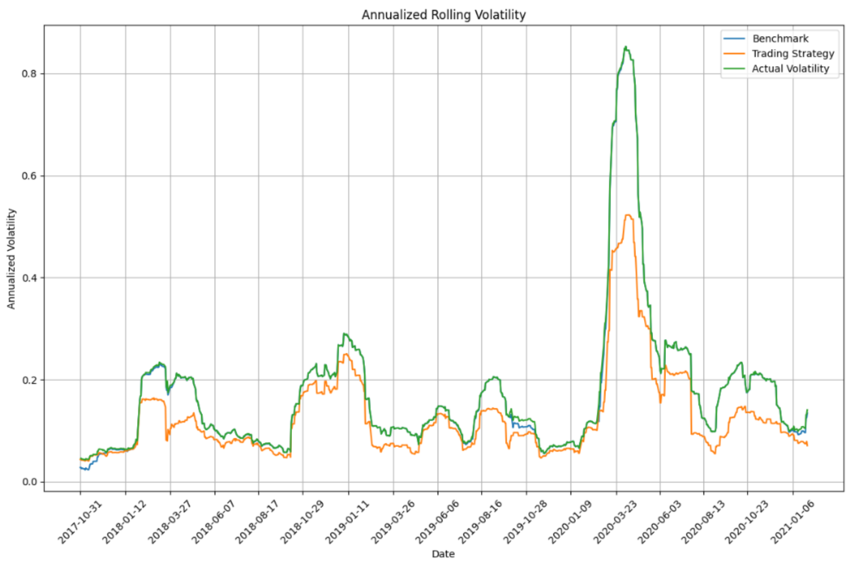

Figure 3: Volatility Prediction

5 Improvement Attempt with Decision Tree

As a next step in our research, we decided to combine other technical indicators to enhance the performance of the Skewness trading strategy by exploring the potential of the classic decision tree machine learning model. This decision was driven by the observation that the characteristics of decision trees align well with the inherent categorical nature of our existing strategies. Specifically, our current approaches revolve around making buy or sell decisions based on whether certain indicators surpass predefined threshold values. This process resonates with the fundamental principles of decision, or more specifically, classification trees.

At their core, decision trees operate by recursively splitting the feature space into increasingly homogeneous subsets. At each level, the model selects the variables and threshold values that minimize a predetermined impurity measure to split the data, and the process continues until further splitting cannot reduce impurity. Among the multiple choices for impurity measures, we opted for the Gini index, a widely used metric for quantifying the degree of class mixture within a subset. Mathematically, the Gini index for each decision region is expressed as $\sum_v \hat{p}_v(1-\hat{p}_v)$, where v denotes the distinct labels within the region, and $\hat{p}_v$ denotes the probability of drawing an element with label v randomly from the region (with replacement). The lower the Gini index, the more pure the region is, with a value of 0 indicating a perfectly homogeneous region without any misclassification.

The model was trained to predict the directional movement of the index price one day ahead. A straightforward trading strategy was then implemented, whereby an "up" prediction triggered a buy signal, while a "down" prediction prompted a sell action.

After determining the model to use, the next step was feature selection. This process largely relies on empirical trials and evaluations. We conducted a number of experiments based on the data on hand and settled on the following set of features:

1. **Skewness**: The main trading strategy in this research is based largely on skewness, so here it is also included for comparability,

2. $Pr($**index increases by 10%**$) - Pr($**index decreases by 10%**$)$: As highlighted in the preceding sections, the upward and downward probability estimates frequently exhibited synchronous movements, complicating the efforts to discern the implied directional movement. To address this challenge, the difference between these two estimates were used, aiming to leverage their relative relationship as a potential indicator of the anticipated market trajectory.

3. **Trading signals (buy, sell & hold) outputs from the main strategy**: In order to augment the capabilities of our primary strategy, we employed a bootstrapping methodology. Specifically, we supplied the signals generated by the main strategy as an input feature to the decision tree model, with the objective of enhancing the accuracy and precision of these signals through the model's learning process.

4. **MACD (Moving Average Convergence Divergence)**: A trend-following momentum indicator composed of two lines: the MACD line and the signal line. The MACD line is derived by subtracting the 26-period exponential moving average (EMA) from the 12-period EMA, while the signal line is typically the 9-period EMA of the MACD line itself. The use of EMAs places greater emphasis on recent price changes, allowing the MACD to react more dynamically to market movements. A bullish signal is generated when the MACD line crosses above the signal line (and vice versa).

5. **RSI (Relative Strength Index)**: Another momentum oscillator designed to quantify the overbought or oversold condition of an asset. The RSI value fluctuates between 0 and 100, with higher readings typically indicating an overbought market condition, implying that the price may be due for a correction or pullback (and vice versa).

Due to the limited availability of data, no further feature selection or parameter tuning processes were conducted. Utilizing the same backtesting methodology introduced in the previous chapter, the final performance comparison yielded the following results:

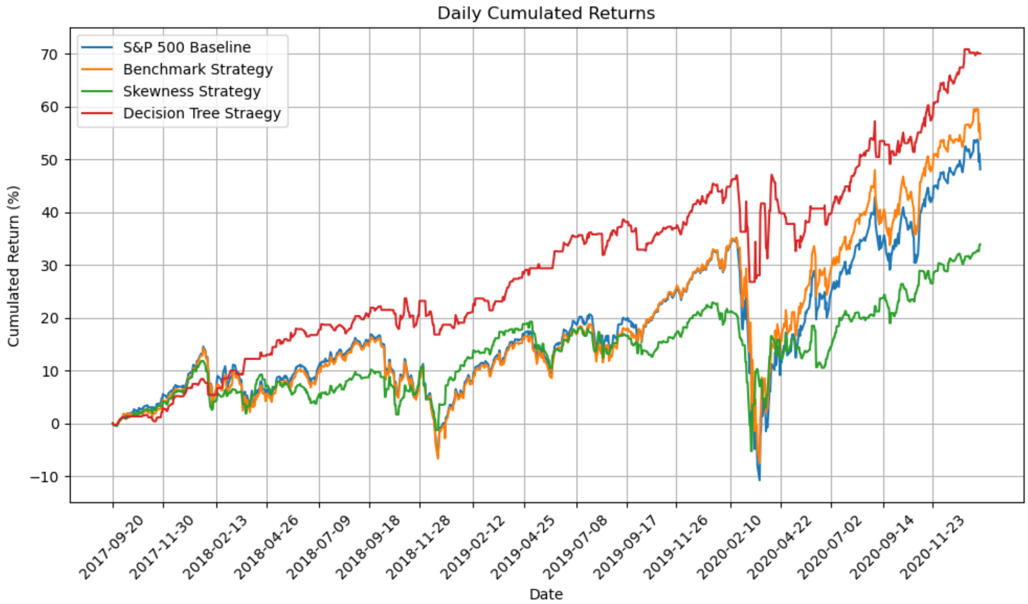

Figure 4: Daily Cumulated Returns

As can be observed from the plot, the decision tree model broke away from the stalemate market condition early in the test set period and managed to maintain a consistent outperformance over the market and other strategies throughout the entire testing period. This demonstrates the potential effectiveness of incorporating the decision tree model into our trading strategy.

6 Conclusion & Future Directions

Our research strategies progressed from leveraging probability distribution metrics to incorporating a decision tree model with a bootstrapping approach, ultimately optimizing performance by refining trading signals.

However, in this report, a polynomial curve of degree two with mean absolute loss function was used to fit the daily volatility curve. This curve focused on fitting the implied volatilities at strikes which are close to the current underlying price. It underestimated the effects of outliers at low strikes and therefore may underestimates the probability of having a drop in the underlying price.

Moreover, the limited data set (around 10 years, 2,500 trading days), constrained the depth of feature and model selection and parameter tuning. The sequential nature of the time series data rendered it incompatible with k-fold cross-validation, which typically benefits from enhancing model validation by averaging results over multiple folds to reduce overfitting.

Furthermore, limited data availability also ruled out the application of more sophisticated models that might help capture more underlying market trends. One such example is the family of gradient boosting methods (e.g., XGBoost) that have found wide applications in, amongst many other fields, predicting volatility (Teller et al., 2022[9]) and forming trading strategies (Qin et al., 2013[10]). These models rely on a series of weak learners (usually decision trees) to sequentially improve the prediction performance.

Looking ahead, future research should focus on obtaining more extensive and diverse datasets to improve model training, enable robust validation techniques, and explore more methods for fitting volatility curves, including different spline methods and loss functions to address spikes in low strike volatilities.

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