Multilevel Modeling (Part 1)

Basic Random-Intercept and Random-Slope Modeling

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Regression Highlights

- A way to summarize the relationship between variables.
- Assuming there is a relationship between Y and the independent variable(s).
- Relationship may be linear (OLS) or non-linear (CLDV).
- Regression helps our understanding of how our dependent variable of interest changes when one or more independent variables vary, while holding remaining variables fixed.
- PRF: $y_i = \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \varepsilon_i$
- SRF: $y_i = b_0 + b_1 x_1 + \cdots + b_k x_k + e_i$

Model Specification

1. Linearity in the parameters
2. The number of observations n must be greater than the number of parameters to be estimated
3. The regression model is correctly specified

Independent Variable(s)

1. X values are fixed in repeated sampling
2. Variability in X values
3. There is no perfect multicollinearity

Error Term

1. Zero mean value of error (e_i)
2. **Homoscedasticity or equal variance** of e_i
3. No autocorrelation between the errors
4. Zero covariance between e_i and X_i

Additional Assumptions

1. Errors are normally distributed
2. **Errors for any two observations are independent of one another**

Multi-Stage Sampling

- OLS assumptions imply utilization of Simple Random Sampling (SRS)
- However, due to cost-efficiency, multi-stage sampling approaches may be utilized instead.
- Researcher may randomly sample grouping units instead of individuals (cluster sampling)
- Examples
  - Students nested in schools
  - Respondents nested in states (countries)
  - Patients nested in hospitals

Applying OLS to Multilevel Data

- Biased standard errors
- Model Misspecification

New Approach

- Best approach to analyzing nested data is a statistical approach that accounts for both within-group and between-group variation *simultaneously*
- One approach is to conceive within-group and between-group variation as random variability
- One can achieve this by including *random coefficient(s)* in the statistical model

Multilevel Model

- Multilevel Model (MLM) is a model where the parameters vary at more than one level
- Features more than one error term
- Variation can occur wit respect to the intercept (*random intercept*) and/or the slope (*random slope*)
- This approach leads to corrected standard errors and correct model specification

Multilevel Model

- Model coefficients are now a combination of both fixed and random components
  - Fixed Coefficient – An unknown constant of nature
  - Random Coefficient – One which varies from sample of groups to sample of groups
- Random coefficients are not estimated, and are instead predicted
- Instead of BLUE (Best Linear Unbiased Estimator) coefficients, we know have BLUP (Best Linear Unbiased Prediction) coefficients

Null Model

- Predicting the outcome from only an intercept that varies between groups
- The null model takes the following form:
  - Level-1 Model: $y_{ij} = \beta_{0j} + \varepsilon_{ij}$
  - Level-2 Model: $\beta_{0j} = \gamma_{00} + U_{0j}$ where
    - β_{0j} – Average (general) intercept holding across all groups (fixed effect)
    - U_{0j} – Group-specific effect on the intercept (random effect)
  - Full Specification: $y_{ij} = \gamma_{00} + U_{0j} + \varepsilon_{ij}$
- Interested in general mean value for y_{ij} (γ_{00}) and deviation between overall mean and group-specific effects for the intercept (U_{0j})

Null Model Assumptions

1. Groups are a random sample from the population of all possible groups
2. U_{0j} is randomly drawn from a population distribution with mean 0 and variance τ_0^2
3. τ_0^2 (Variance of U_{0j}) and σ^2 (variance of ε_{ij}) are uncorrelated

Variance of y_{ij}

- Variance of y_{ij} is just the sum of the level-two and level-one variances
  - $Var(y_{ij}) = Var(U_{0j}) + Var(\varepsilon_{ij}) = \tau_0^2 + \sigma^2$
- Covariance between two observations from the same group (ij and $i'j$) is equal to the variance of the contribution U_{0j} that is shared by these observations
  - $Cov(y_{ij}, y_{i'j}) = Var(U_{0j}) = \tau_0^2$

Intraclass Correlation Coefficient

- A measure of the proportion of variation in the dependent variable that occurs *between* groups versus the total variation (*between* and *within*)
- Correlation between two randomly drawn observations from in one randomly drawn group
- Ranges between 0 (no variation between groups) and 1 (all between-group variation but no within-group variation)
- $\rho(y_{ij}, y_{i'j}) = \frac{\tau_0^2}{\tau_0^2 + \sigma^2}$

Coefficient Plots

- Sometimes, regression models feature many variables
- Also, showing many numbers and stars can be difficult for some readers
- An alternative to reporting a table is a plot of the regression results

`coefplot`

- `coefplot` is another user-written Stata program
- Plots regression results in "dot-whisker" format
  - "Dot" – Coefficient Estimate
  - "Whisker" – Confidence Interval
- Basic Syntax: `coefplot`
- `coefplot` command is executed AFTER regression model is estimated

coefplot Example

```
reg realrinc age i.female
coefplot, title("Model Results") xline(0)
```

coefplot Example

coefplot Example

```
reg realrinc age i.female
coefplot, title("Model Results") xline(0) drop(_cons)
```

`coefplot` Example

Interpreting Coefficients

- Can directly interpret coefficient estimates.
- *A one unit change in X_k leads to a β_k change in Y (holding all other variables constant).*
- Assumes X_k is not a constituent term for an interaction variable.

Predicted (Fitted) Values

- The result of substituting values of interest for the independent variable(s).
- $E[Y|X] = X\hat{\beta}$
- Can calculate standard errors to determine if $E[Y|X = x]$ is statistically significantly different from zero.
- Multiple ways to calculate fitted values in Stata.

Marginal and Discrete Change

- Measuring the change in the dependent variable for a change in one independent variable, holding remaining independent variables constant.
    - *Marginal Change* is the partial derivative, or instantaneous rate of change, in the dependent variable w.r.t. an independent variable, holding remaining variables constant.
    - *Discrete Change* or *First Difference* is the difference in the prediction from one specified value of an independent variable to another specified value, holding remaining variables constant.

Marginal and Discrete Change

- Marginal Change: $\frac{\partial E[Y|X]}{\partial x_k} = \frac{\partial X\beta}{\partial x_k} = \beta_k$
- Discrete Change: $\frac{\Delta E[Y|X]}{\Delta x_k} = E[Y|X, x_k + 1] - E[Y|X, x_k] = \beta_k$

Marginal and Discrete Change

- $\frac{\partial E[Y|X]}{\partial x_k} = \frac{\Delta E[Y|X]}{\Delta x_k} = \beta_k$, assuming there is no interaction terms.
- The standard error of the marginal effect is the same as the standard error of the estimated beta coefficient.
- *For a unit increase in x_k, the expected change in Y equals β_k, holding all other variables constant.*
- *Having characteristic x_k (as opposed to not having the characteristic) results in an expected change of β_k in Y, holding all other variables constant.*
- When there is no interaction term present,
  Marginal Change = Discrete Change

Marginal Effects

margins

- Computes predicted values and marginal effects from last estimated regression model
- Reports computed statistic, standard error, test statistic, *p*-value and 95% CI.
- `at(atspec)` option allows for the calculation of predicted values and marginal effects at specific values of independent variable(s).
- `dydx()` option allows for calculating marginal effects.
- Factor variables (`i.varname`) can go after the `margins` command or within the `at(atspec)` option.
- Continuous variables can only be specified within the `at(atspec)` option.
- `atmeans` option sets variables not specified to be held at their mean value.

Predicted (Fitted) Values – `margins` Syntax

- `margins` – Overall predicted value with all independent variables held at their mean value.
- `margins, at(varname=#)` – Predicted value when one or more independent variables are fixed to a specific value and remaining independent variables held at their mean value.
- `margins, at(varname=numlist)` – Predicted value(s) when one or more independent variables are fixed to multiple values and remaining independent variables held at their mean value.
- `margins varname` – Overall predicted value(s) for categories of `varname` with remaining independent variables held at their mean value.

Marginal Change – `margins` Syntax

- `margins, dydx(varname)` – Average marginal effect a one-unit increase in `varname` has on the dependent variable, holding all other variables constant.

Discrete Change – `margins` Syntax

- `margins, at(varname=(`*start end*`)) post` – Calculates predicted values at specified values, and treats results as estimation results.
- `lincom 2._at − 1._at` – Calculates the difference between the prediction of the ending value and the prediction of the starting value.

marginsplot

- Graphs the results of last estimated `margins` command
- **Needs to be executed immediately after** `margins`
- Resulting graph includes an overall title, a title for the y-axis, x-axis features the name of the variable (variable label if one is included).
- The featured values on the x-axis are the values specified from the `margins` command.
- Can use the `recast` and `recastci` options to change how results are graphed.

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Any Questions?