Work-study report
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Multiple Vehicle Routing Problem (MVRP)

Introduction:

Basically, MVRP is a traditional Travelling Salesman Problem (TSP) with multiple vehicles on route. A typical VRP model consists of a depot and multiple customers in known distances from depot. Challenge is to find optimal route for vehicle which start the tour at depot, to visit all the customers and coming back to depot with minimum possible distance covered. We can make problem more realistic by considering multiple vehicles available at depot and routes for each vehicle to visit all the customers. Hence, in MVRP, multiple vehicles available at depot will complete their respective tours starting and ending at depot so total distance travelled is minimum. MVRP model is very helpful on saving time and cost of fuel of vehicles in any purpose or business by formulating minimum distance coverage tours. Some of the problems can be formulated considering multiple depot as well, but in this project, I am only focusing single depot scenario. In terms of computational complexity, MVRP is a NP hard problem. Hence, computational time rise massively with increasing size of problem. Consequently, I chose a problem that can give result in matter of one to two minutes.

Problem formulation:

No of depots: 1
No of nodes: n
No of vehicles: m
Vehicle capacity: Q
Demand at nodes: D
(i, j): Nodes indices

dij = Distance between i & j.
dii = M

1. **Variables:**
   Xij = {0, 1}, binary decision on going from node ‘i’ to ‘j’
   Y(j) = Cumulative demand satisfied at node ‘j’ by a vehicle.
2. $\boldsymbol{z} = min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij*X_{ij}}$

   s.t.

3. $\sum_{i=1}^{n} X_{0j} = 5$ $(i = 0;\ depot)$: start of ‘m’ tours at depot.
4. $\sum_{i=1}^{n} x_{i0} = 5$ $(j = 0;\ depot)$: end of ‘m’ tours at depot.
5. $\sum_{j=2}^{n} x_{ij} = 1$ $\forall i$ (i = 2, 3, …., n)

6. $\sum_{i=2}^{n} x_{ij} = 1$ $\forall j$ (j = 2, 3, …., n)

7. $x_{ij} + x_{ji} \leq 1$ $\forall i, j$ $(i, j = 1, 2, \ldots., n)$

8. Y[j] >= Y[i] + D[j]* X[i,j] – Q*(1-X[i,j]) (i, j = 2, 3, …., n)

9. D[j] <= Y[j] <= Q (j = 2, 3, ….,n)

Problem Data:
I chose a single depot MVRP problem with ‘4’ vehicles and ‘25’ total nodes, one of which is a depot. Each customer has a demand of units to be delivered by vehicle. There are ‘4’ routes with each vehicle starting and ending their tours at depot. A vehicle has capacity of ‘85’ units and total demands for all customers is ‘326’ units. Following represent problem data in CSV format with order of Node, X coordinate, Y coordinate, Demand at node.

1 35 35

2 41 49 10

3 35 17 7

4 55 45 13

5 55 20 19

6 15 30 26

7 25 30 3

8 20 50 5

9 10 43 9

10 55 60 16

11 30 60 16

12 20 65 12

13 50 35 19

14 30 25 23

15 15 10 20

16 30 5 8

17 10 20 19

18 5 30 2

19 20 40 12

20 15 60 17

21 45 65 9

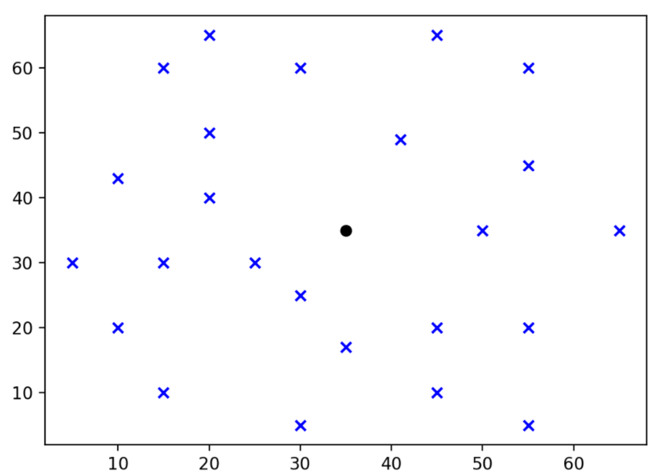
22 45 20 11

23 45 10 18

24 55 5 29

25 65 35 3

Following graph shows the coordinates of the nodes in miles distance. Black ‘o’ mark represents depot and blue ‘x’ mark represent customers.



Solution:

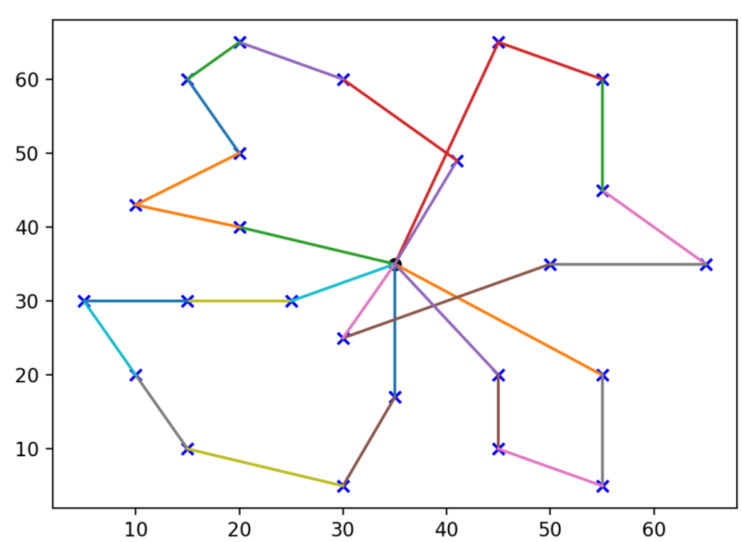
Solution method used is Mixed Integer Programming (MIP) with the help of simplex solver of Gurobi software in Python-Gurobi interface. In Python programming, distance between each node is calculated with Pythagoras law. A model is started in Gurobi platform, and decision variables are defined. Nature and dimension of variables are added to model as mentioned in formulation, with Gurobi's format of syntax. All the constraints are defined and added to model and finally objective is added. Finally, model is optimized with Gurobi optimizer v9.0.0. Final solution is retributed and stored in table array. Final decisions, objective, computational time and routes showing each vehicle's tour achieved are as following:

```
Objective (minimum distance covered): 398.725
```

Variable	x
x0,2	1
x0,4	1
x0,18	1
x0,20	1
x1,0	1
x2,15	1
x3,24	1
x4,23	1
x5,6	1
x6,0	1
x7,19	1
x8,7	1
x9,3	1
x10,1	1
x11,10	1
x12,13	1
x13,0	1
x14,16	1
x15,14	1
x16,17	1
x17,5	1
x18,8	1
x19,11	1
x20,9	1
x21,0	1
x22,21	1
x23,22	1
x24,12	1

y1	85
y2	7
y3	38
y4	19
y5	82
y6	85
y7	30
y8	21
y9	25
y10	75
y11	59
y12	60
y13	85
y14	35
y15	15
y16	54
y17	56
y18	12
y19	47
y20	9
y21	77
y22	66
y23	48
y24	41

```
Explored 268128 nodes (3704223 simplex iterations) in 84.84 seconds
Thread count was 8 (of 8 available processors)
```



Other Experiments:
I tried to implement other data sets with different no of customers, no of vehicles and vehicle capacity, and following resulting routes are obtained:

Case 2: 10 vehicles, 34 nodes, vehicle capacity: 65. This gives the optimal solution for bigger problem.

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Explored 277437 nodes (4305877 simplex iterations) in 306.08 seconds
Thread count was 8 (of 8 available processors)
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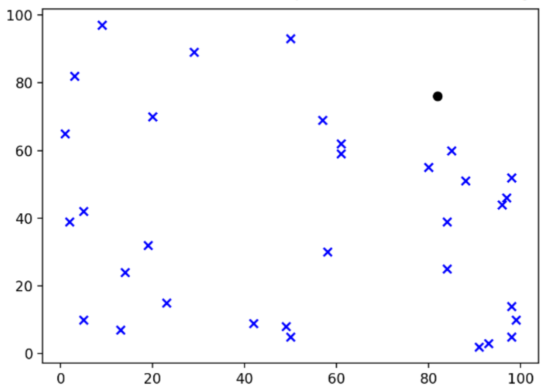

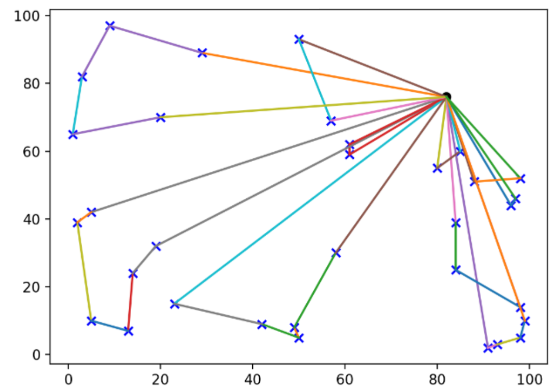

Case 3: 1 vehicle, 100 nodes, unlimited vehicle capacity.
When no. of vehicle is equals to ‘1’, this model is a TSP model and is capable of solving TSP problems.

```
Explored 26694 nodes (978942 simplex iterations) in 59.02 seconds
Thread count was 8 (of 8 available processors)
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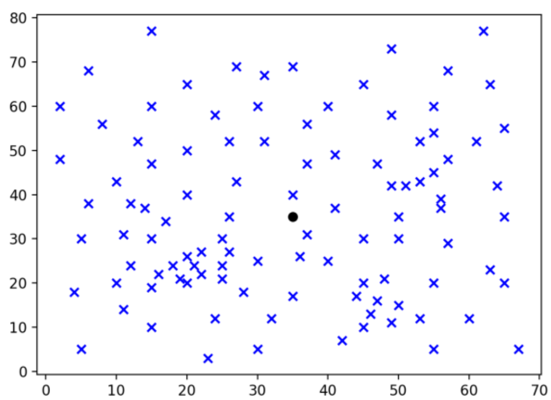

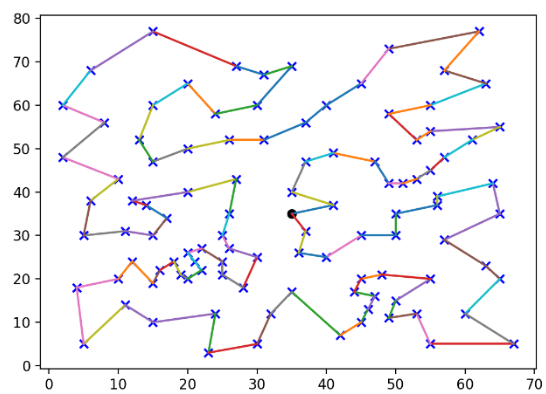

Conclusion:
MVRP modeling is very useful to determine routes for vehicles to reduce cost of travel. I formulated and solved a MVRP model with single depot, 25 nodes and 4 vehicles with Gurobi solver in Gurobi-Python interface finding that all vehicles can finish tours by covering minimum distance 399 miles. MVRP is a NP hard problem and computational time rises with raising size of the problem. This model is capable of solving MVRP problem for bigger scenarios as well as TSP problem when no of vehicle is just one. For real life businesses and other routing purposes, Optimal route plans with this model will minimize the fuel cost and travel duration of vehicles on day to day operation which is a huge economic and time saving advantage.