Vehicle routing problem with flexible time window

Problem statement: The problem encountered at the NAFTAL company is exactly a vehicle routing problem with flexible time windows. From a central depot CHIFFA (fuel distribution center), there are 30 trucks of homogeneous and limited capacity, the role of these trucks is to distribute and deliver fuels (gasoline-normal, without -plumb, diesel) to their customers. NAFTAL's customers are either: (service stations, police stations, hospitals, etc..) each customer is characterized by geographic coordinates (x, y), and by a demand d, defined, and by a time window ai, bi every customer desire to be served in his windows of time. it happens where the trucks can arrive earlier or before the opening of the service at this customer so he must wait for the start of the time window before starting the service. The objective of this problem is to find all the routes that can be carried out to meet customer demands while minimizing the total distance traveled by the trucks, while respecting the following constraints:

- Each customer must be served once and only once by a single vehicle.
- Each routing begins and ends at the depot.
- The capacity of vehicles must not be exceeded.
- The time windows for each client must be respected.

Problem formulation with mixed integer programming:

Minimize cost= Cost which consist of travelling costs + vehicle activation cost + tardiness penalty cost.

Indices

K- number of identical vehicles.

Cn- capacity of each identical

vehicle for n.fuel

N- number of customers.

L-number of Fuel

Parameters

c : the cost of traveling one unit distance of distance.

c_k: the cost of activating the vehicle k $\in K$.
e_i: earliest service time start time of customer i.

l_i: latest service start time of customer i.

P_{max}: the maximum allowance for violation of time windows of customer.

c_{ei}: the unit penalty for the service that begins before its earliest start time

c_{li}: the unit penalty for the service that begins after its latest start time

$q_{i,n}$: the demand of customer i. for n. fuel

d_{ij}: the distance between two different customers i and j.

t_{ij}: the travel time between two different customers i and j.

u_i: the service time for loading / unloading activities at customer i.

s_i: the start time at customer i.

Decision variables

The model formulation requires three groups of variables:

a. The first group of variables is binary and determines the sequence that vehicles visit customers:

$$X_{ij}^k = \begin{cases} 1 \text{ if customer } i \text{ is followed by customer } j \text{ in the sequence visited by } k \\ 0 \quad \text{otherwise} \end{cases}$$

b. The second group of variables is also binary and checks if the vehicle is active:

$$z_k = \begin{cases} 1 \text{ if vehicle } k \text{ is active} \\ 0 \quad \text{otherwise} \end{cases}$$

c. The third group determines the service start time of each customer and is represented with s_i.

s_i: the start time at customer i.

y_i: Variable that prevents subtours from occurring

Objective function

$$\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} cd_{ij}x_{ij}^{k} + \sum_{k=1}^{K} c_k\, z_k + \sum_{i=1}^{N}\left(c_{ei}\Delta_i^e + c_{li}\Delta_i^l\right)$$

Subject to

$$\sum_{i=1}^{N} x_{i0}^{k} = 1 \quad \forall\, k \qquad (1)$$

$$\sum_{i=1}^{N} x_{0i}^{k} = 1 \quad \forall\, k \qquad (2)$$

$$\sum_{i=0}^{N}\sum_{k=1}^{K} x_{ij}^{k} = 1 \quad \forall\, j \quad (3)$$

$$\sum_{j=0}^{N}\sum_{k=1}^{K} x_{ij}^{k} = 1 \quad \forall\, i \quad (4)$$

$$\sum_{i=0}^{N} x_{im}^{k} - \sum_{j=0}^{N} x_{mj}^{k} = 0 \quad \forall\, k, m \quad (5)$$

$$\sum_{i=0}^{N}\sum_{j=0}^{N} q_{in} * x_{i,j,k} \le C_n \qquad \forall \boldsymbol{n}, \forall \boldsymbol{k} \quad (6)$$

$$s_i \ge e_i - P_{max} \qquad \forall\, i \qquad (7)$$

$$s_i \le l_i + P_{max} \qquad \forall\, i \qquad (8)$$

$$\sum_{i=0}^{N}\sum_{j=0}^{N} x_{i,j,k} * \left(s_i + u_i + t_{ij}\right) = s_j \qquad \forall \boldsymbol{j} > \boldsymbol{0} (9)$$

$$s_0 = e_0 \qquad (9.1)$$

$$\Delta_i^e \ge e_i - s_i \quad \forall\, i \qquad (10)$$

$$\Delta_i^l \ge s_i - l_i \quad \forall\, i \qquad (11)$$

$$Y_j \geq Y_i + 1 - N\left(1 - \sum_{k=1}^{M} X_{ijk}\right), \qquad i \neq j, \quad \begin{matrix} i = 1, \dots, N, \\ j = 1, \dots, N, \end{matrix} \tag{12}$$

$$\sum_{i=0}^{N} \sum_{j=0}^{N} x_{i,j,k} \leq z_k \qquad \forall k \ (13)$$

$$\boldsymbol{x_{ij}^k, z_k \in \{0, 1\}}$$

$$\boldsymbol{\Delta_i^e, \Delta_i^l \geq 0}$$

Constraints (1) and (2) make sure that each route starts and terminates at the depot, other words at customer zero. Constraints (3) to (5) assure that exactly one vehicle enters, serves and leaves each customer. Constraints (6) indicates that vehicle capacity is not exceeded. Constraints (7) and (8) determine the lower and upper boundaries for extended service start time of each customer. Constraint (9) ensures that if the vehicle travels from i to j, service at j cannot start earlier than that at i. Constraint (9.1) allows us to set the start time. Constraints (10) and (11) determine the tardiness that will be penalized in the objective function. constraint (12) constraints the formation of sub-tours. are the constraints that prevent it. The decision variable Yj included in the constraints (12) is within the route.j = 1, . . . ,N can be interpreted as the position of the node.