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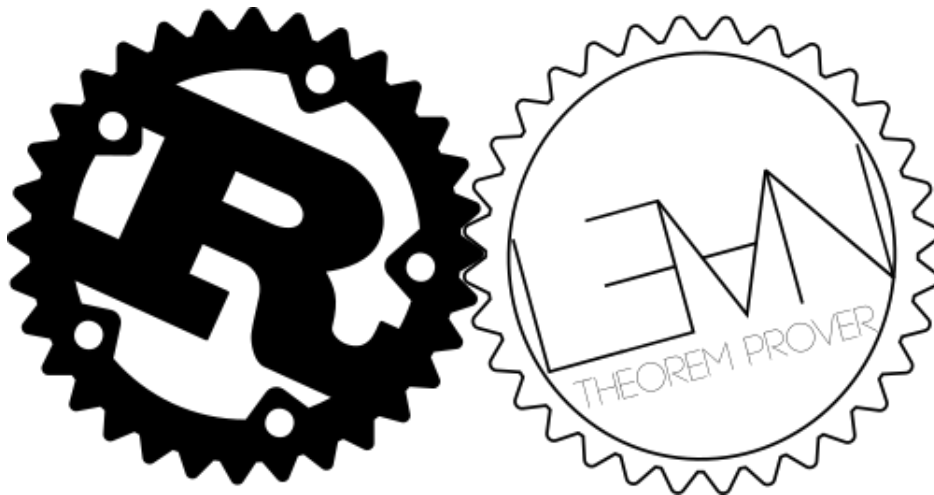und Datenorganisation (IPD)

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Simple Verification of Rust Programs via Functional Purification

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Einfache Verifikation von Rust-Programmen

Imperative Programmiersprachen sind in der modernen Softwareentwicklung allgegenwärtig, stellen aber ein Hindernis für formale Softwareverifikation dar durch ihre Verwendung von veränderbaren Variablen und Objekten. Programme in diesen Sprachen können normalerweise nicht direkt auf die unveränderliche Welt von Logik und Mathematik zurückgeführt werden, sondern müssen in eine explizit modellierte Semantik der jeweiligen Sprache eingebettet werden. Diese Indirektion stellt ein Problem für die Benutzung von interaktiven Theorembeweisern dar, da sie die Entwicklung von neuen Werkzeugen, Taktiken und Logiken für diese "innere" Sprache bedingt.

Die vorliegende Arbeit stellt einen Compiler von der imperativen Programmiersprache Rust in die pur funktionale Sprache des Theorembeweisers Lean vor, der nicht nur generell das erste Werkzeug zur Verifikation von Rust-Programmen darstellt, sondern diese insbesondere auch mithilfe der von Lean bereitgestellten Standardwerkzeugen und -logik ermöglicht. Diese Transformation ist nur möglich durch spezielle Eigenschaften von allen validen Rust-Programmen, die die Veränderbarkeit von Werten auf begrenzte Geltungsbereiche einschränken und statisch durch Rusts Typsystem garantiert werden. Die Arbeit demonstriert den Einsatz des Compilers anhand der Verifikation von Realbeispielen und zeigt die Erweiterbarkeit des Projekts über reine Verifikation hinaus am Beispiel von asymptotischer Laufzeitanalyse auf.

Abstract

Imperative programming, and aliasing in particular, represents a major obstacle in formally reasoning about everyday code. By utilizing restrictions the imperative programming language Rust imposes on mutable aliasing, we present a scheme for transforming a substantial part of the Rust language into the purely functional language of the Lean theorem prover. We use this scheme to verify the correctness of real-world examples of Rust code without the need for special semantics or logics. We furthermore show the extensibility of our transformation by incorporating an analysis of asymptotic runtimes.

Contents

1 Introduction

Imperative programming languages are ubiquitous in today's software development, making them prime targets for formal reasoning. Unfortunately, their semantics differ from those of mathematics and logic – the languages of formal methods – in some significant details, starting with the very concept of "variables". The problem of mutability is only exacerbated for languages that allow references to *alias*, or point to the same memory location, enabling non-local mutation.

The standard way of verifying programs in such languages with the help of an interactive theorem prover is to explicitly model the semantics of the language in the language of the theorem prover, then translate the program to this representation (a "deep" embedding) and finally prove the correctness of its formalized behavior. This general approach is very flexible and allows for the verification of meta programs such as program transformations. The downside of the approach is that the theorem prover's tools and tactics may not be directly applicable to the embedded language, defeating many amenities of modern theorem provers. Alternatively, programs can be "shallowly" embedded by directly translating them into terms in the theorem prover's language without the use of an explicit inner semantics. This simplifies many semantic details such as the identification and substitution of bound variables, but is harder to accomplish the more the semantics of the source language differs from the theorem prover's own semantics.

Regardless of the type of embedding, an explicit heap that references can point into must generally be modeled and passed around in order to deal with the aliasing problem. References in this model may be as simple as indices into a uniform heap, but various logics such as separation logic [15] have been developed to work on a more abstract representation and to express aliasing-free sets of references.

> verification condition generators? Move to Related Work?

Languages with more restricted forms of aliasing exist, however. Rust [12], a new, imperative systems programming language, imposes on mutable references the restriction of never being aliased by any other reference, mutable or immutable. This restriction eliminates the possibility of data races and other common bugs created by the presence of mutable sharing such as iterator invalidation. It furthermore enables more aggressive optimizations.

While the full Rust language also provides raw pointers, which are not bound by the aliasing restriction, and other "unsafe" operations, a (informal or formal) memory model for Rust has yet to be proposed. We therefore focus on the "safe" subset of Rust that has no unsolved semantic details.

We utilize safe Rust's aliasing restriction to design a monadic shallow embedding of a substantial subset of Rust into the purely functional language

of the Lean [6] theorem prover without the need for any heap-like indirection. This allows us to reason about unannotated, real-world Rust code in mostly the same manner one would reason about native Lean definitions. The monadic approach gives us further flexibility in modeling additional effects such as function runtime.

We first discuss the simpler cases of the translation, notably excluding mutable references, in Section 4 and show their application by giving a formal verification of Rust's `std::[T]::binary_search` method in Section 5. Section 6 discusses the translation of most usages of mutable references, which is used in Section 7 for a verification of the `fixedbitset` crate.

2   Related Work

While this thesis presents the first verification tool for Rust programs, tools for many other imperative languages have been developed before.

The Why3 project [3] is notable for its generality. It provides an imperative ML-like language *WhyML* together with a verification condition generator that can interface with a multitude of both automatic and interactive theorem provers. While WhyML supports advanced language features such as type polymorphism and exceptions, it does not support higher-order functions, which are ubiquitous in Rust code. WhyML provides a reference type `ref` that can point to a fresh cell on the heap and is statically checked not to alias with other `ref` instances, but cannot point into some existing datum like Rust references can. For example, the first of the following two WhyML functions fails to type check because the array elements are not known to be alias-free, while the second one will return a reference to a *copy* of `a[i]`.

```
let get_mut (a : array (ref int)) (i : int) : ref int = a[i]
let get_mut (a : array int) (i : int) : ref int = ref a[i]
```

WhyML is also being used as an intermediate language for the verification of programs in Ada [10], C [5] and Java [7]. For the latter two languages, aliasing is reintroduced by way of an explicit heap.

The remarkable SeL4 project [11] delivers a full formal verification of an operating system microkernel by way of multiple levels of program verification and refinement steps. The C code that produces the final kernel binary is verified by embedding it into the theorem prover Isabelle/HOL [13], using a deep embedding for statements and a shallow one for expressions. The C memory model used allows type-unsafe operations by use of a byte-size heap, but as with Why3, higher-order functions are not supported. The AutoCorres [8, 9] tool then transforms this representation into a shallow monadic embedding, dealing with the 'uninteresting complexities of C' [9] on the way. The result is an abstracted representation that is quite similar to ours (and in fact inspired it in part, as we shall note below), but doesn't go the last mile of completely eliminating the heap where possible. Without explicit no-alias annotations, the semantics of C would allow that in far fewer places than those of Rust in any case.

It should be noted that our work, like most verification projects based on either embedding or code extraction, relies on both an unverified compiler and an unverified embedding tool, effectively making both part of the trusted computing base. SeL4 is a notable exception in this, providing (at lower optimization levels) a direct equivalence proof [16] between the produced

kernel binary and the verified embedded code, thus completely removing the original C code from the trusted computing base.

```rust
struct Point { x: u32, y: u32 }
enum Option<T> { None, Some(T) }

fn map<S, T, F: Fn(S) -> T>(opt: Option<S>, f: F) -> Option<T> {
    match opt {
        Option::None    => Option::None,
        Option::Some(s) => Option::Some(f(s)),
    }
}
```

Figure 1: A first example of functional programming in Rust, showing algebraic data types, polymorphic and higher-order functions, pattern matching, type inference and the expression-oriented syntax

3 Background

We start by giving a basic introduction to our source and target languages, focusing on the parts relevant to our work. We will discuss finer semantic details where needed in Section 4 and Section 6.

3.1 Rust

Rust [12] is a modern, multi-paradigm systems programming language sponsored by Mozilla Research and developed as an open-source community effort. Rust is still a quite young language, with its first stable version having been released on May 15, 2015. The two biggest Rust project as of this writing are the Servo[1] [1] web browser engine and the Rust compiler `rustc`[2] itself.

As a partly functional language, Rust is primarily inspired by ML and shares much of its syntax, as evidenced in Figure 1. However, the syntax also shows influences by C, the dominant systems programming language of the present. Finally, Rust also features a *trait* system modeled after Haskell's type classes.

Many features of Rust other than the syntax can be explained by Rust's desire to feature an ML-like abstraction level while still running as efficiently as C, even on resource-constrained systems that may not allow dynamic allocation at all. Most prominently, Rust uses manual memory management just like C and C++, but guarantees memory safety through its *ownership* and *borrowing* systems. Rust also features an *unsafe* language subset that allows everything-goes programming on the level of C, but which is usually

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[1]https://github.com/servo/servo
[2]https://github.com/rust-lang/rust

reserved for implementing low-level primitives on which the *safe* part of the language can then build. In general, safe Rust is (thought to be) a type-safe language like ML and unlike either C or C++. We focus on safe Rust in the following and in our work in order to peruse these guarantees.

Ownership describes the application of *linear types* to memory management as proposed by Wadler [17]. The owner of a Rust object is the binding that is responsible for freeing the object's resources (by calling a method of the `Drop` trait), which generally happens at the end of the binding's scope. Because an object managing resources should only ever have one owner, types that implement `Drop` are linear types: A value may only be used once, at which point it is consumed and ownership is transferred to its new binding[3]. In the following example, we extract an element from a `Vec` (a dynamically-sized array type that has to free heap space in its `Drop` implementation), after which we are not permitted to use the `Vec` again.

```rust
fn get<T>(v: Vec<T>, idx: usize) -> T {
    v[idx]
    // v will be freed here
}

let v: Vec<u32> = vec![1];
let x = get(v, 0);
// get(v, 1); // error[E0382]: use of moved value: `v`
```

One way to retain access to the `Vec` would be to also return it from the function, regaining ownership. However, since `T` in general is an linear type too, `get` would have to remove the indexed element before returning the `Vec`.

A much better alternative is to use *references*, which provide standard pointer indirection. Because a reference does not take ownership of the pointee, creating it is also called *borrowing*.

```rust
fn get<T>(v: &Vec<T>, idx: usize) -> &T {
    &v[idx]
}

let v: Vec<u32> = vec![1];
let x = get(&v, 0); // x: &u32
```

Here `&T` represents an immutable reference to a value of type `T`. Note that the compiler would stop us if we tried to return `v[idx]` by value:

---

[3]Technically, because leaking resources (i.e. not consuming the object at all) is a safe operation in Rust, such types are merely *affine*. However, the distinction is not relevant for our purposes.

```
error[E0507]: cannot move out of indexed content
```

Still, coming from other languages with manual memory management, this might look like a potentially unsafe thing to do: The function signature does not tell the callee that the returned reference is only valid as long as the `Vec`. Even Wadler tells us that a temporary reference to a linear value must be checked not to escape from the local scope. Indeed, it seems like the following program should produce a dangling pointer.

```rust
fn dangling() -> u32 {
    let x = {
        let v: Vec<u32> = vec![1];
        get(&v, 0)
        // v will be freed here
    };
    *x
}
```

However, the Rust compiler will stop us from doing this, printing an elaborate error message:

```
error: `v` does not live long enough
|
|         get(&v, 0)
|              ^ does not live long enough
|     };
|     - borrowed value only lives until here
|     *x
| }
| - borrowed value needs to live until here
```

The compiler must have had some information about the relationship of `x` and `v` in order to deduce this without resorting to inter-procedural analysis. It turns out that the full signature of the `get` function is as follows:

```rust
fn get<'a, T>(v: &'a Vec<T>, idx: usize) -> &'a T
```

`'a` is called a *formal lifetime parameter*. It specifies that the returned reference is indeed only valid as long as the first argument. By integrating lifetimes into the type system like this, Rust can reason about references even when confronted with complex, inter-procedural, higher-order reference lifetime relations.

While we have solved the dangling pointer problem for immutable data, mutability as so often aggravates the problem.

```rust
fn dangling2() -> u32 {
    let mut v: Vec<u32> = vec![1];
    let x = get(&v, 0);
    // remove all elements from v
    v.clear(); // shorthand for (&mut v).clear();
    *x
    // v will be freed here
}
```

By clearing the vector while we still hold a reference to its content, we should again produce a dangling pointer – even though this time, v indeed outlives x. Fortunately, the Rust compiler will again stop us:

```
error[E0502]: cannot borrow `v` as mutable because it is also borrowed
↪  as immutable
|
|     let x = get(&v, 0);
|                 - immutable borrow occurs here
|     // remove all elements from v
|     v.clear(); // shorthand for (&mut v).clear();
|     ^ mutable borrow occurs here
|     *x
| }
| - immutable borrow ends here
```

We have finally arrived at the aliasing problem: In a language with manual memory management, we can create type unsafety through the mere existence of two pointers, at least one of them mutable, to the same datum. Thus, Rust detects and forbids any occurrences of mutable aliasing, as shown above.

The beauty of forbidding mutable aliasing is that it solves many sources of bugs in imperative programs even outside of managed memory management. Indeed, as Wadler notes, it makes mutable references safe even in a referentially transparent language: "In order for destructive updating of a value to be safe, it is essential that there be only one reference to the value when the update occurs" [17]. While Rust does introduce APIs such as for I/O that break referential transparency, the absence of mutable aliasing still provides safety guarantees that are usually only attributed to purely functional languages, first and foremost among them the elimination of data races. By focusing on a subset of Rust and its APIs that is truly referentially transparent, we obtain a sufficiently narrow gap between Rust and the purely functional language Lean that our transformation between them becomes feasible.

3.2 Lean

The Lean [6] theorem prover is an open source, dependently typed, interactive
theorem prover developed jointly at Microsoft Research and Carnegie Mellon
University. The first official release of Lean was announced at CADE-25 in
August 2015, making it just a few months younger than Rust. As of this
writing, development on Lean is focused on the next, unreleased version that
will feature powerful automation written in Lean itself.

Lean supports two different interpretations of Martin-Löf type theory: a
purely constructive one based on Homotopy Type Theory, and one based on
the Calculus of Inductive Constructions [4, 14] as championed by the Coq [2]
theorem prover, which supports both constructive and classical reasoning.
We use the latter in our work.

The primitive type in dependent type theory is the dependent function
(or product) type $\Pi x : A, B$, where x may occur in B; if it does not, we obtain
the standard function type $A \rightarrow B$. Function abstraction and application
extend naturally to dependent functions, as perhaps best described by their
formal typing rules.

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash (\lambda x : A, e) : \Pi x : A, B} \qquad \frac{\Gamma \vdash f : \Pi x : A, B \qquad \Gamma \vdash e : A}{\Gamma \vdash f e : [e/x]B}$$

The Calculus of Inductive Constructions extends basic dependent type
theory with a type scheme for inductive types, which are described by a set
of (possibly dependent) functions, their *constructors*. Figure 2 shows basic
inductive definitions from the Lean standard library.

As we can see in Figure 2, inductive types themselves are instances of
a type, namely `Type`. This turns out to be a slight simplification, however.
More specifically, Lean has a whole hierarchy of indexed types or *universes*
`Type.{i}`, with `Type.{i}` : `Type.{i+1}`[4]. The universe hierarchy is needed to
avoid the type theoretic equivalent of Russell's paradox. When we write
just `Type` for the type of an inductive definition like in Figure 2, a correct
universe level $i \geq 1$ (possibly dependent on argument universe levels) will
automatically be inferred. The reason for skipping `Type.{0}` is that it has
a special function that is suggested by its more common name, `Prop`: It
is the universe we normally declare types in that are to be interpreted as
propositions.

Under the Curry-Howard isomorphism, an objects of a type can be inter-
preted as a proof of a proposition. The reason Lean uses a separate universe

---

[4]The hierarchy is, however, not cumulative: It is not true that `Type.{i}` :
`Type.{i+2}`.

```
inductive empty : Type

inductive unit : Type :=
star : unit

inductive prod (A B : Type) : Type :=
mk : A → B → prod A B

inductive sum (A B : Type) : Type :=
| inl : A → sum A B
| inr : B → sum A B

-- the dependent sum type
inductive sigma (A : Type) (B : A → Type) : Type :=
mk : Π(x : A), B x → sigma A B

inductive bool : Type :=
| ff : bool
| tt : bool

inductive option (A : Type) : Type :=
| none : option A
| some : A → option A

inductive nat : Type :=
| zero : nat
| succ : nat → nat
```

Figure 2: The most basic inductive types as well as some basic types from functional programming

type name (notation)	
in **Type**	in **Prop**
empty	false
unit	true
prod (×)	and (∧)
sum (+)	or (∨)
sigma (Σ x : A, B)	Exists (∃ x : A, B)
Π x : A, B	∀ x : A, B
A → B	A → B

Table 1: The basic Curry-Howard correspondence. The table lists types from Figure 2 and the corresponding types from the standard library with the same constructors, but declared in **Prop**. We also show their notations as well as the special universal quantifier notation for dependent functions into **Prop**. Nondependent functions and implications are not distinguished by notation.

for this interpretation is that **Prop** is given a specific property that would not make sense for the other universes: By *proof irrelevance*, any two objects of a type in **Prop** are definitionally equal. In other words, proofs are irrelevant for computation. Finally, **Prop** is also *impredicative*: If B : **Prop**, then also (Π x : A, B) : **Prop** for any A. This property ensures that predicates and universal quantifications are still propositions. The separation of inductive types and inductive propositions can lead to some duplication, which however turns out to be very useful in ensuring suggestive names and notations for each side (Table 1).

On top of its interpretation of dependent type theory, Lean includes many notational amenities. On the type level, in addition to basic and inductive definitions, it features syntactic **abbreviation**s as well as **structure**s. The latter are single-constructor inductive types that automatically define projections to each of their constructor parameters (or *fields*) and furthermore support inheriting fields from other structures.

```
structure point2 :=
(x : ℕ)
(y : ℕ)

structure point3 extends point2 :=
(z : ℕ)

example : point2 := ⦃point2, x := 0, y := 1⦄
check point3.x -- point3.x : point3 → ℕ
```

In addition to the standard parameter syntax `(x : A)`, Lean also supports two more binding modes, `{x : A}` and `[x : A]`. In the first one, `x` is an *implicit* parameter and will be inferred from other parameters or the expected result type, such as in the constructor of the ubiquitous type `eq` modeling Leibniz equality:

```
inductive eq {A : Type} (a : A) : A → Prop :=
refl : eq a a -- explicit form: @eq A a a
```

The binding mode `[x : A]` instructs Lean to infer `x` by *type class inference.* Type classes are arbitrary definitions annotated with the `[class]` attribute. Type class inference synthesizes instances of a class by a Prolog-like search through definitions of the class type marked with `[instance]`.

```
structure inhabited [class] (A : Type) : Type :=
(value : A)

definition default (A : Type) [inhabited A] : A :=
inhabited.value A

definition nat.is_inhabited [instance] : inhabited ℕ :=
⦃inhabited, value := 0⦄
definition prod.is_inhabited [instance] (A B : Type)
  [inhabited A] [inhabited B] : inhabited (A × B) :=
⦃inhabited, value := (default A, default B)⦄

eval default (ℕ × ℕ) -- (0, 0)
```

extend ad
nauseam when
needed

I have no idea where to put the complexity analysis

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Erklärung

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