Model Interpretation and Visualization I Supplemental
Iowa Social Research Center (ISRC) Workshop

Desmond D. Wallace & Scott J. LaCombe

Department of Political Science
The University of Iowa
Iowa City, IA

October 24, 2018

OLS Predicted Values Introduction

- Definition – The value predicted for Y when X takes on a specific value, or set of values.
- Formula – $E[Y|x^*] = x^*\beta$

OLS Predicted Values Types

- Average Predicted Value (APV) – The predicted value $E[Y|X]$ for each observation at specified value(s) x^*, and taking the average of these values.
  - $\overline{E[Y|X = x^*]} = \frac{1}{N} \sum_{i=1}^{N} x^* \beta$
  - *The average expected value of Y is...*
- Predicted Value at the Mean (PVM) – The predicted value $E[Y|X]$ for each observation at its mean values.
  - $E[Y|X = \bar{x}] = \bar{x}\beta$
  - *For someone who is average on all characteristics, the expected value of Y is...*
- Predicted Value at Representative Values (PVR) – The predicted value of $E[Y|X]$ for each observation at specified value(s) x^*.
  - Specify values that are instructive for the substantive questions under consideration.
  - PVM is a special case of PVR.
  - If not all variables are specified, PVRs will be calculated for the variables specified, and averaged across the values for the unspecified variables.

OLS Marginal Effects Definitions

- Measuring the change in the dependent variable for a change in one independent variable, holding remaining independent variables constant.
  - *Marginal Change* is the partial derivative, or instantaneous rate of change, in the dependent variable w.r.t. an independent variable, holding remaining variables constant.
  - *Discrete Change* is the difference in the prediction from one specified value of an independent variable to another specified value, holding remaining variables constant.

OLS Marginal Effects Formulas

- Marginal Change: $\frac{\partial E[Y|X]}{\partial x_k} = \frac{\partial X\beta}{\partial x_k} = \beta_k$
- Discrete Change: $\frac{\Delta E[Y|X]}{\Delta x_k \left(x_k^{start} \to x_k^{end}\right)} = E[Y|X, x_k^{end}] - E[Y|X, x_k^{start}]$

OLS Marginal Effects Interpretations

- $\frac{\partial E[Y|X]}{\partial x_k} = \frac{\Delta E[Y|X]}{\Delta x_k \left(x_k^{start} \to x_k^{end} \right)} = \beta_k$ when $x_k^{start} \to x_k^{end} = 1$, assuming there is no interaction terms.

- The standard error of the marginal effect is the same as the standard error of the estimated beta coefficient.

- *For a unit increase in x_k, the expected change in Y equals β_k, holding all other variables constant.*

- *Having characteristic x_k (as opposed to not having the characteristic) results in an expected change of β_k in Y, holding all other variables constant.*

OLS Marginal Effects
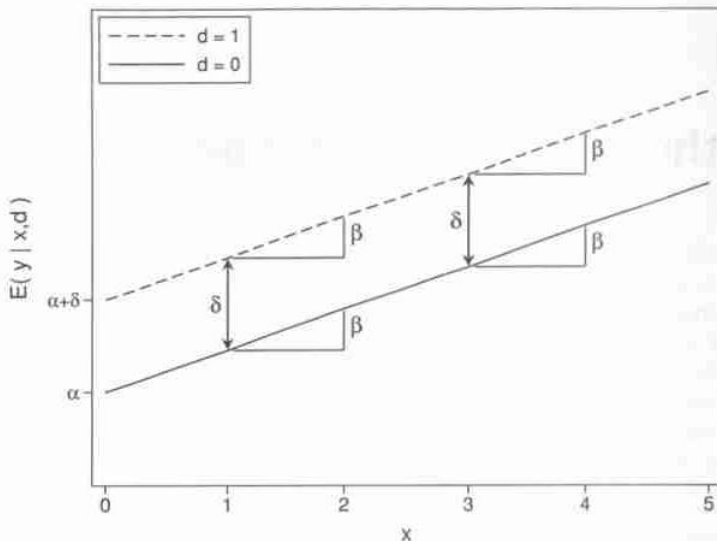
BRM Predicted Probabilities Introduction

- Definition – The probability predicted for $Pr(y = 1|X)$ when X takes on a specific value, or set of values.
- Formula – $Pr(y = 1|X = x^*) = F(x^*\beta)$
  - Probit – $F = $ Normal CDF
  - Logit – $F = $ Logistic CDF

BRM Predicted Probabilities Types

- Average Predicted Probability (APP) – The predicted probability $Pr(y = 1|X)$ for each observation at specified value(s) x^*, and taking the average of these probabilities.
  - $\overline{Pr(y = 1|X = x^*)} = \frac{1}{N} \sum_{i=1}^{N} F(x^*\beta)$
  - *The average predicted probability of Y is...*
- Predicted Probability at the Mean (PPM) – The predicted probability $Pr(y = 1|X)$ for each observation at its mean values.
  - $Pr(y = 1|X = \bar{x}) = F(\bar{x}\beta)$
  - *For someone who is average on all characteristics, the predicted probability of Y is...*

BRM Predicted Probabilities Types

- Predicted Probability at Representative Values (PPR) – The predicted probability of $Pr(y = 1|X)$ for each observation at specified value(s) x^*.
  - Specify values that are instructive for the substantive questions under consideration.
  - PPM is a special case of PPR.
  - If not all variables are specified, PPRs will be calculated for the variables specified, and averaged across the values for the unspecified variables.

BRM Marginal Effects Definitions

- Measuring the change in the probability of an outcome for a change in one independent variable, holding remaining independent variables constant at specific values.
  - *Marginal Change* is the rate of change in the probability for an infinitely small change in x_k, holding other variables at specific values.
  - *Discrete Change* is the actual change in the predicted probability for a given change in x_k, holding other variables at specific values.

BRM Marginal Effects Formulas

- Marginal Change:
  - General: $\frac{\partial Pr(y=1|X=x^*)}{\partial x_k} = f(X\beta)\beta_k$
    - Probit – f = Normal PDF
    - Logit – f = Logistic PDF
  - Logit only:
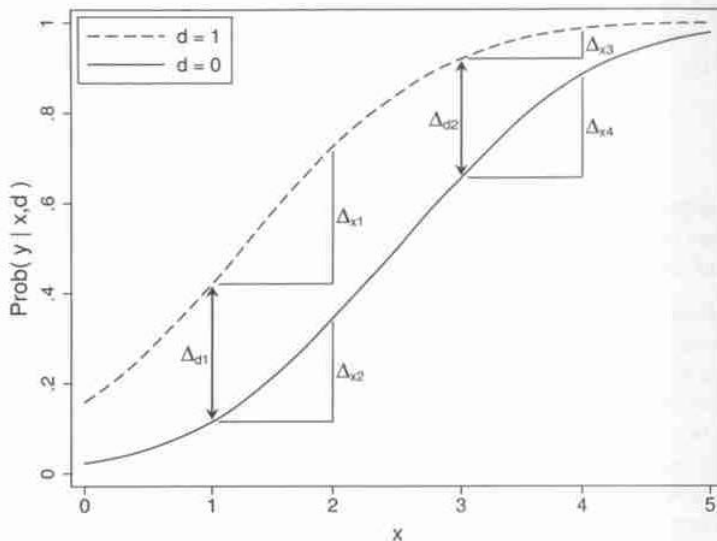 $\frac{\partial Pr(y=1|X=x^*)}{\partial x_k} = Pr(y=1|X=x^*)[1 - Pr(y=1|X=x^*)]\beta_k$
- Discrete Change: $\frac{\Delta Pr(y=1|X=x)}{\Delta x_k(x_k^{start} \rightarrow x_k^{end})} =$
  $Pr(y=1|X=x, x_k=x_k^{end}) - Pr(y=1|X=x, x_k=x_k^{start}) =$
  $F(X\beta, x_k=x_k^{end}) - F(X\beta, x_k=x_k^{start})$

BRM Marginal Effects Interpretation

- $\frac{\partial Pr(y=1|X=x^*)}{\partial x_k} \approx \frac{\Delta Pr(y=1|X=x)}{\Delta x_k\left(x_k^{start} \rightarrow x_k^{end}\right)}$, the more linear the probability curve is in the region where the change is occurring.

- In general, $\frac{\partial Pr(y=1|X=x^*)}{\partial x_k} \neq \frac{\Delta Pr(y=1|X=x)}{\Delta x_k\left(x_k^{start} \rightarrow x_k^{end}\right)}$

BRM Marginal Effects

BRM Marginal Effects Types

- Average Marginal Effect (AME) – The marginal effect of x_k for each observation at specified values x^*, and taking the average of these effects.
  - Marginal Change
    - $\frac{1}{N} \sum_{i=1}^{N} \frac{\partial Pr(y=1|X=x^*)}{\partial x_k}$
    - *The average marginal effect of x_k is...*
  - Discrete Change
    - $\frac{1}{N} \sum_{i=1}^{N} \frac{\Delta Pr(y=1|X=x^*)}{\Delta x_k(x_k^{start} \rightarrow x_k^{end})}$
    - *On average, increasing x_k by δ increases the probability by...*
    - *On average, increasing x_k from <u>start-value</u> to <u>end-value</u> increases the probability by...*

BRM Marginal Effects Types

- Marginal Effect at the Mean (MEM) – The marginal effect of x_k with all independent variables held at their means.
  - Marginal Change
    - $\frac{\partial Pr(y=1|X=\bar{x_k})}{\partial x_k}$
    - *For someone who is average on all characteristics, the marginal change of x_k is...*
  - Discrete Change
    - $\frac{\Delta Pr(y=1|X=\bar{x_k})}{\Delta x_k \left(x_k^{start} \to x_k^{end} \right)}$
    - *For someone who is average on all characteristics, increasing x_k by δ changes the probability by...*

BRM Marginal Effects Types

- Marginal Effect at Representative Values (MER) – The marginal effect of x_k with independent variables held at specific values.
  - Specify values that are instructive for the substantive questions under consideration.
  - MEM is a special case of MER.
  - If not all variables are specified, MERs will be calculated for the variables specified, and averaged across the values for the unspecified variables.

Email: desmond-wallace@uiowa.edu
Any Questions?