

# Digital Image Processing

## CS390S

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# Day 2

- Image (color gray binary), dimension
- Pixel, intensity
- Pixel relationship, distance
- **Matlab, Matlab tutorial**
- [https://www.tutorialspoint.com/matlab/matlab\\_environment.htm](https://www.tutorialspoint.com/matlab/matlab_environment.htm)
- **Digital Image Processing using Matlab:** Chapter 2 2.1-2.8
- Start Basic Image Enhancement Tools (pdf)

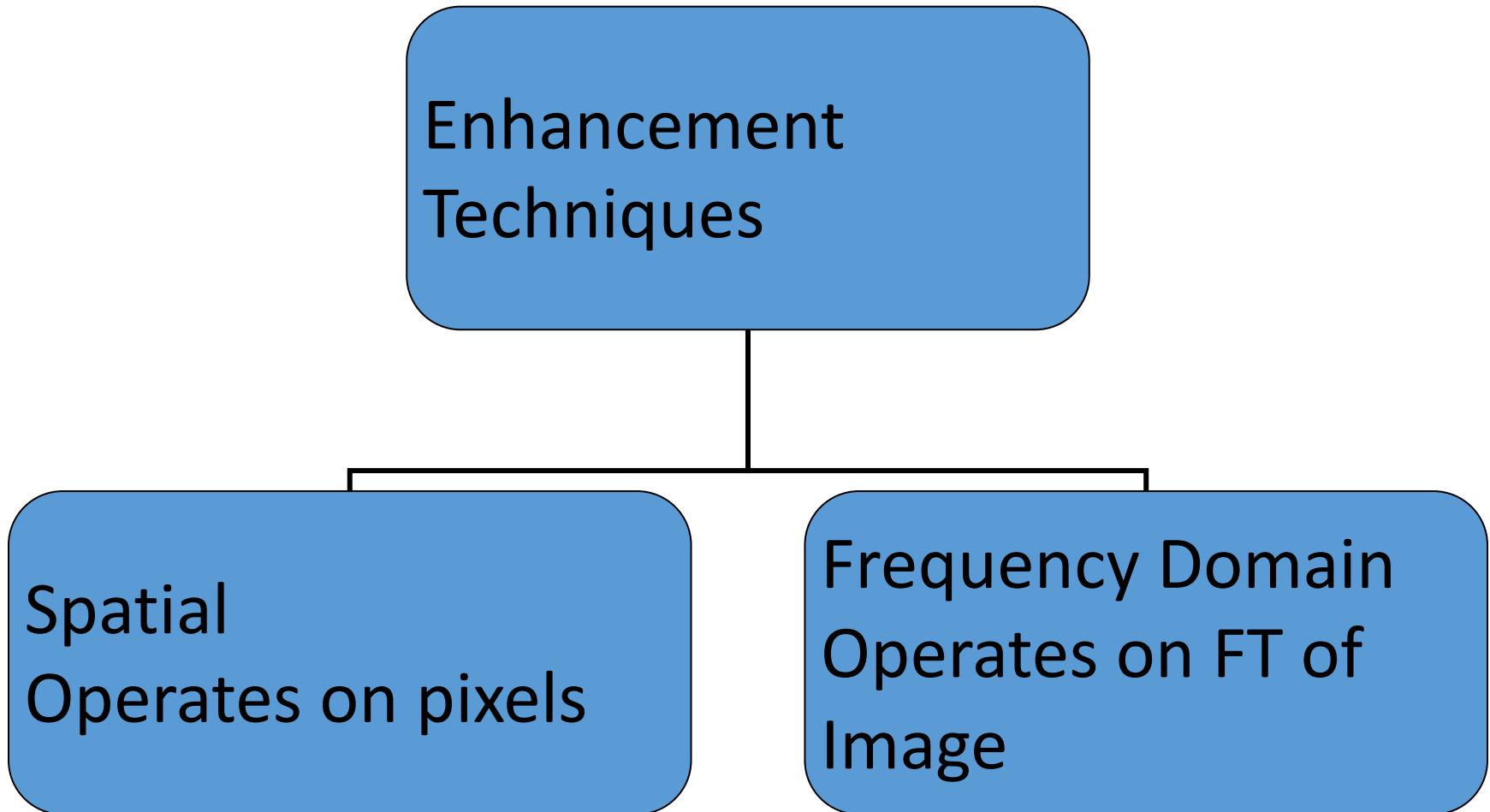
# Day 3

- Image (color gray binary), dimension
- Pixel, intensity
- Pixel relationship, distance
- **Matlab, Matlab tutorial**
- [https://www.tutorialspoint.com/matlab/matlab\\_environment.htm](https://www.tutorialspoint.com/matlab/matlab_environment.htm)
- **Digital Image Processing using Matlab: Chapter 2 2.1-2.8**
- **Basic Image Enhancement Tools (pdf)**
- **Assignment 1**

# Image Enhancement in Spatial Domain

- Image Histogram
- Image Negative
- Contrast Stretching
- Power-Law Transformation
- Histogram Equalization
- Local Enhancement
- Image Subtraction
- Image Averaging
- Image Smoothing
- Image Sharpening

# Image enhancement



# Spatial Domain Methods

- In these methods a operation (linear or non-linear) is performed on the pixels in the neighborhood of coordinate (x,y) in the input image F, giving enhanced image F'
- Neighborhood can be any shape but generally it is rectangular ( 3x3, 5x5, 9x9 etc)

$$g(x,y) = T[f(x,y)]$$

# Image negative

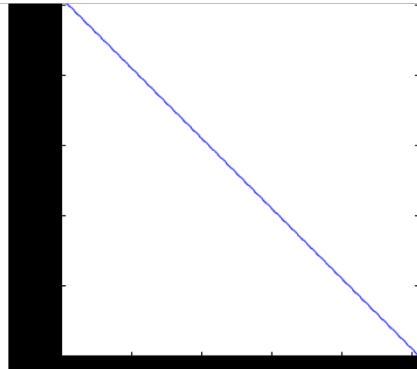


a b

**FIGURE 3.4**  
(a) Original  
digital  
mammogram.  
(b) Negative  
image obtained  
using the negative  
transformation in  
Eq. (3.2-1).  
(Courtesy of G.E.  
Medical Systems.)

$$\text{Image Negative: } s = L - 1 - r$$

# Image negative



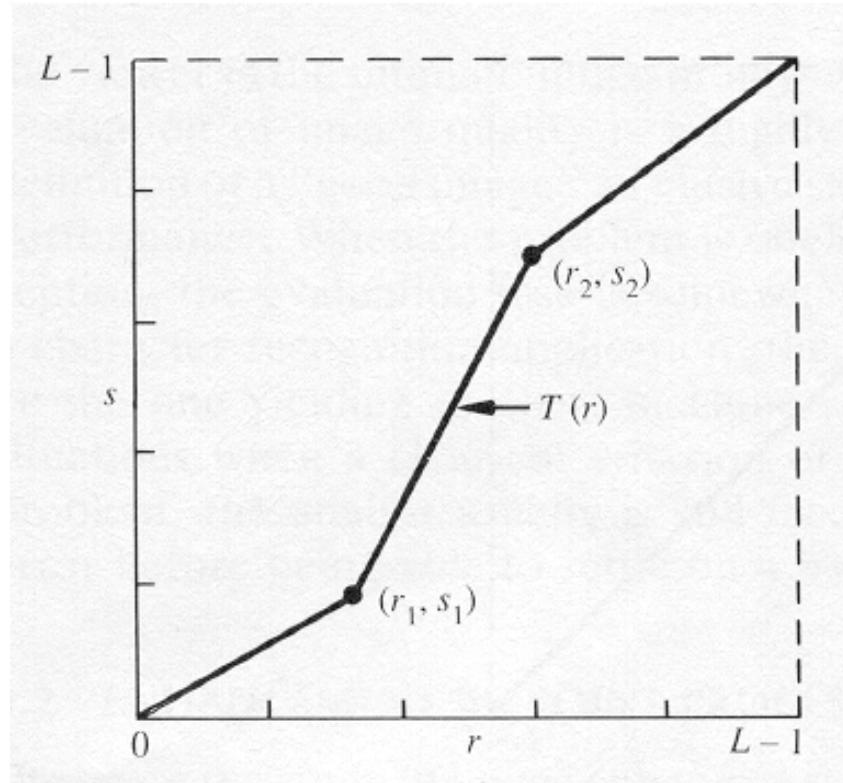
S=255-r

Demo

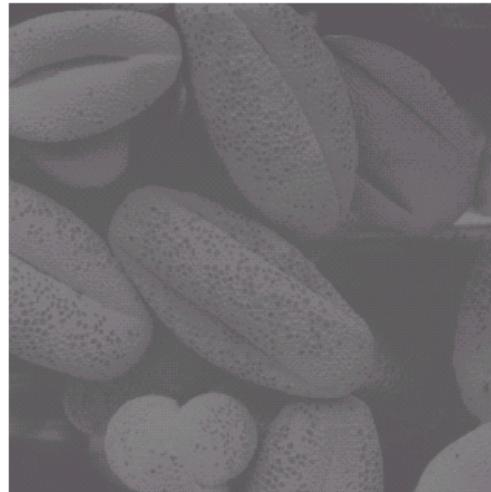
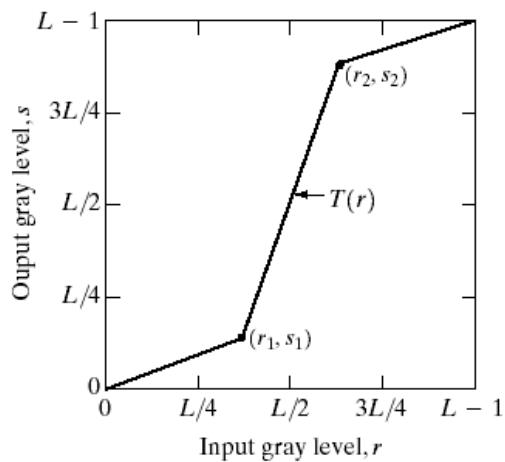
Image Negative:  $s = L - 1 - r$

# Contrast Stretching

- To increase the dynamic range of the gray levels in the image being processed.



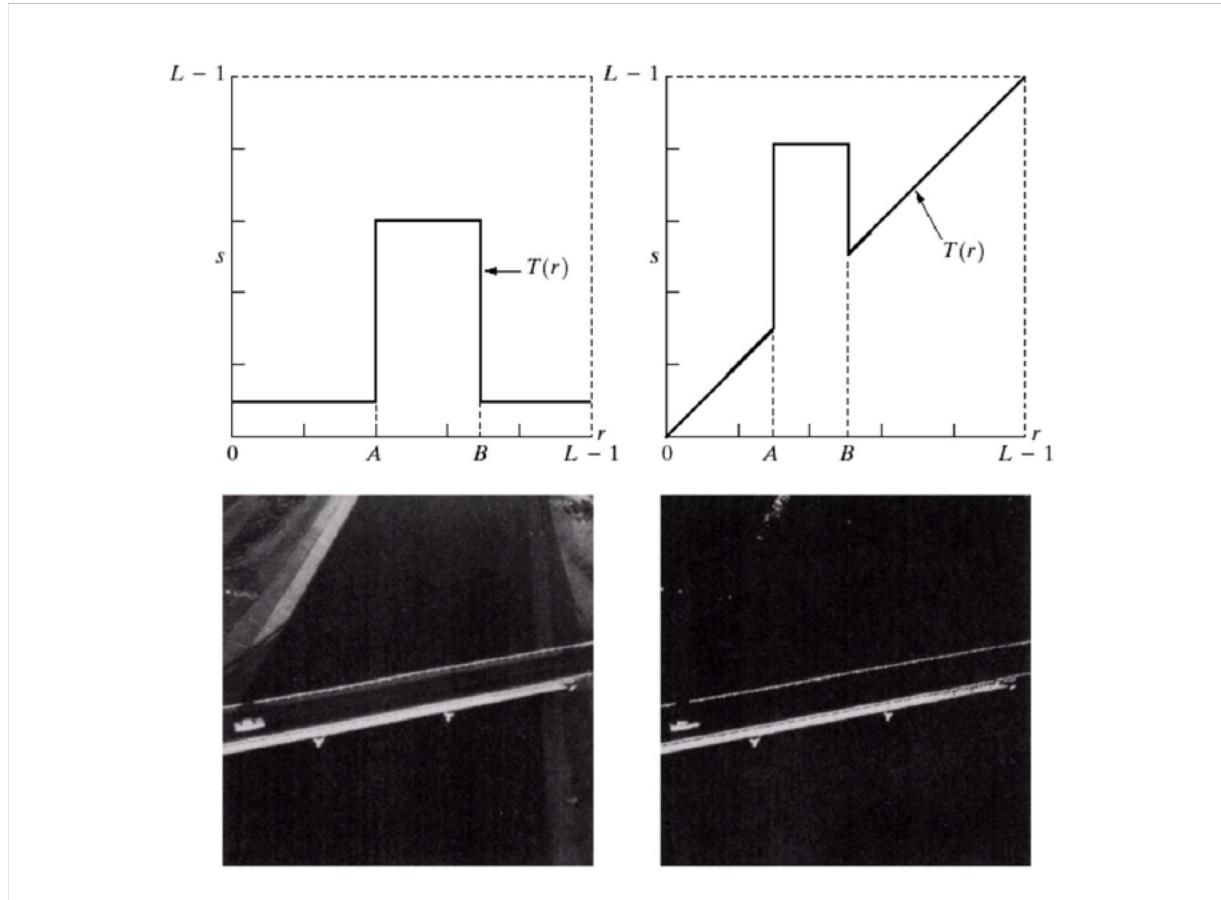
# Example



a  
b  
c  
d

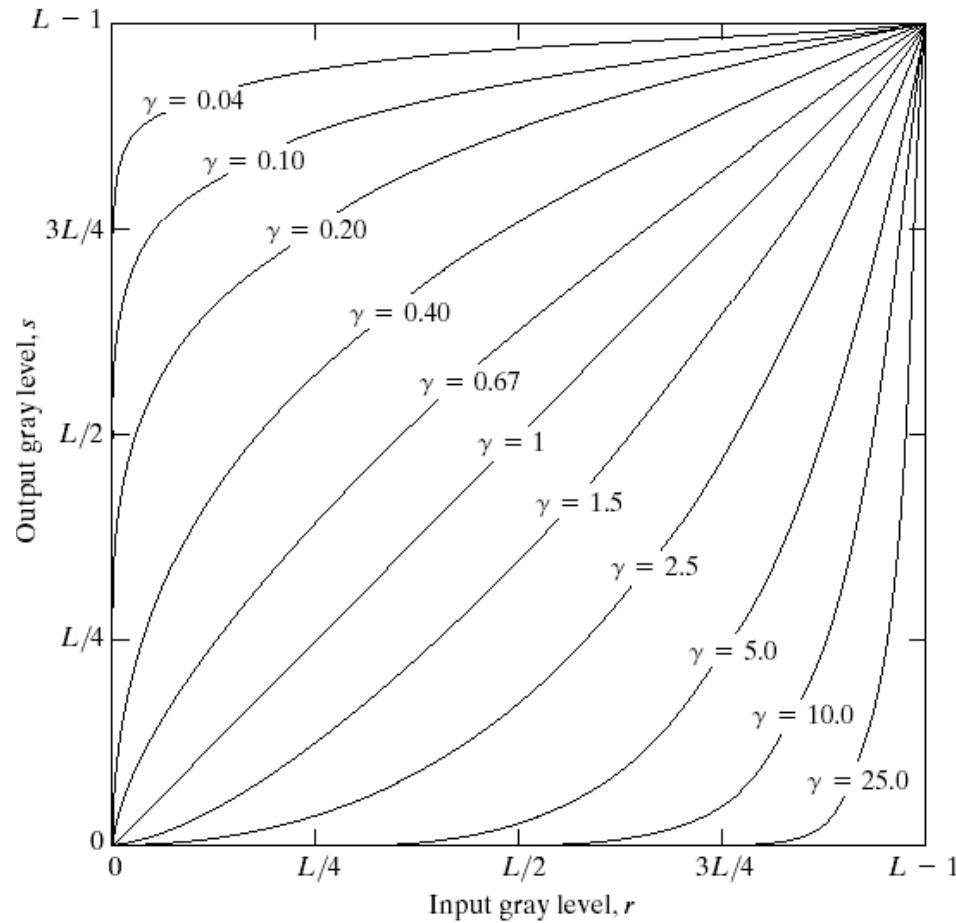
**FIGURE 3.10**  
Contrast stretching.  
(a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

# Contrast Stretching for image highlighting



# Power Law Transformation

- $s = cr^\gamma$
- $C, \gamma$  : positive constants
- Gamma correction



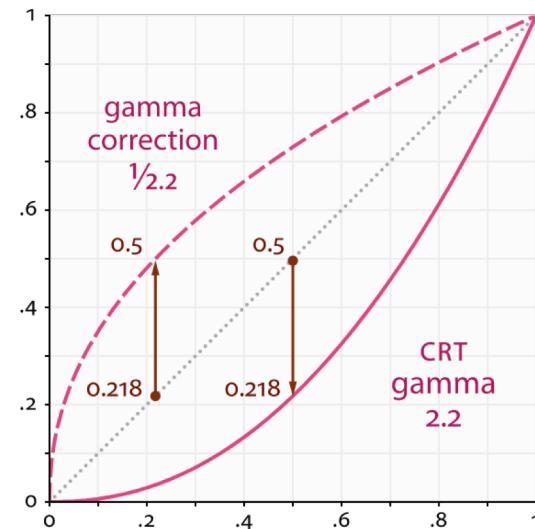
**FIGURE 3.6** Plots of the equation  $s = cr^\gamma$  for various values of  $\gamma$  ( $c = 1$  in all cases).

# Power Law Transformation



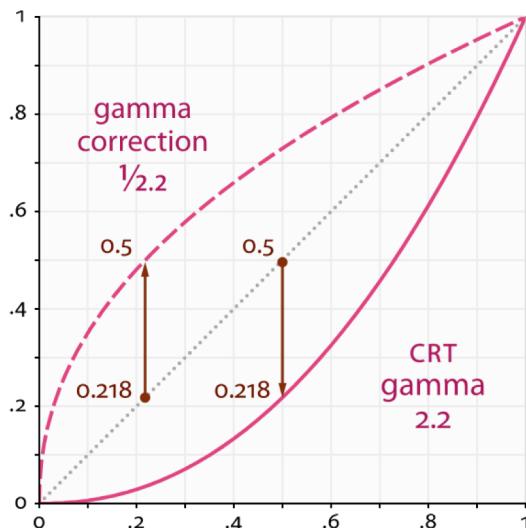
Almost every computer monitor, from whatever manufacturer, has one thing in common. They all have a intensity to voltage response curve which is roughly a 2.5 power function.

The effect can be overcome by applying an end-to-end power function whose exponent is about 1.1 or 1.2.



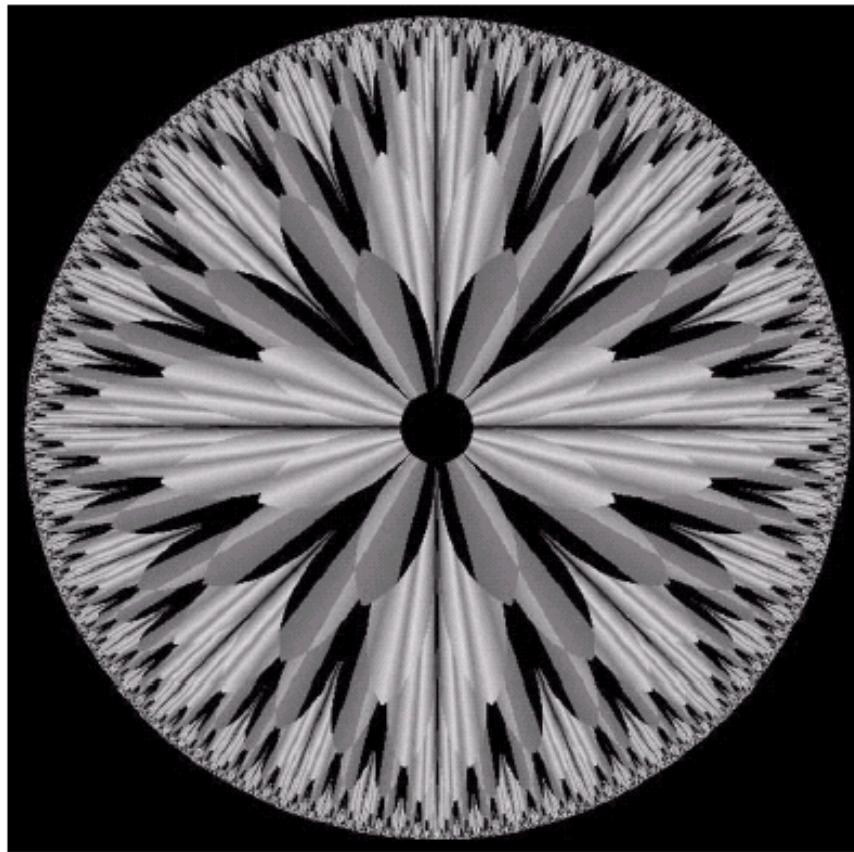
# Power Law Transformation

Gamma correction function is a function that maps luminance levels to compensate the non-linear luminance effect of display devices (or sync it to human perceptive bias on brightness).

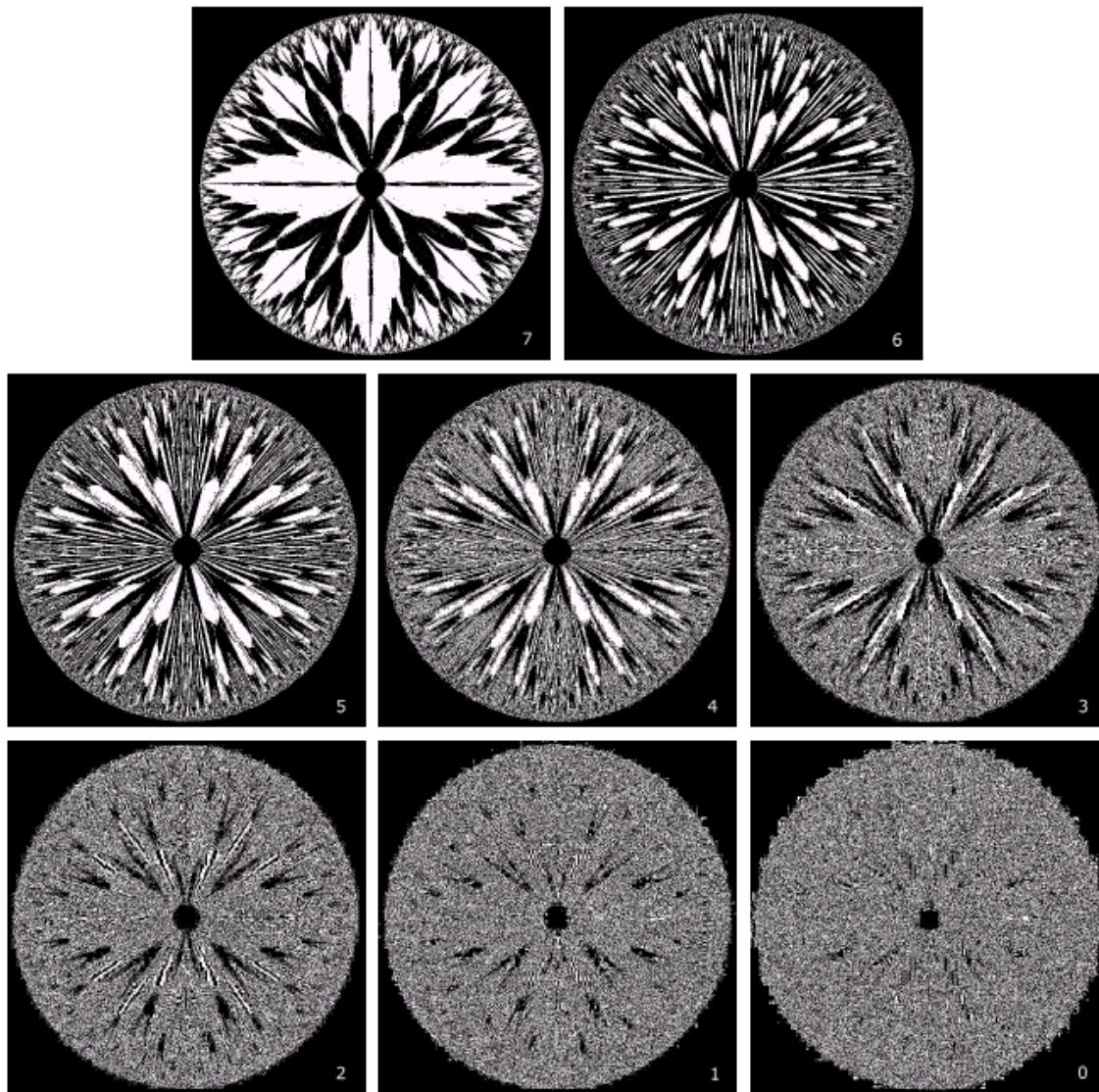


# Bit-Plane Slicing

- To highlight the contribution made to the total image appearance by specific bits.
  - i.e. Assuming that each pixel is represented by 8 bits, the image is composed of 8 1-bit planes.
  - Plane 0 contains the least significant bit and plane 7 contains the most significant bit.
  - Only the higher order bits (top four) contain visually significant data. The other bit planes contribute the more subtle details.



**FIGURE 3.13** An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

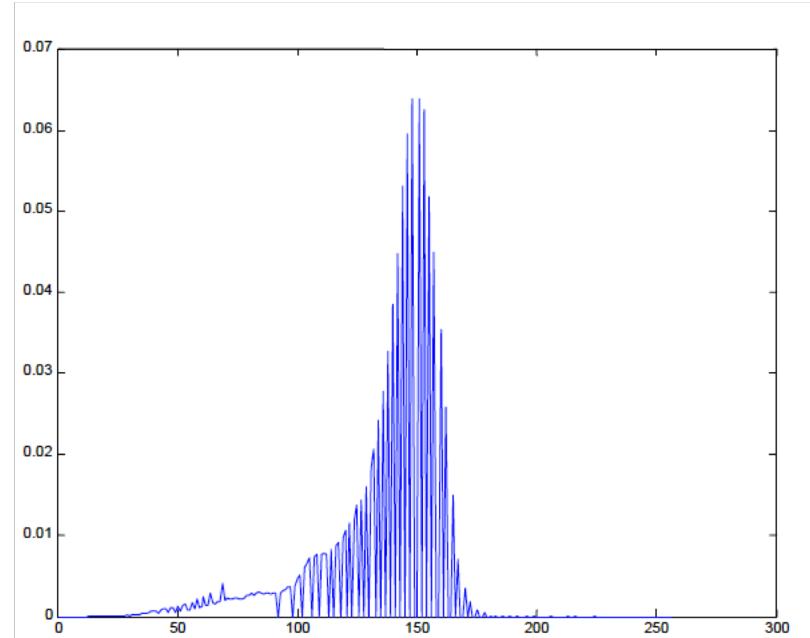


**FIGURE 3.14** The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

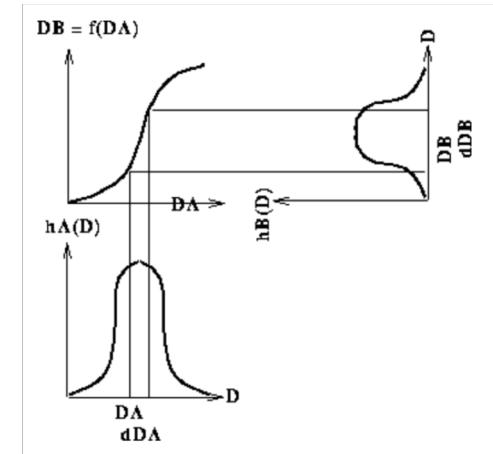
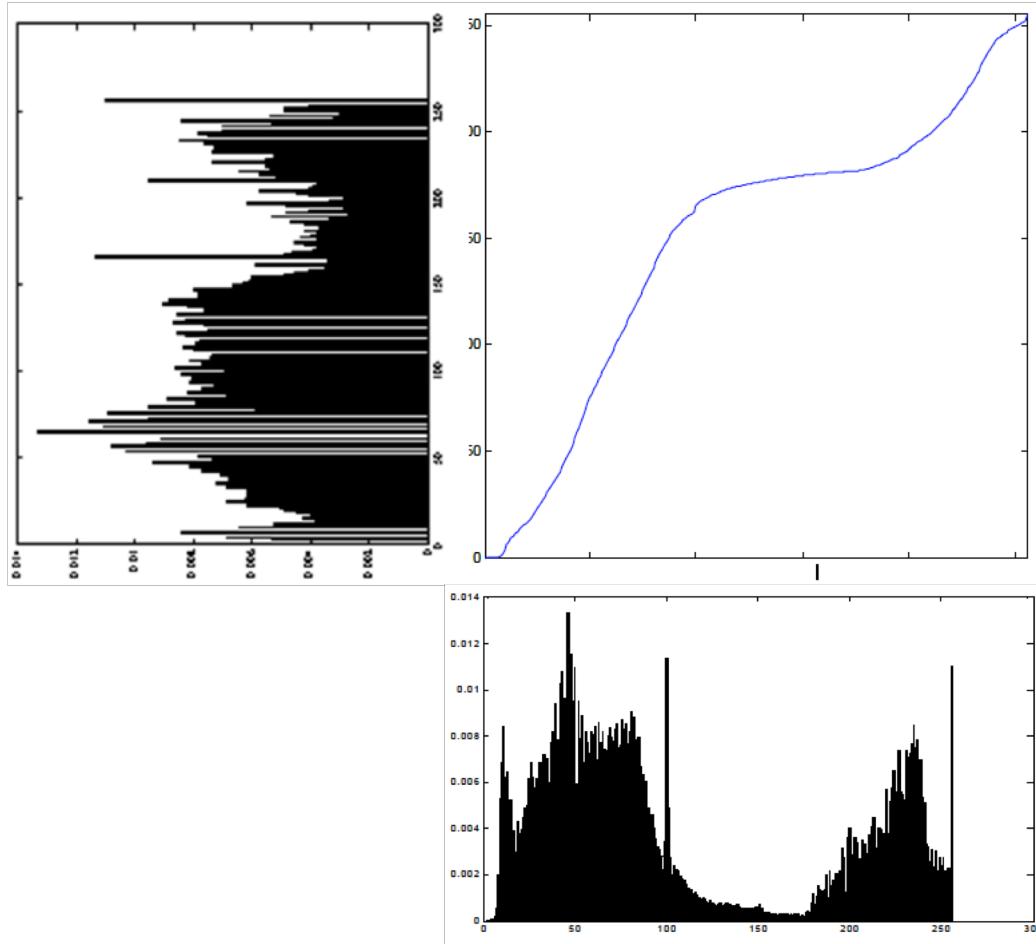
# Histogram Processing

- The histogram of a digital image with gray levels from 0 to L-1 is a discrete function  $h(r_k)=n_k$ , where:
  - $r_k$  is the kth gray level
  - $n_k$  is the # pixels in the image with that gray level
  - n is the total number of pixels in the image
  - $k = 0, 1, 2, \dots, L-1$
- Normalized histogram:  $p(r_k)=n_k/n$ 
  - sum of all components = 1

# Histogram Processing



# Histogram Equalization



<http://homepages.inf.ed.ac.uk/rbf/HIPR2/histeq.htm>

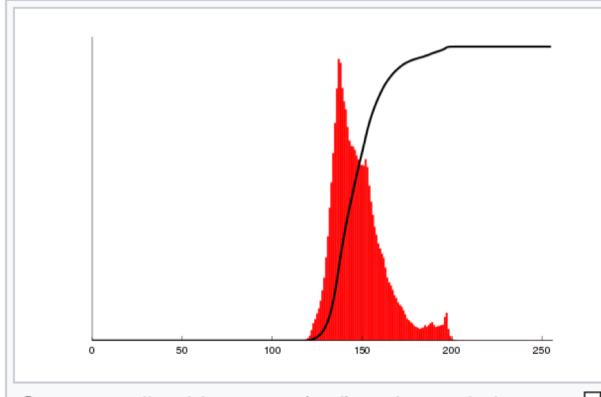


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# Histogram Equalization



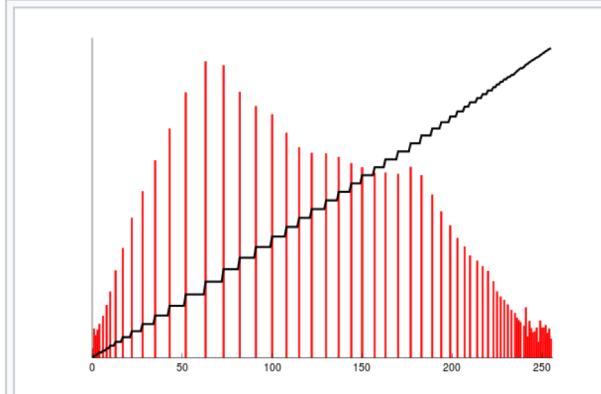
Before Histogram Equalization



Corresponding histogram (red) and cumulative histogram (black)



After Histogram Equalization



Corresponding histogram (red) and cumulative histogram (black)

# Histogram Equalization

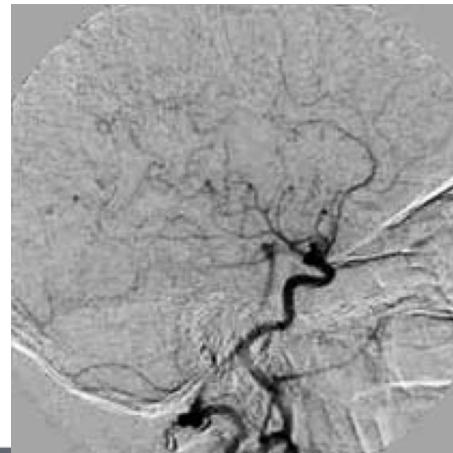
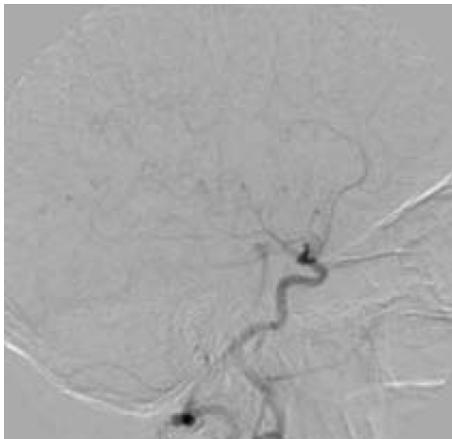
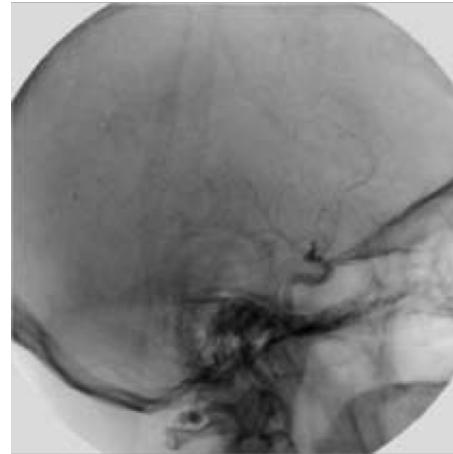
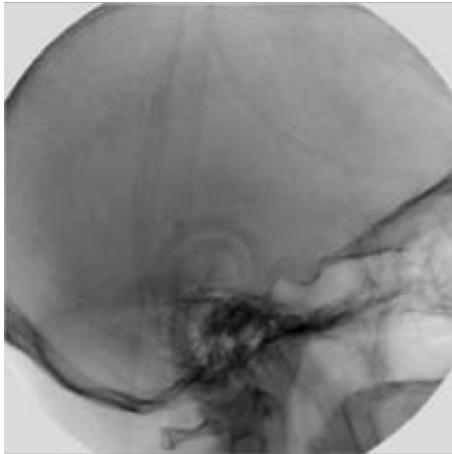


Original Image



After Histogram Equalization

# Image subtraction



(a) Mask image.  
(b) A live image.  
(c) Difference  
between (a) and  
(b). (d) Enhanced  
difference image.  
(Figures (a) and  
(b) courtesy of  
The Image  
Sciences Institute,  
University  
Medical Center,  
Utrecht, The  
Netherlands.)



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# Day 4

- **Basic Image Enhancement Tools**
- **About noise, filter**
- **Common applications of image filtering**

# Review

## Random Variable (RV)

- Variable (number) associated with the outcome of an random experiment
- Dice
  - E.g. Assign 1-6 to the faces of dice
- Urn
  - Assign 0 to black and 1 to white (or vise versa)
- Cards
  - Lots of different schemes - depends on application
- A function of a random variable is also a random variable

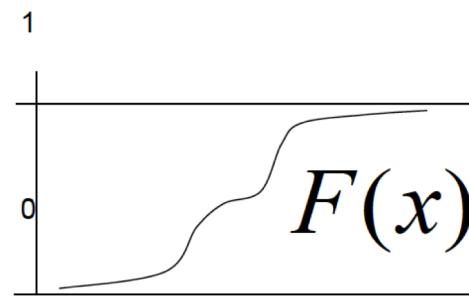
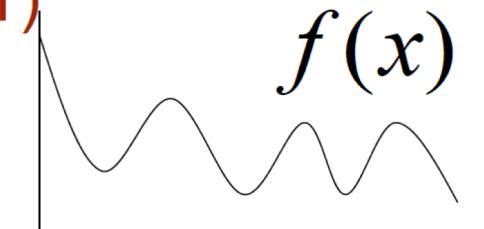
# Review

## Continuous Random Variables

- $f(x)$  is pdf (normalized to 1)
- $F(x)$  – cdf continuous  
— $\rightarrow x$  is a continuous RV

$$F(x) = \int_{-\infty}^x f(q)dq$$

$$f(x) = \left. \frac{dF(q)}{dq} \right|_x = F'(x)$$



# Review

## Probability Density Functions

- $f(x)$  is called a probability density function (pdf)

$$\int_{-\infty}^{\infty} f(x) = 1 \quad f(x) \geq 0 \quad \forall x$$

- A probability density is not the same as a probability
- The probability of a specific value as an outcome of continuous experiment is (generally) zero
  - To get meaningful numbers you must specify a range

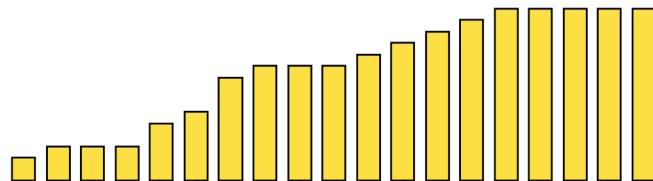
$$P(a \leq x \leq b) = \int_a^b f(q)dq = F(b) - F(a)$$

# Review

## Cumulative Distribution Function (cdf)

- $F(x)$ , where  $x$  is a RV
- $F(-\infty) = 0, F(\infty) = 1$
- $F(x)$  non decreasing

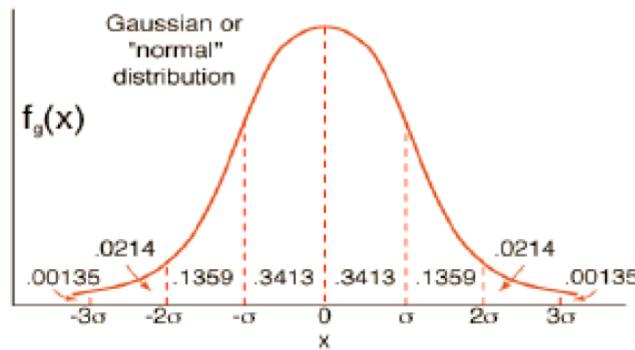
$$F(x) = \sum_{i=-\infty}^x P(i)$$



# Review

## Gaussian Distribution

- “Normal” or “bell curve”
- Two parameters:  $\mu$  - mean,  $\sigma$  – standard deviation



$$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

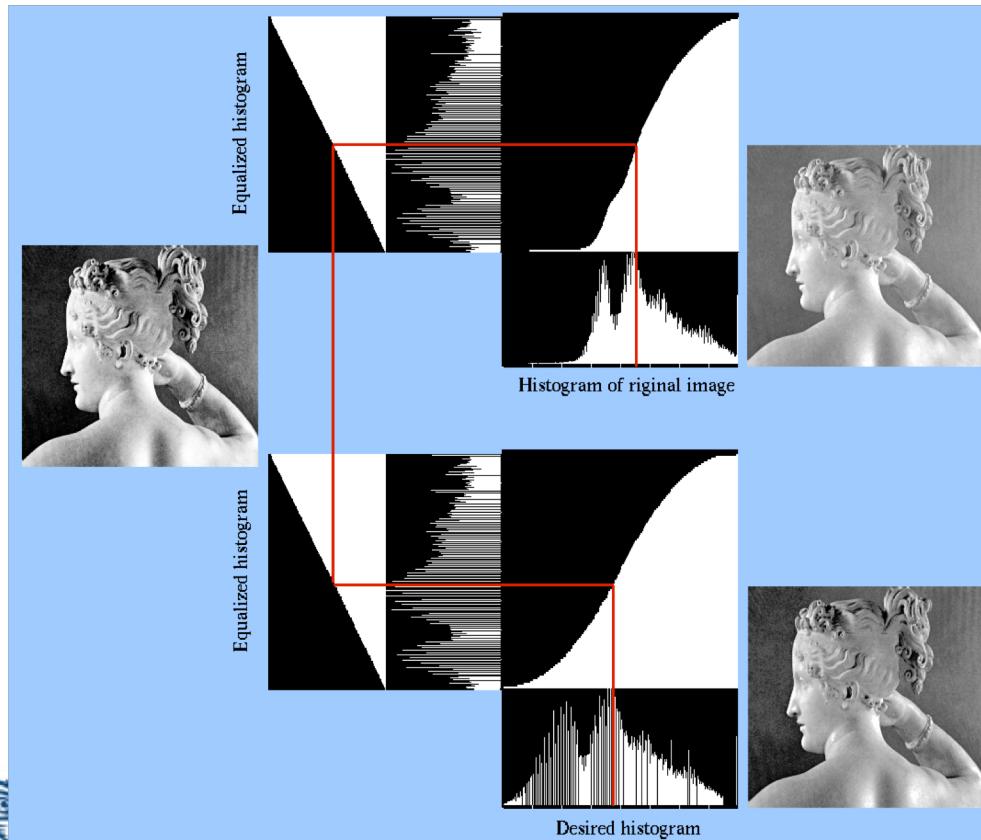
# Review

## Gaussian Properties

- Best fitting Gaussian to some data is gotten by mean and standard deviation of the samples
- Occurrence
  - Central limit theorem: result from lots of random variables
  - Nature (approximate)
    - Measurement error, physical characteristic, physical phenomenon
    - Diffusion of heat or chemicals

# Histogram

- Histogram equalization
- Histogram specification



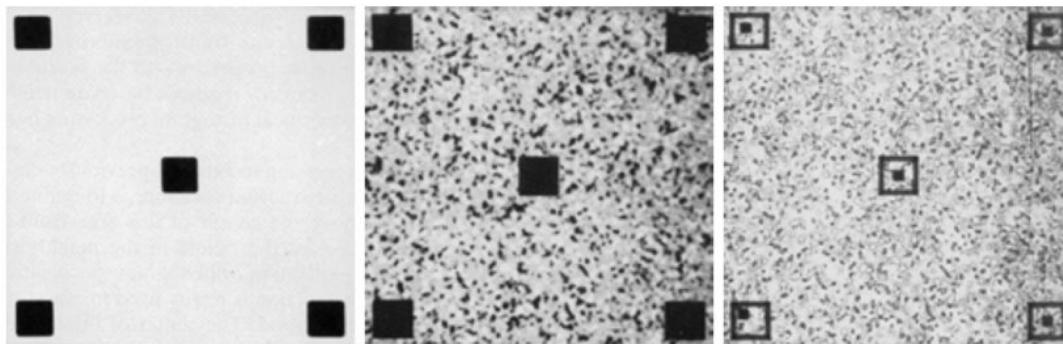
**G=histeq(f,p);**



# Histogram

- **Histogram equalization**
- **Histogram specification**
- **Local histogram equalization**

Define a neighborhood and move its center from pixel to pixel. At each location, the histogram of the points in the neighborhood is computed and histogram equalization transformation is obtained.



a b c

**FIGURE 3.23** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a  $7 \times 7$  neighborhood about each pixel.

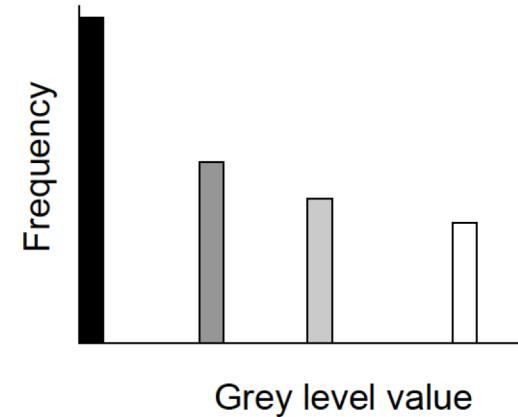
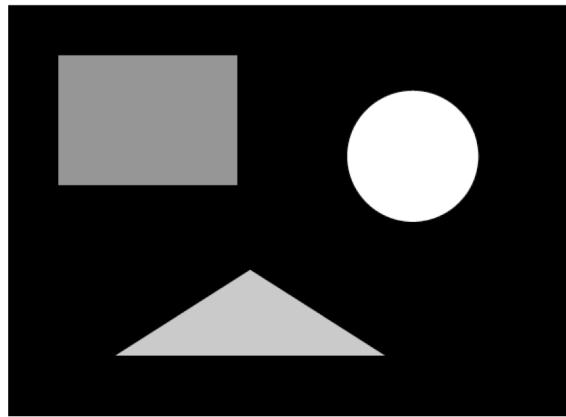
# Histogram Applications

- <http://www.sci.utah.edu/~gerig/CS6640-F2012/CS6640-F2012.html>

# Histogram Applications

## Histogram of Image Intensities

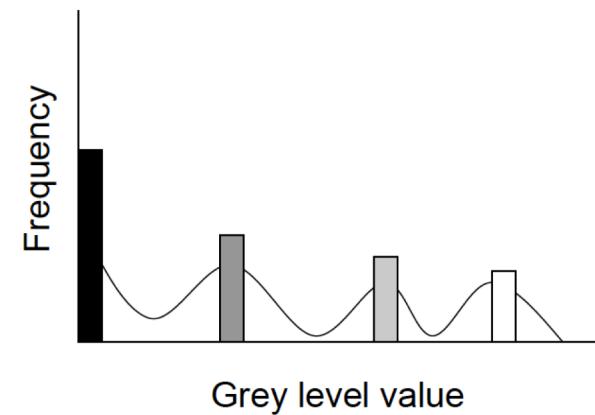
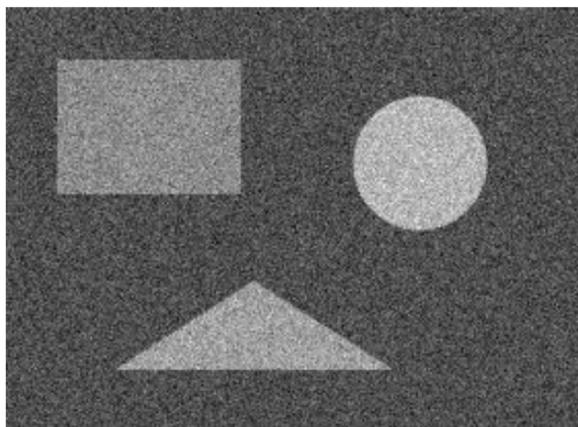
- Create bins of intensities and count number of pixels at each level
  - Normalize or not (absolute vs % frequency)



# Histogram Applications

## Histograms and Noise

- What happens to the histogram if we add noise?
  - $g(x, y) = f(x, y) + n(x, y)$



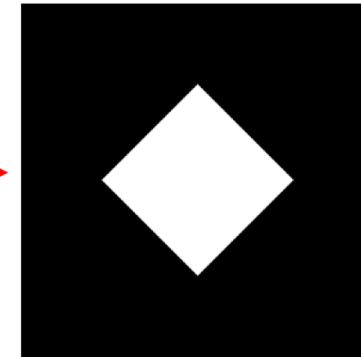
# Histogram Applications

## What is image segmentation?

- Image segmentation is the process of subdividing an image into its constituent regions or objects.
- Example segmentation with two regions:



Input image  
intensities 0-255

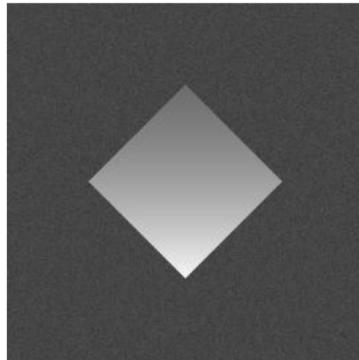


Segmentation output  
0 (background)  
1 (foreground)

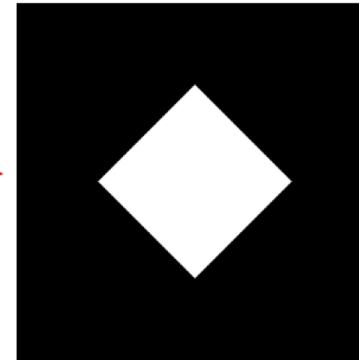
# Histogram Applications

## Thresholding

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T \\ 0 & \text{if } f(x, y) \leq T \end{cases}$$



Input image  $f(x, y)$   
intensities 0-255

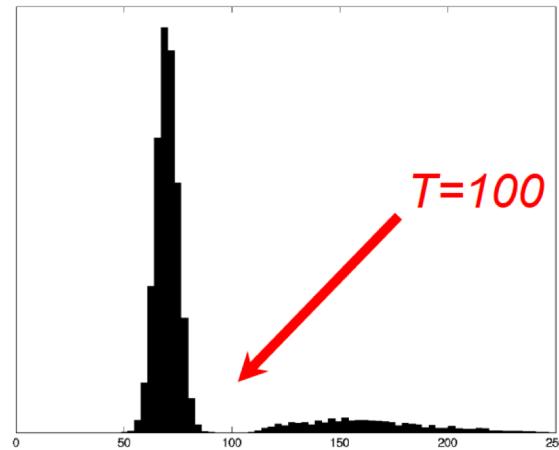
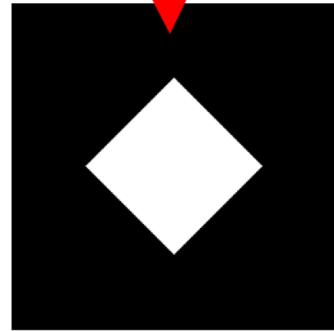
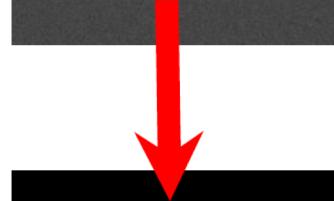
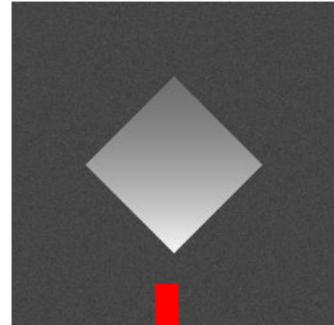


Segmentation output  $g(x, y)$   
0 (background)  
1 (foreground)

- How can we choose  $T$ ?
  - Trial and error
  - Use the histogram of  $f(x, y)$

# Histogram Applications

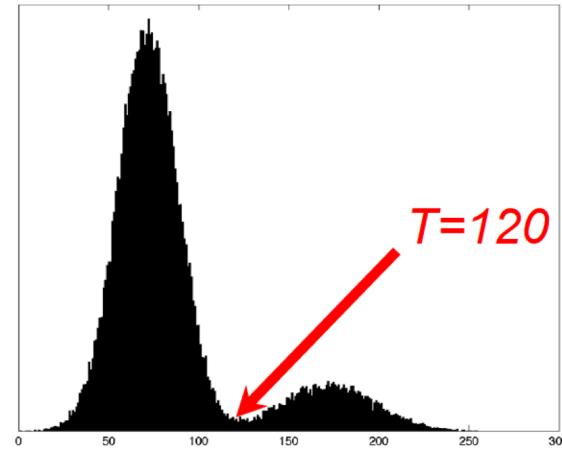
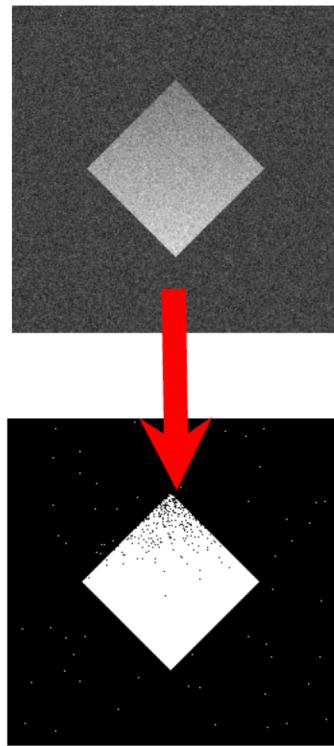
## Choosing a threshold



Histogram

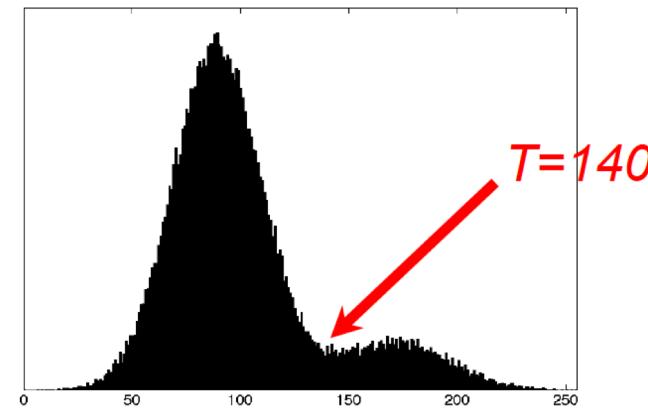
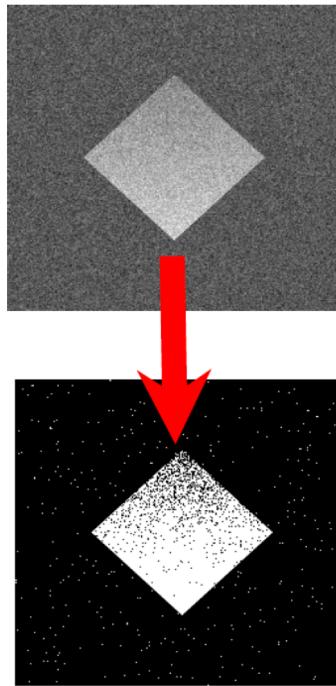
# Histogram Applications

## Role of noise



# Histogram Applications

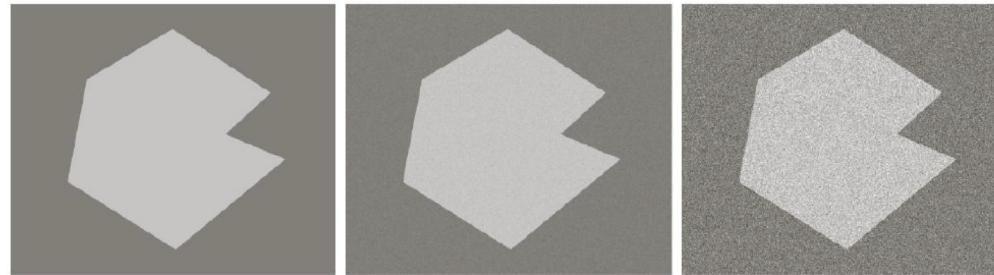
Low signal-to-noise ratio



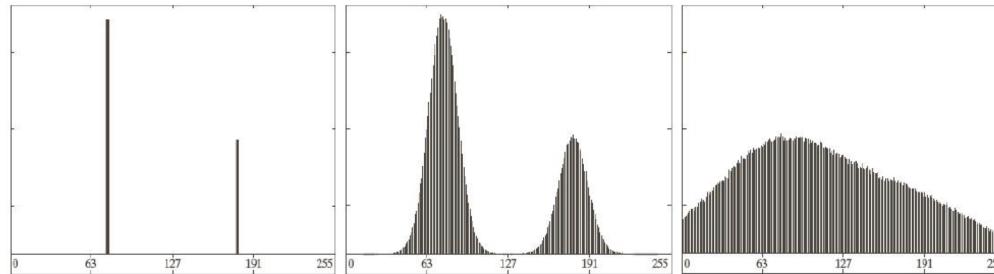
# Histogram Applications

## Effect of noise on image histogram

Images



Histograms



No noise

With noise

More noise

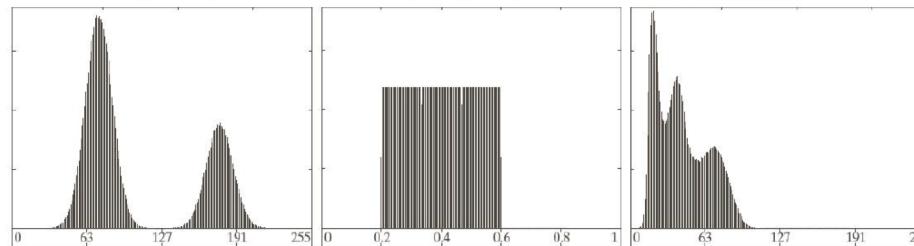
# Histogram Applications

## Effect of illumination on histogram

Images



Histograms



$f$   
Original  
image

$x$

$g$   
Illumination  
image

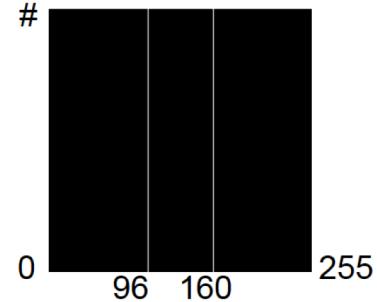
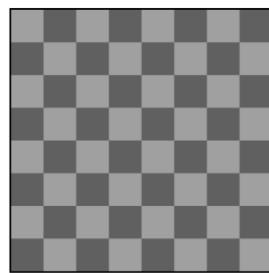
$=$

$h$   
Final  
image

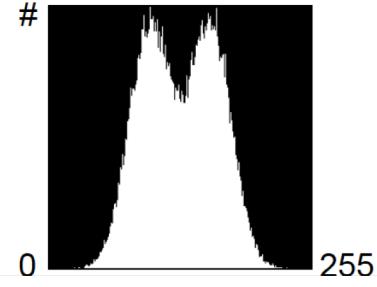
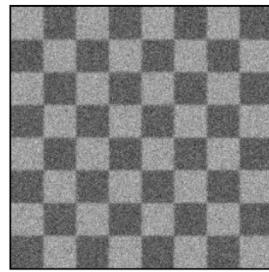
# Histogram Applications

## Histogram of Pixel Intensity Distribution

**Histogram:** Distribution of intensity values  $p(v)$   
(count #pixels for each intensity level)



Checkerboard with  
values 96 and 160.



Histogram:  
- horizontal: intensity  
- vertical: # pixels

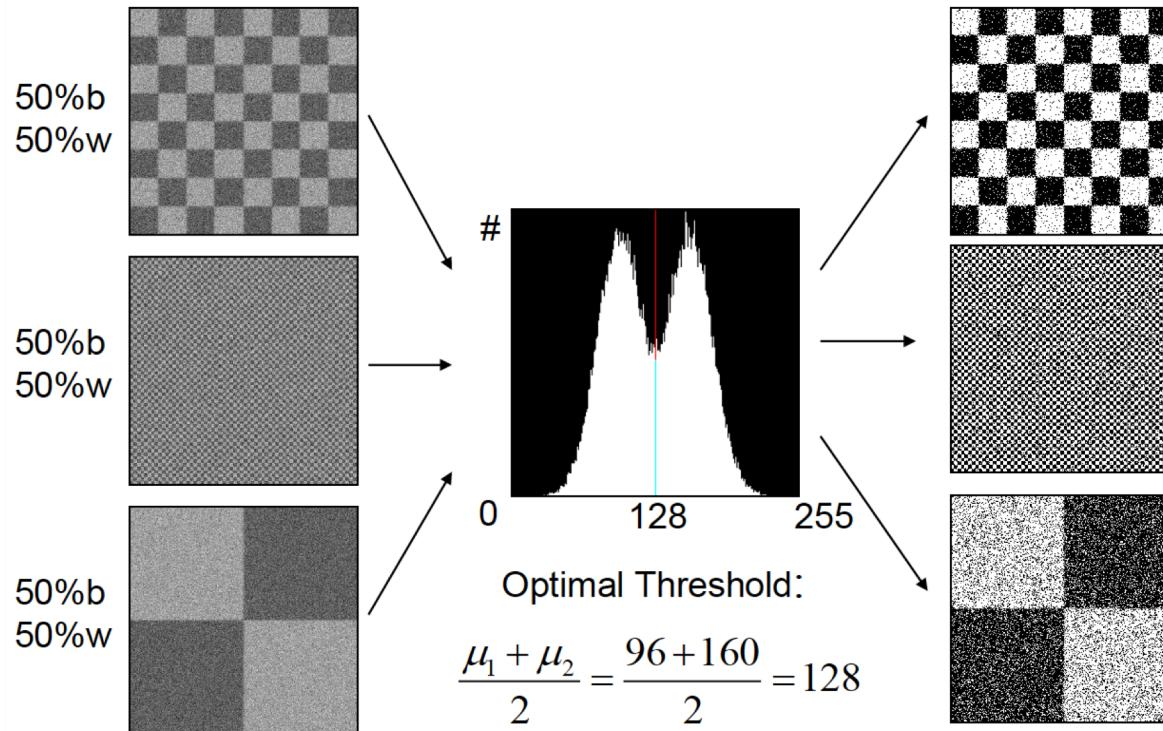
Checkerboard with  
additive Gaussian noise  
(sigma 20).

Regions: 50%b,50%w

36

# Histogram Applications

## Classification by Thresholding

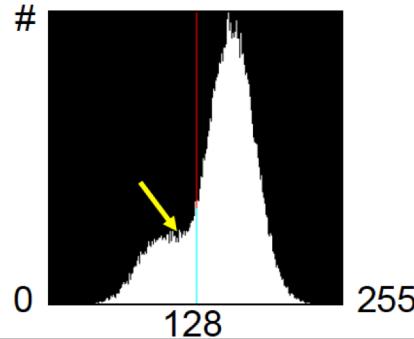
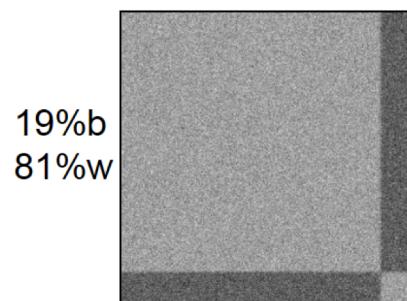
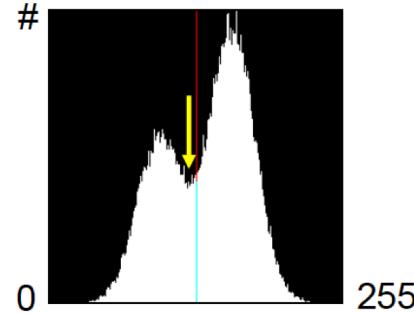
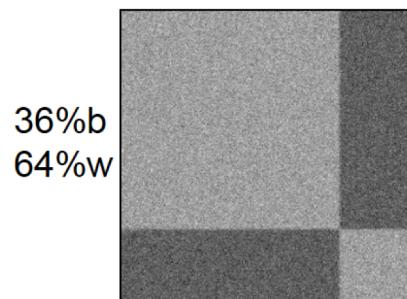


# Histogram Applications

- Histogram does not represent image structure such as regions and shapes, but only distribution of intensity values
- Many images share the same histogram

# Histogram Applications

Is the histogram suggesting  
the right threshold?

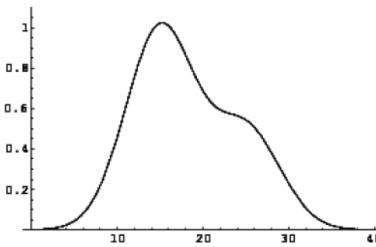
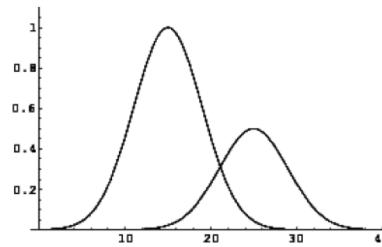
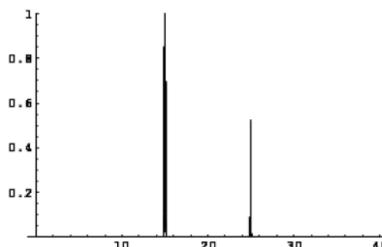
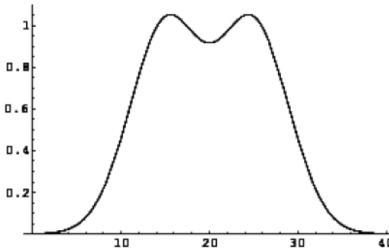
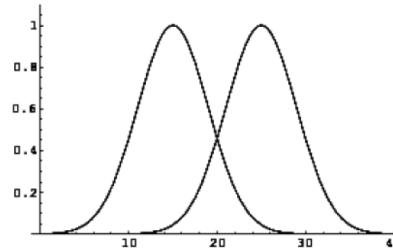
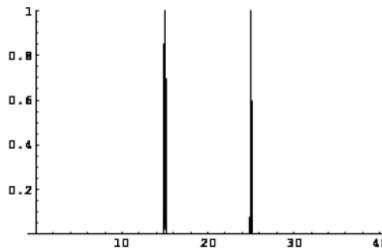


Proportions of  
bright and dark  
regions are  
different  $\Rightarrow$  Peak  
presenting bright  
regions becomes  
dominant.

Threshold value 128  
does not match with  
valley in distribution.

# Histogram Applications

## Histogram as Superposition of PDF's (probability density functions)



Regions with  
2 brightness levels,  
different proportions

Corruption with  
Gaussian noise,  
individual distributions

Histogram:  
Superposition of  
distributions

# Histogram Applications

## Gaussian Mixture Model

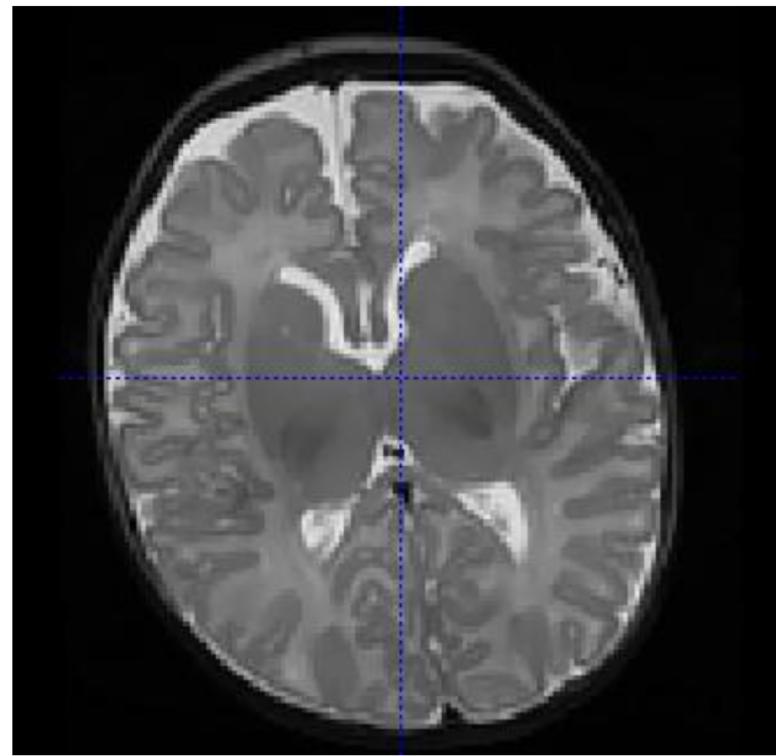
$$hist = a_1 G(\mu_1, \sigma_1) + a_2 G(\mu_2, \sigma_2)$$

*more general with  $k$  classes :*

$$hist = \sum_k a_k G(\mu_k, \sigma_k)$$

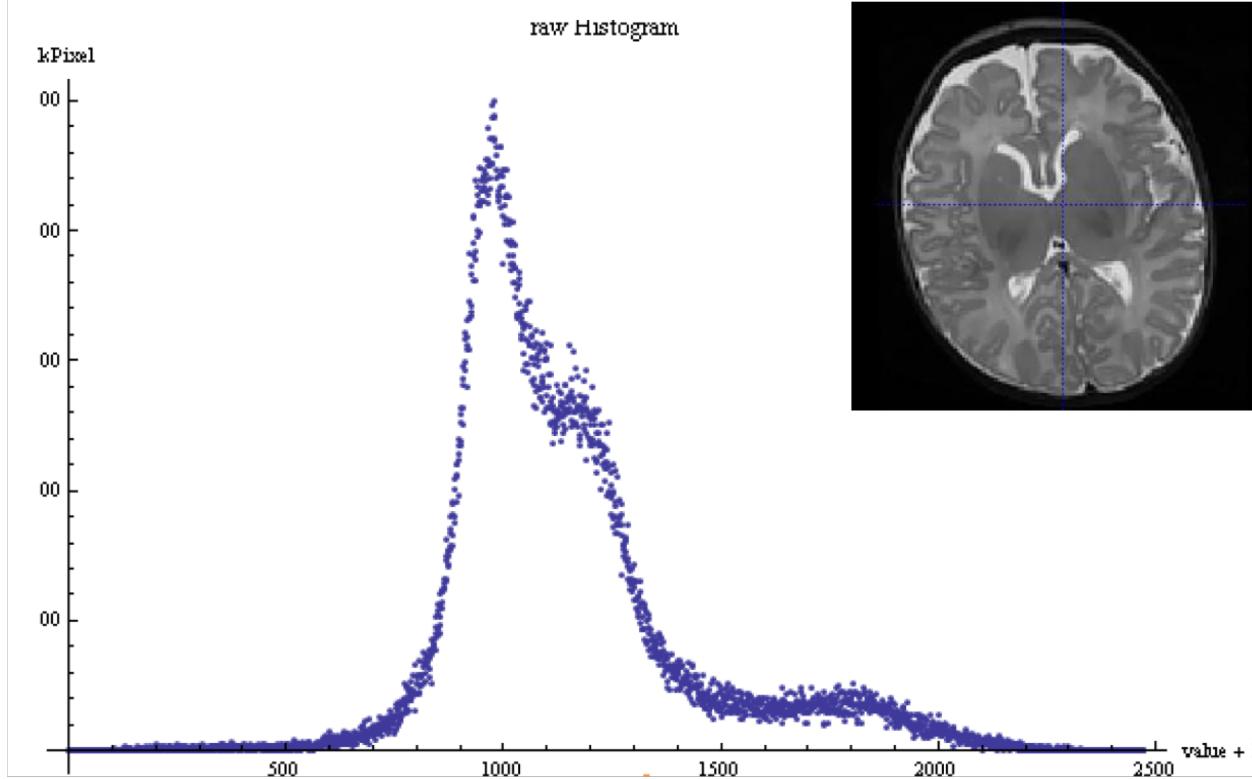
# Histogram Applications

Example: MRI



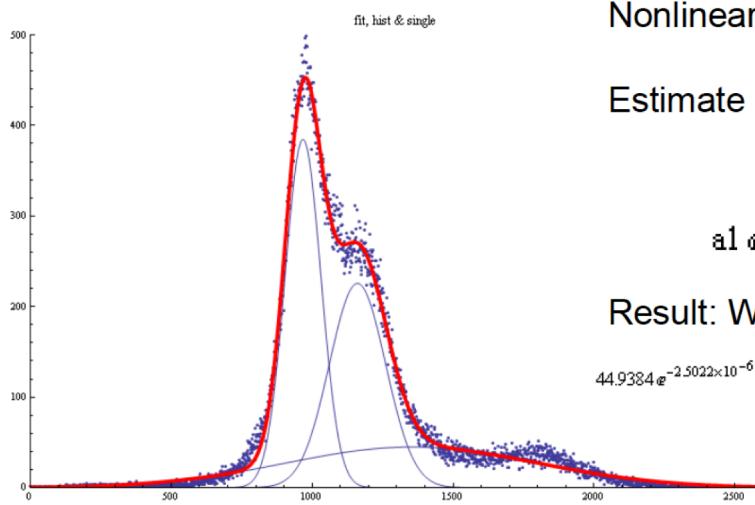
# Histogram Applications

## Example: MRI



# Histogram Applications

## Fit with 3 weighted Gaussians



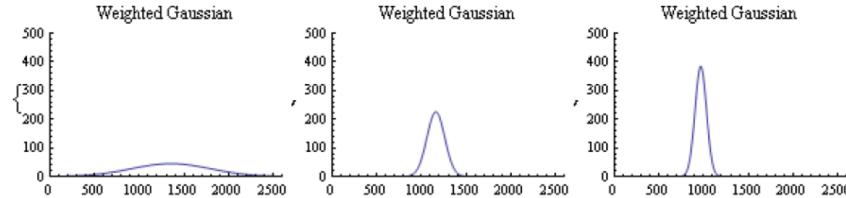
Nonlinear optimization

Estimate 9 parameters for best fit:

$$a_1 e^{-\frac{(x-my_1)^2}{2 \sigma_{y1}^2}} + a_2 e^{-\frac{(x-my_2)^2}{2 \sigma_{y2}^2}} + a_3 e^{-\frac{(x-my_3)^2}{2 \sigma_{y3}^2}}$$

Result: Weighted sum of Gaussians (pdf's):

$$44.9384 e^{-2.5022 \times 10^{-6} (x-1353.63)^2} + 225.575 e^{-0.0000503733 (x-1160.5)^2} + 384.58 e^{-0.000122748 (x-967.112)^2}$$



# Histogram Applications

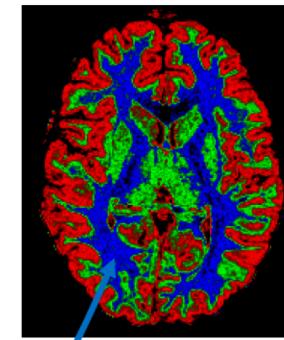
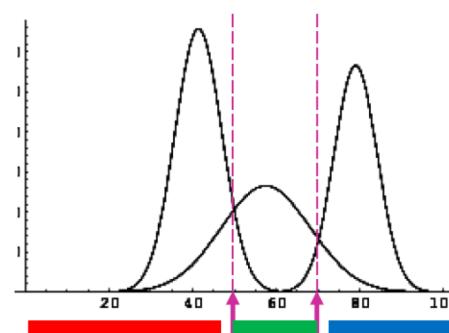
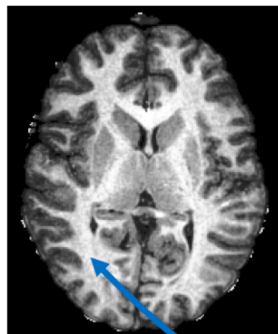
## Segmentation: Learning pdf's

*set of pdf's:*

$$G_k(\mu_k, \sigma_k | k), \quad (k = 1, \dots, n)$$

*calculate thresholds*

*assign pixels to categories*



Classification

# Image Enhancement in Spatial Domain

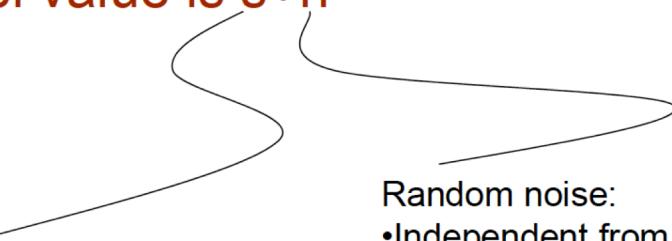
- Image Histogram
- Image Negative
- Contrast Stretching
- Power-Law Transformation
- Histogram Equalization
- Local Enhancement
- Image Subtraction
- Image Averaging
- Image Smoothing
- Image Sharpening

# Noise

## Application: Noisy Images

- Imagine  $N$  images of the same scene with random, independent, zero-mean noise added to each one
  - Nuclear medicine—radioactive events are random
  - Noise in sensors/electronics
- Pixel value is  $s+n$

True pixel  
value



Random noise:  
•Independent from one image to the next  
•Variance =  $\sigma$

# Image Smoothing or Averaging

- A noisy image:

$$g(x, y) = f(x, y) + n(x, y)$$

■ Let  $g(x, y)$  denote a corrupted image formed by the addition of noise,  $\eta(x, y)$ , to a noiseless image  $f(x, y)$ ; that is,

$$g(x, y) = f(x, y) + \eta(x, y) \quad (2.6-4)$$

**EXAMPLE 2.5:**  
Addition  
(averaging) of  
noisy images for  
noise reduction.

# Image Smoothing or Averaging

- A noisy image:

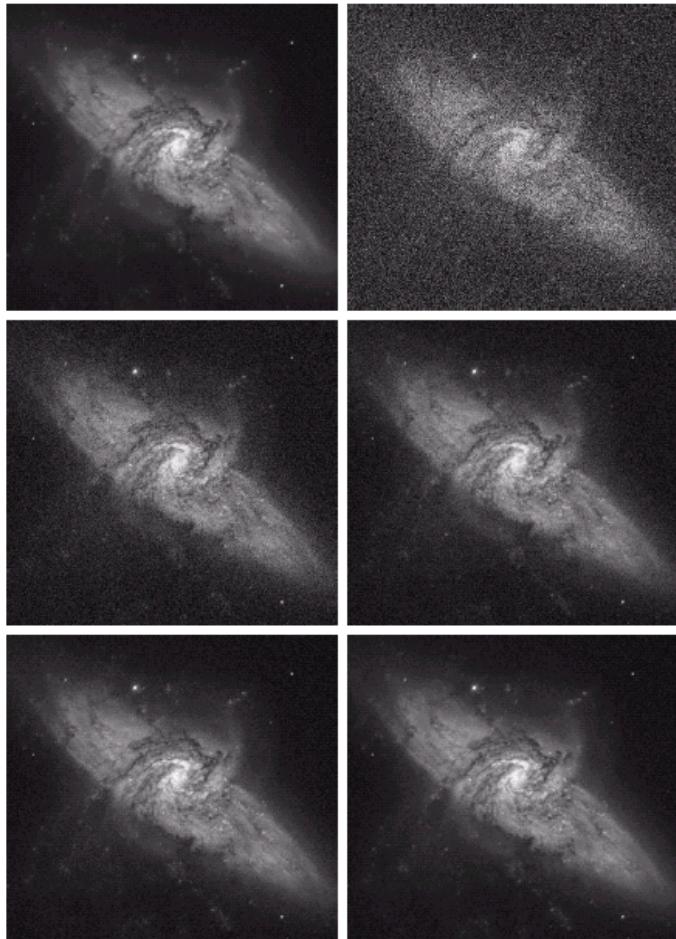
$$g(x, y) = f(x, y) + n(x, y)$$

- Averaging M different noisy images:

$$\bar{g}(x, y) = \frac{1}{M} \sum_{i=1}^M g_i(x, y)$$

- As M increases, the variability of the pixel values at each location decreases.
  - This means that  $\bar{g}(x, y)$  approaches  $f(x, y)$  as the number of noisy images used in the averaging process increases.

# Example



a b  
c d  
e f

**FIGURE 3.30** (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging  $K = 8, 16, 64$ , and  $128$  noisy images. (Original image courtesy of NASA.)