# Single Matrix – Hamiltonian minimization with gradient

$\Omega$ is a $l_{max} \times l_{max}$ matrix.
It will include loops up to $2 l_{max} - 2 \equiv M$

Loops are normalized as follows:

$$\boxed{\phi_i = \frac{1}{N^{\frac{i}{2}+1}} \sum_{n}^{N} x_n^{\,i} \,, \quad \text{where } x_n \text{ are eigenvalues}}$$

Note
- $i = 0,1,2,\dots,M$ has $M-1 = 2l_{max} - 1$ loops including $\phi_0 = 1$
- $l_{max}$ = Omegasize in program

$$\Omega_{ij} = (i+1)(j+1)\,\phi_{i+j} \qquad , \; i,j = 0,1,2,\dots (l_{max}-1)$$

$$w_i = (i+1) \sum_{l=0}^{i-1} \phi_l \, \phi_{i-1-l} \,, \quad i = 1,2,\dots (l_{max}-1) \;;\; w_0 = 0$$

$(\ln J)_i$ solves $\Omega_{ij} (\ln J)_j = w_i \qquad i,j = 0,1,2,\dots,(l_{max}-1)$

Then

$$\boxed{V_{eff} = \frac{1}{8} \sum_i w_i (\ln J)_i + \frac{1}{2}\phi_2 + g\,\phi_4}$$

Note that with normalization above $V_{eff}$ is already of order 1

## Derivatives / Gradient

Note:
$$\frac{1}{8}\,\delta\left(w_i\, \Omega^{-1}_{ij}\, w_j\right) = \frac{1}{4}\,\delta w_i\, \Omega^{-1}_{ij} w_j - \frac{1}{8}\left(w\Omega^{-1}\right)_i \delta\Omega_{ij} \left(\Omega^{-1} w\right)_j$$

$$= \frac{1}{4}\,\delta w_i\,(\ln J)_i - \frac{1}{8}(\ln J)_i\,(\delta\Omega)_{ij}\,(\ln J)_j$$

So ②

(A) $$\Omega_{ij} = (i+1)(j+1)\,\phi_{i+j} = (i+1)(j+1)\sum_n \frac{x_n^{\,i+j}}{N^{\frac{i+j}{2}+1}}$$

$$\frac{\partial \Omega_{ij}}{\partial x_n} = (i+1)(j+1)(i+j)\,\frac{x_n^{\,i+j-1}}{N^{\frac{i+j+2}{2}}}$$

and $-\frac{1}{8}(\ln J)_i\,(\delta\Omega)_{ij}\,(\ln J)_j =$

$$\boxed{= -\left[\sum_{\substack{i,j=0,1,\ldots(l_{max}-1)\\ (i=j=0 \text{ excluded})}} (\ln J)_i\,\frac{(i+1)(j+1)(i+j)\,x_n^{\,i+j-1}}{N^{\frac{i+j+2}{2}}}\,(\ln J)_j\right]\Big/ 8}$$

(B) $\frac{1}{8}\,2\,\delta w_i\,(\ln J)_i$ :

$$w_i = (i+1)\sum_{l=0}^{i-1} \phi_l\,\phi_{i-1-l} \; , =$$

$$= (i+1)\sum_{l=0}^{i-1}\sum_n \frac{x_n^{\,l}}{N^{\frac{l+2}{2}}}\,\phi_{i-1-l}\,; \quad i=1,2,\ldots(l_{max}-1)$$
$$w_0 = 0$$

Then:

$$\frac{\partial w_i}{\partial x_n} = 2(i+1)\sum_{l=1}^{i-1} \frac{l\,x_n^{\,l-1}}{N^{\frac{l+2}{2}}}\,\phi_{i-1-l}$$

↑ note ($l=1$)

and

$$\boxed{\frac{1}{8}\,2\,\delta w_i\,(\ln J)_i = \frac{1}{8}\left\{\sum_{i=2}^{l_{max}-1} 4(i+1)\sum_{l=1}^{i-1}\frac{l\,x_n^{\,l-1}}{N^{\frac{l+2}{2}}}\,\phi_{i-1-l}\,(\ln J)_i\right\}}$$

↑ note (as $\delta w_1 = 0$) ($i=2$)

(C) Potential term.

$$\frac{\partial}{\partial x_n}\left(\frac{1}{2N^2}\sum_n x_n^2 + \frac{g}{N^3}\sum_n x_n^4\right) =$$

$$\boxed{= \frac{1}{N^2}x_n + \frac{4g}{N^3}x_n^3} \longrightarrow$$