

Đề 1

Câu 1

$$A = \begin{pmatrix} 2 & 1 & 1 & x \\ 1 & 2 & 1 & y \\ 1 & 1 & 2 & z \\ 1 & 1 & 1 & t \end{pmatrix} \xrightarrow{h_1 = h_4} \begin{pmatrix} 1 & 1 & 1 & t \\ 1 & 2 & 1 & y \\ 1 & 1 & 2 & z \\ 2 & 1 & 1 & x \end{pmatrix} \xrightarrow{\begin{matrix} h_2 = h_2 - h_1 \\ h_3 = h_3 - h_1 \\ h_4 = h_4 - 2h_1 \end{matrix}} \begin{pmatrix} 1 & 1 & 1 & t \\ 0 & 1 & 0 & y-t \\ 0 & 0 & 1 & z-t \\ 0 & -1 & -1 & x-2t \end{pmatrix}$$

$$\underline{h_4 = h_4 + h_3} \rightarrow \begin{pmatrix} 1 & 1 & 1 & t \\ 0 & 1 & 0 & y-t \\ 0 & 0 & 1 & z-t \\ 0 & 0 & 0 & x+y+z-4t \end{pmatrix} = 4t - x - y - z$$

Câu 2

$$A^{bs} = \begin{pmatrix} -1 & -1 & 1 & 1 \\ 2 & 3 & m & 3 \\ 1 & m & 3 & 2 \end{pmatrix} \xrightarrow{\begin{matrix} h_2 = h_2 + 2h_1 \\ h_3 = h_3 + h_1 \end{matrix}} \begin{pmatrix} -1 & -1 & 1 & 1 \\ 0 & 1 & m+2 & 5 \\ 0 & m+1 & 4 & 3 \end{pmatrix} \rightarrow$$

$$h_3 = h_3 - (m+1)h_2 \rightarrow \begin{pmatrix} -1 & -1 & 1 & 1 \\ 0 & 1 & m+2 & 5 \\ 0 & 0 & (m+3)(2-m) & 8-5m \end{pmatrix} (*)$$

$$BL: \rightarrow (m+3)(2-m) = 0 \Leftrightarrow \begin{cases} -m = 2 \\ m = -3 \end{cases} \Rightarrow r(A) = 2 < r(A^k) = 3 \Rightarrow \text{hệ pt V/N}$$

$$\rightarrow (m+3)(2-m) \neq 0 \Leftrightarrow \begin{cases} m \neq 2 \\ m \neq -3 \end{cases} \Rightarrow r(A) = 3 = r(A^k) = \text{số ẩn} \Rightarrow \text{hệ pt có nghiệm}$$

$$(*) \Rightarrow \begin{cases} x_1 = \frac{m-4}{2-m} \\ x_2 = \frac{14-3m}{(2-m)(m+3)} \\ x_3 = \frac{8-5m}{(2-m)(m+3)} \end{cases}$$

Câu 3 Tập \$S = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}\$. Lập ma trận \$A\$ như sau:

$$A = \begin{pmatrix} \alpha_5 \\ \alpha_2 \\ \alpha_1 \\ \alpha_4 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 & 5 & -6 \\ 4 & -2 & 3 & 7 & 2 \\ 5 & -3 & 2 & 4 & 1 \\ 7 & -3 & 7 & 12 & 4 \\ 8 & -6 & -1 & -5 & -2 \end{pmatrix} \xrightarrow{\begin{matrix} h_5 = h_5 - h_4 + h_1 \\ h_4 = h_4 - h_3 + 2h_1 \\ h_3 = h_3 - h_2 + h_1 \\ h_2 = h_2 + 4h_1 \end{matrix}} \begin{pmatrix} -1 & 0 & 1 & 5 & -6 \\ 0 & -2 & 7 & 27 & -22 \\ 0 & -1 & 0 & 2 & -7 \\ 0 & 0 & 7 & 23 & -9 \\ 0 & -3 & -7 & -12 & -12 \end{pmatrix}$$

$$\xrightarrow{h_2 \leftrightarrow h_3} \begin{pmatrix} -1 & 0 & 1 & 5 & -6 \\ 0 & -1 & 0 & 2 & -7 \\ 0 & -2 & 7 & 27 & -22 \\ 0 & 0 & 7 & 23 & -9 \\ 0 & -3 & -7 & -12 & -12 \end{pmatrix} \xrightarrow{\begin{matrix} h_3 = h_3 - 2h_2 \\ h_5 = h_5 - 3h_2 \end{matrix}} \begin{pmatrix} -1 & 0 & 1 & 5 & -6 \\ 0 & -1 & 0 & 2 & -7 \\ 0 & 0 & 7 & 23 & -8 \\ 0 & 0 & 7 & 23 & -9 \\ 0 & 0 & 7 & 23 & -9 \end{pmatrix} \xrightarrow{\begin{matrix} h_5 = h_5 - h_4 \\ h_4 = h_4 - h_3 \end{matrix}} \begin{pmatrix} -1 & 0 & 1 & 5 & -6 \\ 0 & -1 & 0 & 2 & -7 \\ 0 & 0 & 7 & 23 & -8 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow r(A) = 4 \rightarrow r(S) = 4 < 5 = \text{số vectơ} \Rightarrow \text{hệ } S \text{ phụ thuộc}$$

Đề 2

Câu 1

$$|A| = \begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ a & b & c & d \\ -1 & -1 & 1 & 0 \end{vmatrix} \xrightarrow{h_3 = h_3 - ah_1} \begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & b & c+a & d+a \\ 0 & -1 & 0 & -1 \end{vmatrix} \xrightarrow{h_4 = h_4 + h_1} \begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & b & c+a & d+a \\ 0 & 0 & -1 & 0 \end{vmatrix} \xrightarrow{h_3 = h_3 + bh_2} \begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & c+a+b & d+a+b \\ 0 & 0 & -1 & 0 \end{vmatrix}$$

$$\xrightarrow{h_3 \leftrightarrow h_4} \begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & c+a+b & d+a+b \end{vmatrix} \xrightarrow{h_4 = h_4 - (c+a+b)h_3} \begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 3a-b+2c+d \end{vmatrix} = 3a-b+2c+d$$

Câu 2

$$\begin{cases} -2x_3 - x_1 - 2x_2 = 1 \\ 2x_1 + (5-m)x_3 - (m-2)x_2 = -2 \\ x_2 + mx_1 + (m+1)x_3 = -2 \end{cases} \Leftrightarrow \begin{cases} -x_1 - 2x_2 - 2x_3 = 1 \\ 2x_1 + (2-m)x_2 + (5-m)x_3 = -2 \\ mx_1 + x_2 + (m+1)x_3 = -2 \end{cases}$$

$$A^{bs} = \left(\begin{array}{ccc|c} -1 & -2 & -2 & 1 \\ 2 & 2-m & 5-m & -2 \\ m & 1 & m+1 & -2 \end{array} \right) \xrightarrow{\begin{array}{l} h_2 = h_2 + 2h_1 \\ h_3 = h_3 + mh_1 \end{array}} \left(\begin{array}{ccc|c} -1 & -2 & -2 & 1 \\ 0 & -2-m & 1-m & 0 \\ 0 & 1-2m & 1-m & m-2 \end{array} \right)$$

$$\xrightarrow{h_2 \leftrightarrow h_3} \left(\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 1-2m & 1-m & m-2 \\ 0 & -2-m & 1-m & 0 \end{array} \right) \xrightarrow{h_2 = h_2 - 2h_3} \left(\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 5 & m-1 & m-2 \\ 0 & -2-m & 1-m & 0 \end{array} \right) \xrightarrow{h_3 = 5h_3 + (m+2)h_2} \left(\begin{array}{ccc|c} 1 & -2 & -2 & 1 \\ 0 & 5 & m-1 & m-2 \\ 0 & 0 & (1-m)(3-m) & m^2-4 \end{array} \right) (*)$$

ĐL: $\begin{cases} m=1 \\ m=3 \end{cases} \Rightarrow r(A)=2 < r(A^{bs})=3 \Rightarrow \text{hệ pt VV}$

$\Rightarrow \begin{cases} m \neq 1 \\ m \neq 3 \end{cases} \Rightarrow r(A)=3 = r(A^{bs}) = \text{số 'cũ'} \Rightarrow \text{hệ có nghiệm duy nhất}$

$$(*) \Rightarrow \begin{cases} x_1 = \frac{-m^2 - 2m + 8}{(1-m)(3-m)} \\ x_2 = \frac{m-2}{3-m} \\ x_3 = \frac{m^2-4}{(1-m)(3-m)} \end{cases}$$

Câu 3. Dãy $S = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$. Lập ma trận A

$$A = \begin{pmatrix} 2 & -1 & 4 & 0 & 3 \\ -2 & 0 & 1 & -6 & 1 \\ 5 & -3 & -2 & 0 & -4 \\ 3 & -1 & 2 & -2 & -1 \\ -2 & 0 & 0 & m & -3 \end{pmatrix} \begin{array}{l} h_2 = h_2 + h_1 \\ h_3 = 2h_3 - 5h_1 \\ h_4 = 2h_4 - 3h_1 \\ h_5 = h_5 + h_1 \end{array} \rightarrow \begin{pmatrix} 2 & -1 & 4 & 0 & 3 \\ 0 & -1 & 5 & -6 & 4 \\ 0 & -24 & -24 & 0 & -23 \\ 0 & 1 & -8 & -4 & -11 \\ 0 & -1 & 4 & m & 0 \end{pmatrix}$$

$$\begin{array}{l} h_3 = h_3 - h_2 \\ h_4 = h_4 + h_2 \\ h_5 = h_5 - h_2 \end{array} \rightarrow \begin{pmatrix} 2 & -1 & 4 & 0 & 3 \\ 0 & -1 & 5 & -6 & 4 \\ 0 & -29 & 6 & -27 \\ 0 & 0 & -3 & -10 & -7 \\ 0 & 0 & -1 & m+6 & -4 \end{pmatrix} \begin{array}{l} h_3 = 3h_3 - h_4 \\ h_4 = h_4 - 29 - 3h_3 \end{array} \rightarrow \begin{pmatrix} 2 & -1 & 4 & 0 & 3 \\ 0 & -1 & 5 & -6 & 4 \\ 0 & 0 & -29 & 6 & -27 \\ 0 & 0 & 0 & -308 & -122 \\ 0 & 0 & 0 & 3m+28 & -5 \end{pmatrix}$$

$$h_5 = 308h_3 + (3m+28)h_4 \rightarrow \begin{pmatrix} 2 & -1 & 4 & 0 & 3 \\ 0 & -1 & 5 & -6 & 4 \\ 0 & 0 & -29 & 6 & -27 \\ 0 & 0 & 0 & -308 & -122 \\ 0 & 0 & 0 & 0 & -4956 - 366m \end{pmatrix}$$

+> Nếu $-4956 - 366m = 0 \Leftrightarrow m = \frac{826}{61} \rightarrow S$ phụ thuộc tuyến tính

+> Nếu $m \neq \frac{826}{61} : S$ độc lập tuyến tính