

4) Data set  $D$  of size  $|D| = d$   
 $d$  times sampling with replacement  
from  $D$  gives a training set of  $d$  samples

Each data entry of  $D$  has probability  
of  $\frac{1}{d}$  to get sampled.

Sampling  $d$  times results in  $\left(\frac{1}{d}\right)^d$   
probability to get sampled (with  
replacement).

Or  $\left(1 - \frac{1}{d}\right)^d$  probability not being  
sampled.

$$\lim_{d \rightarrow \infty} \left(1 - \frac{1}{d}\right)^d = \frac{1}{e} \approx 0.368 \text{ not being}$$

$$\text{or } 1 - \frac{1}{e} = 0.632 \text{ being selected.}$$

Doubling drawn samples:

$$\Rightarrow \lim_{d \rightarrow \infty} \left(1 - \frac{1}{d}\right)^{2d} \approx \frac{1}{e^2} \approx 0.135 \text{ not being}$$

being selected or  $1 - 0.135 \approx 0.865$   
being selected.