Digital Image Processing
CS390S

Feng Jiang
2018 Spring

METROPOLITAN STATE UNIVERSITY℠
OF DENVER

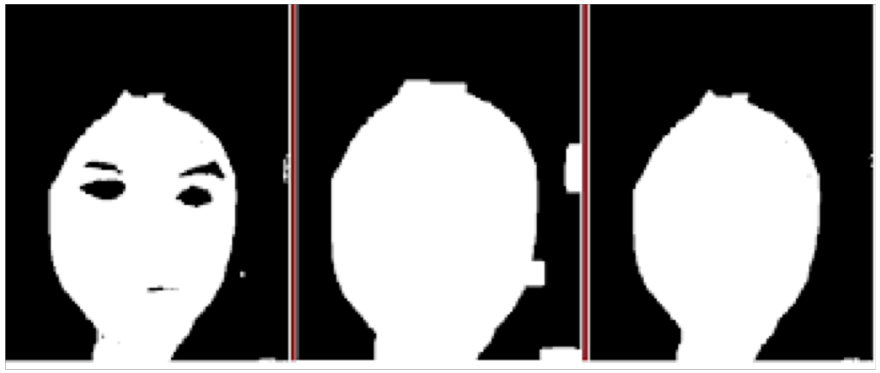
Morphological Image Processing

- Morphology
- Math morphology “set theory”
- Morphology in image processing
  - Structuring element
  - Dilation and erosion, Boundary
  - Fill the hole
  - Convex hull

Morphological Image Processing

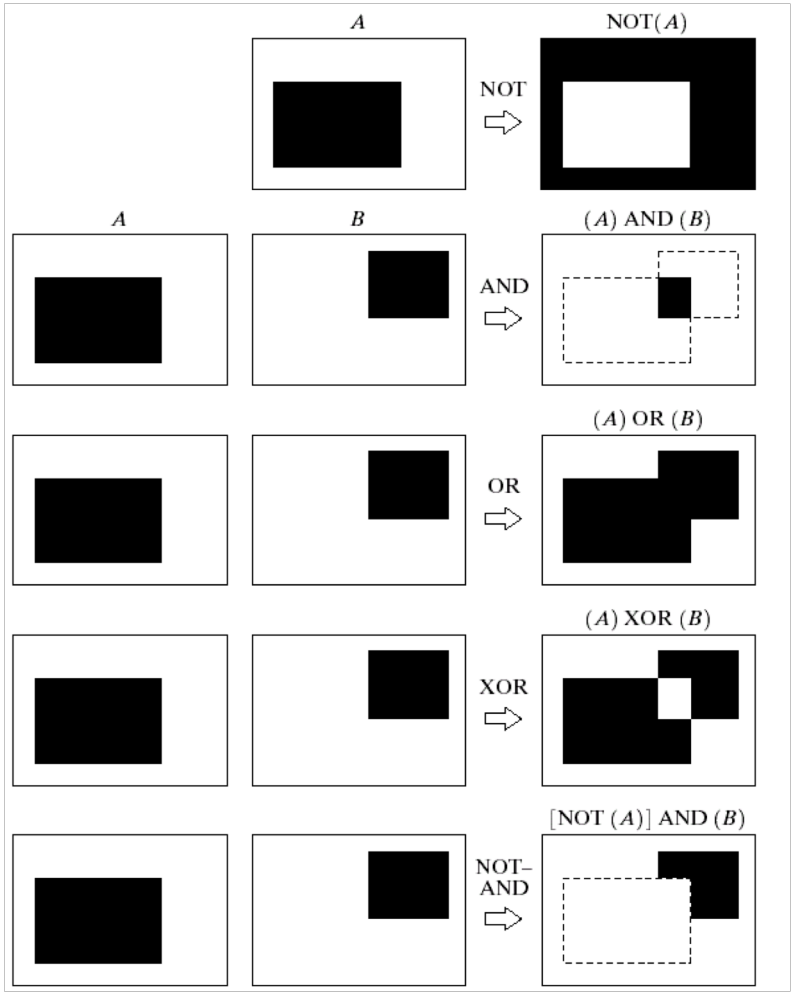
- Morphology: a branch of biology that deals with the form and structure of animals and plants.
- Mathematical morphology is a tool to extract the image components of the region shape, such as boundary, skeleton and convex hull.
- Set theory

Morphological Image Processing



Morphological Image Processing

- Mathematical morphology
- Set theory



Morphological Image Processing

- Dilation and erosion
- SE: **structuring element**

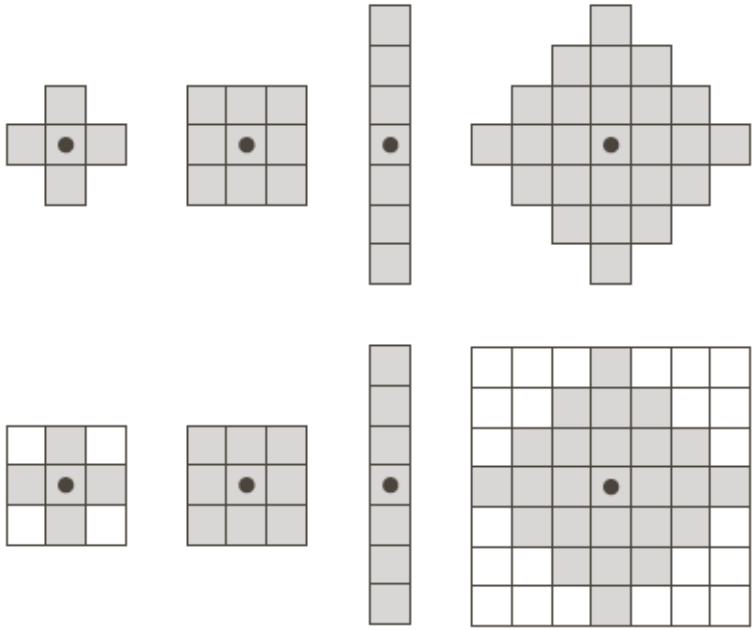


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Morphological Image Processing

- Dilation

With A and B as sets in Z^2, the *dilation* of A by B, denoted $A \oplus B$, is defined as

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}. \qquad (9.2\text{-}1)$$

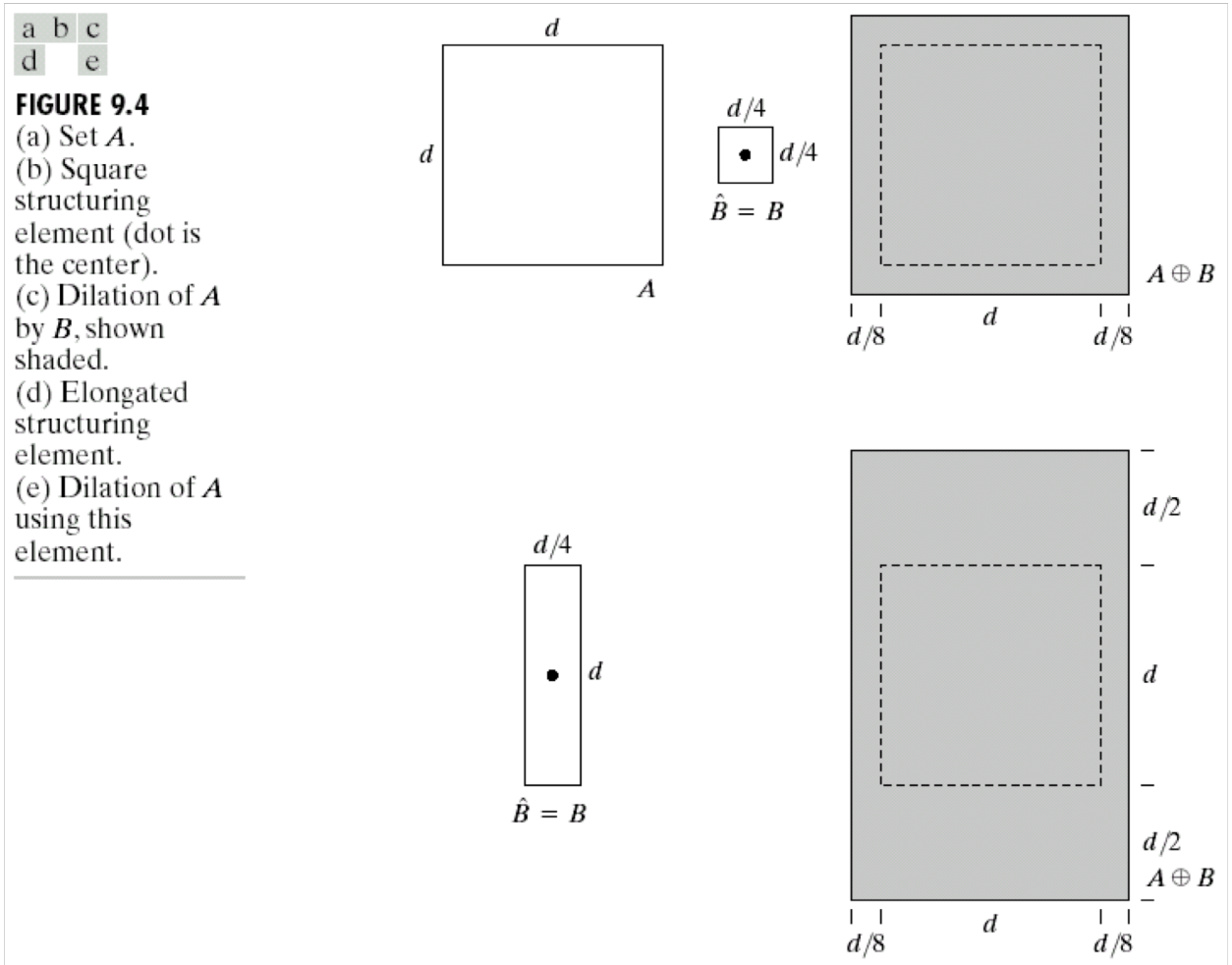
This equation is based on obtaining the reflection of B about its origin and shifting this reflection by z. The dilation of A by B then is the set of all *displacements*, z, such that $\hat{B}$ and A overlap by at least one element. Based on this interpretation, Eq. (9.2-1) may be rewritten as

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}. \qquad (9.2\text{-}2)$$

Set B is commonly referred to as the *structuring element* in dilation, as well as in other morphological operations.

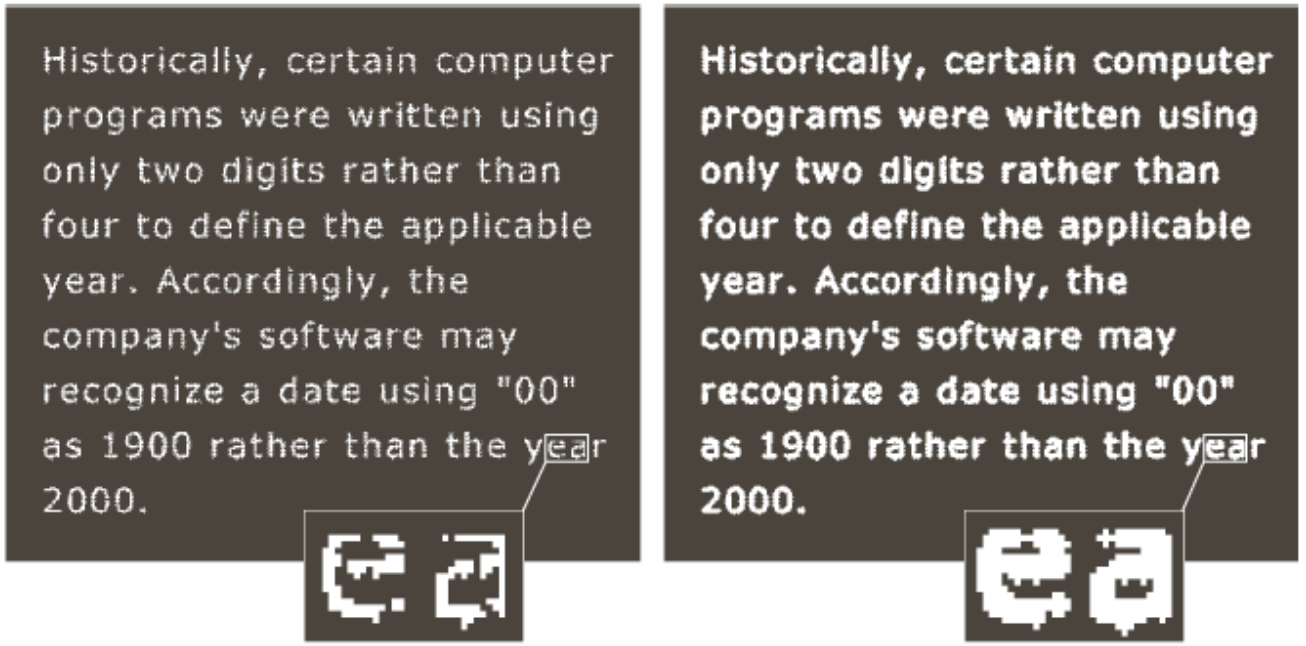
Morphological Image Processing

- Dilation



Morphological Image Processing

- Dilation



a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

0	1	0
1	1	1
0	1	0

Morphological Image Processing

- Erosion

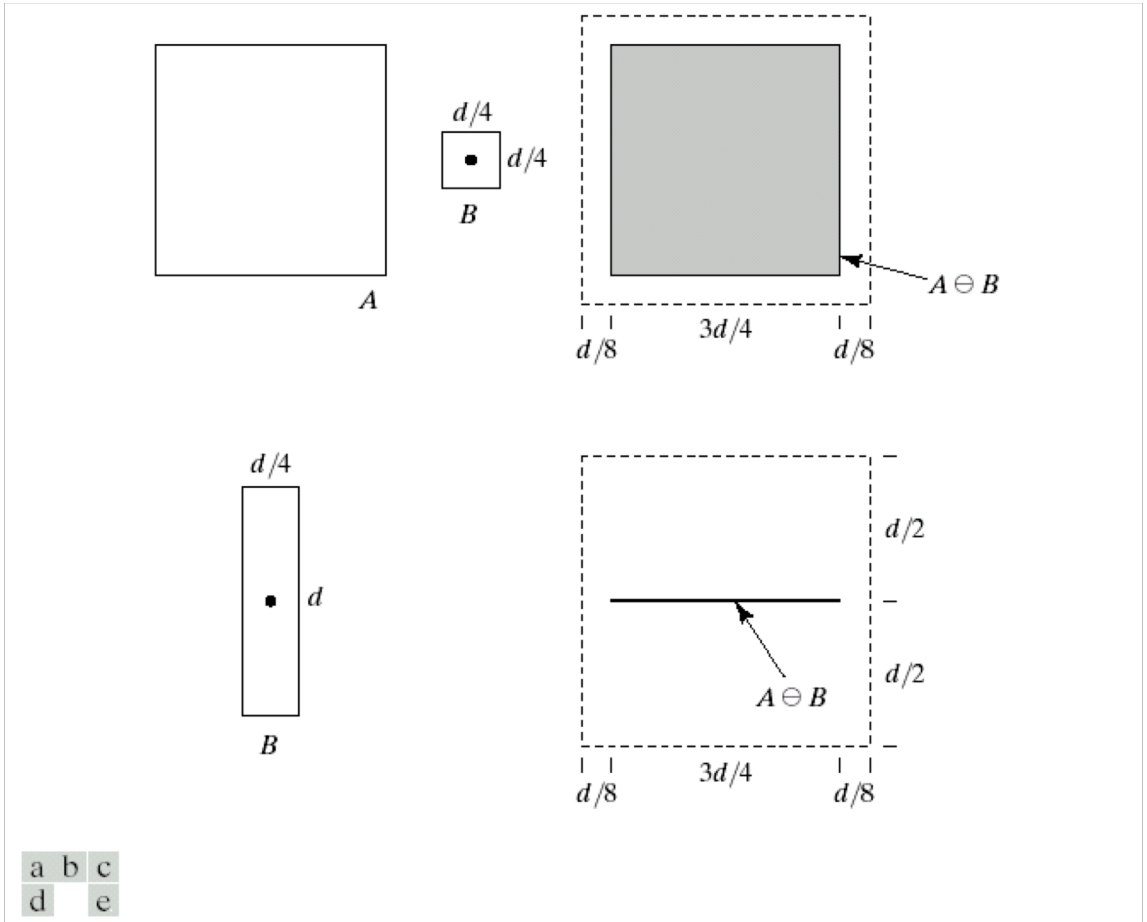
For sets A and B in Z^2 the erosion of A by B, denoted $A \ominus B$, is defined as

$$A \ominus B = \{z \mid (B)_z \subseteq A\}. \qquad (9.2\text{-}3)$$

In words, this equation indicates that the erosion of A by B is the set of all points z such that B, translated by z, is contained in A.

Morphological Image Processing

- Erosion

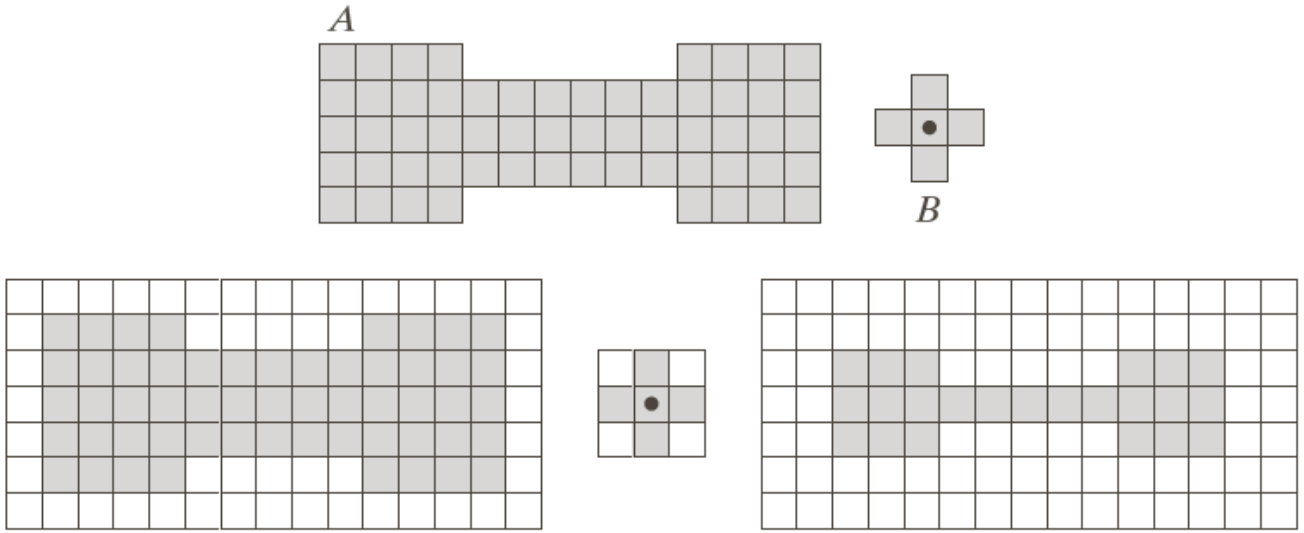


a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

Morphological Image Processing

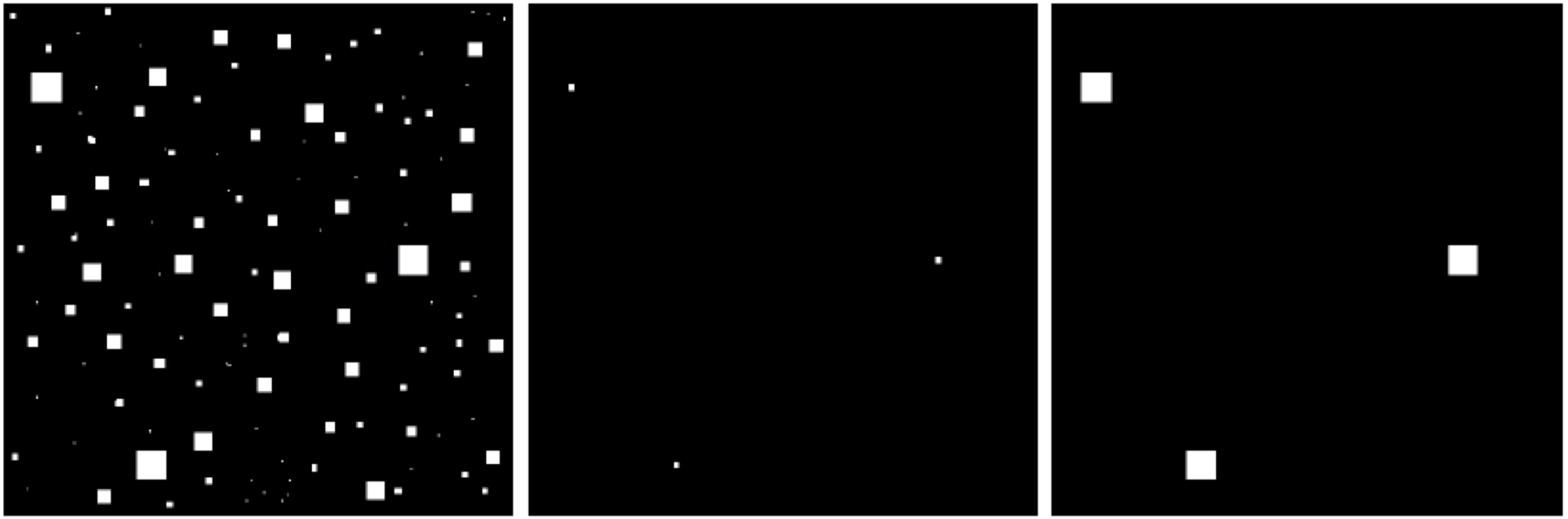
- Erosion



a b
c d e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Morphological Image Processing



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Morphological Image Processing

- Open

As we have seen, dilation expands an image and erosion shrinks it. In this section we discuss two other important morphological operations: opening and closing. *Opening* generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions. *Closing* also tends to smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.

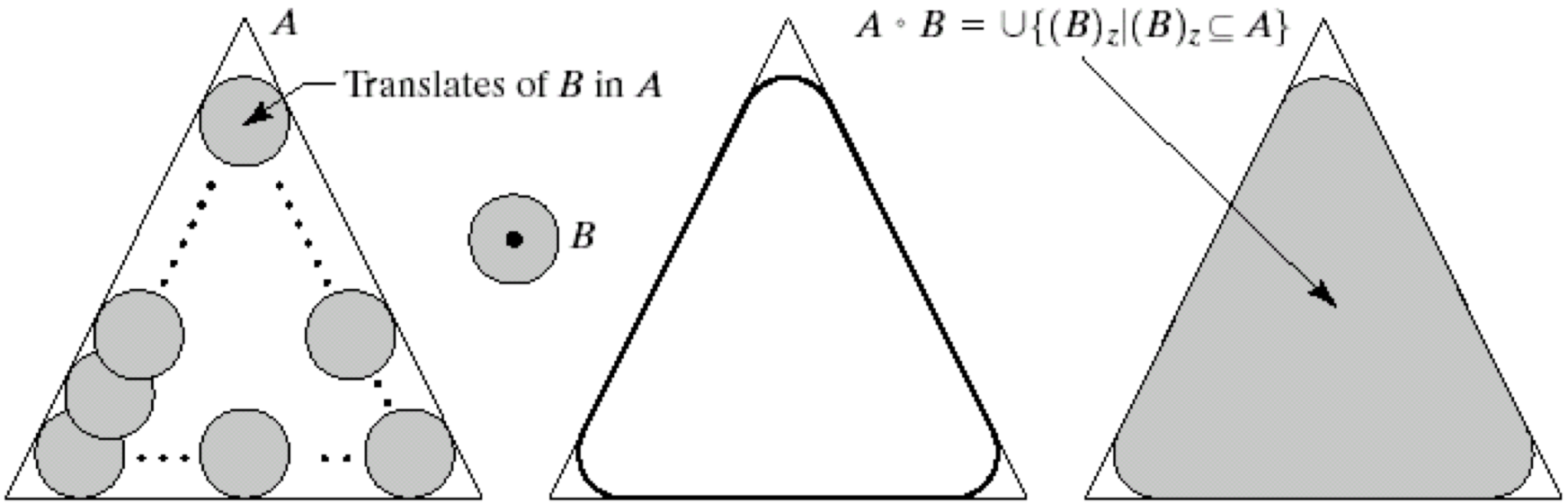
The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = (A \ominus B) \oplus B. \tag{9.3-1}$$

Thus, the opening A by B is the erosion of A by B, followed by a dilation of the result by B.

Morphological Image Processing

- Open



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Morphological Image Processing

- Close

As we have seen, dilation expands an image and erosion shrinks it. In this section we discuss two other important morphological operations: opening and closing. *Opening* generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions. *Closing* also tends to smooth sections of contours but, as opposed to opening, it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.

The opening of set A by structuring element B, denoted $A \circ B$, is defined as

$$A \circ B = (A \ominus B) \oplus B. \tag{9.3-1}$$

Thus, the opening A by B is the erosion of A by B, followed by a dilation of the result by B.

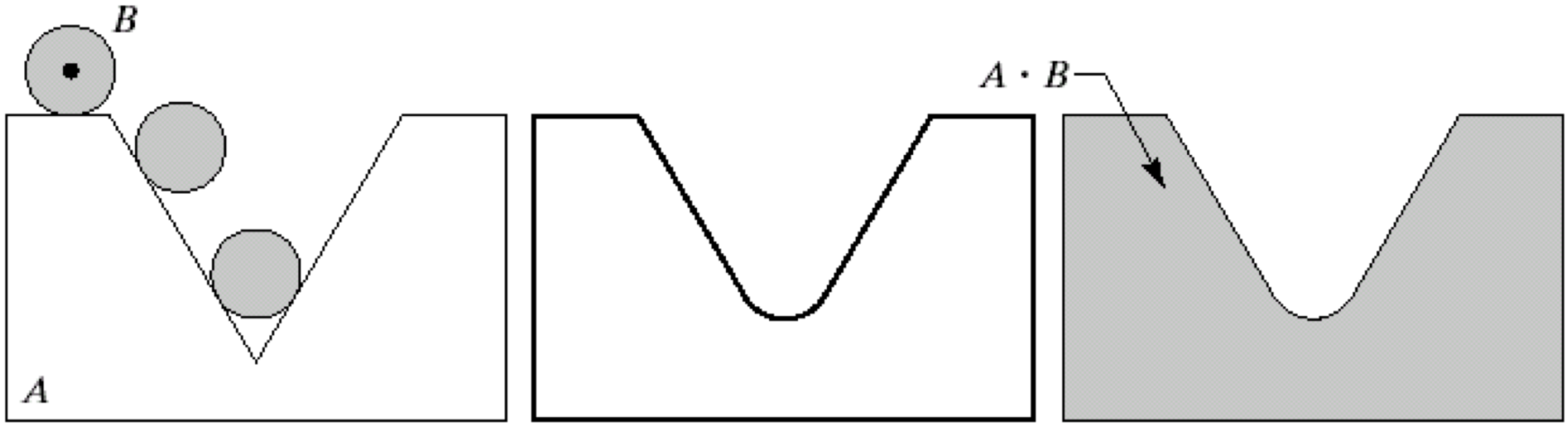
Similarly, the closing of set A by structuring element B, denoted $A \bullet B$, is defined as

$$A \bullet B = (A \oplus B) \ominus B, \tag{9.3-2}$$

which, in words, says that the closing of A by B is simply the dilation of A by B, followed by the erosion of the result by B.

Morphological Image Processing

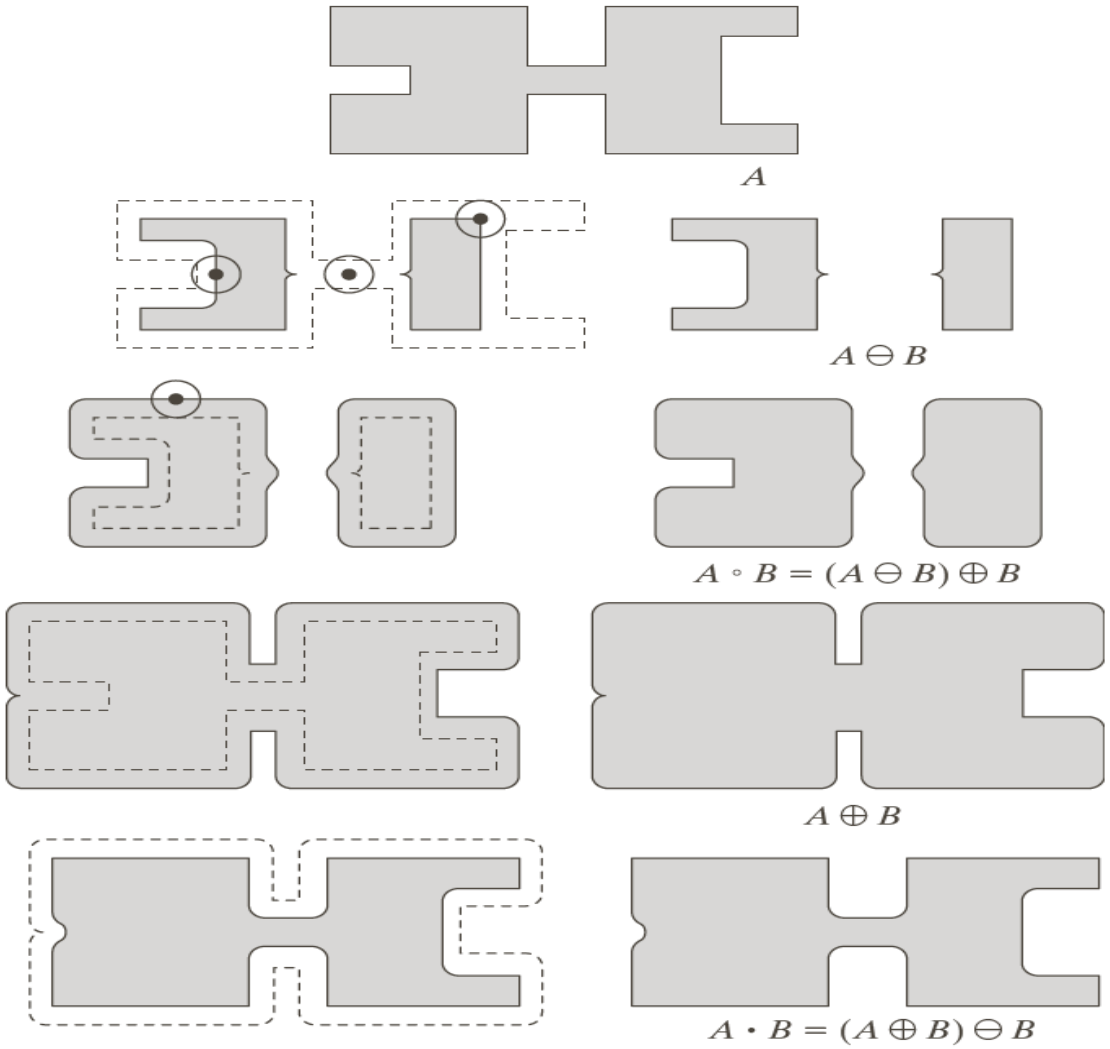
- Close



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

Morphological Image Processing



a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

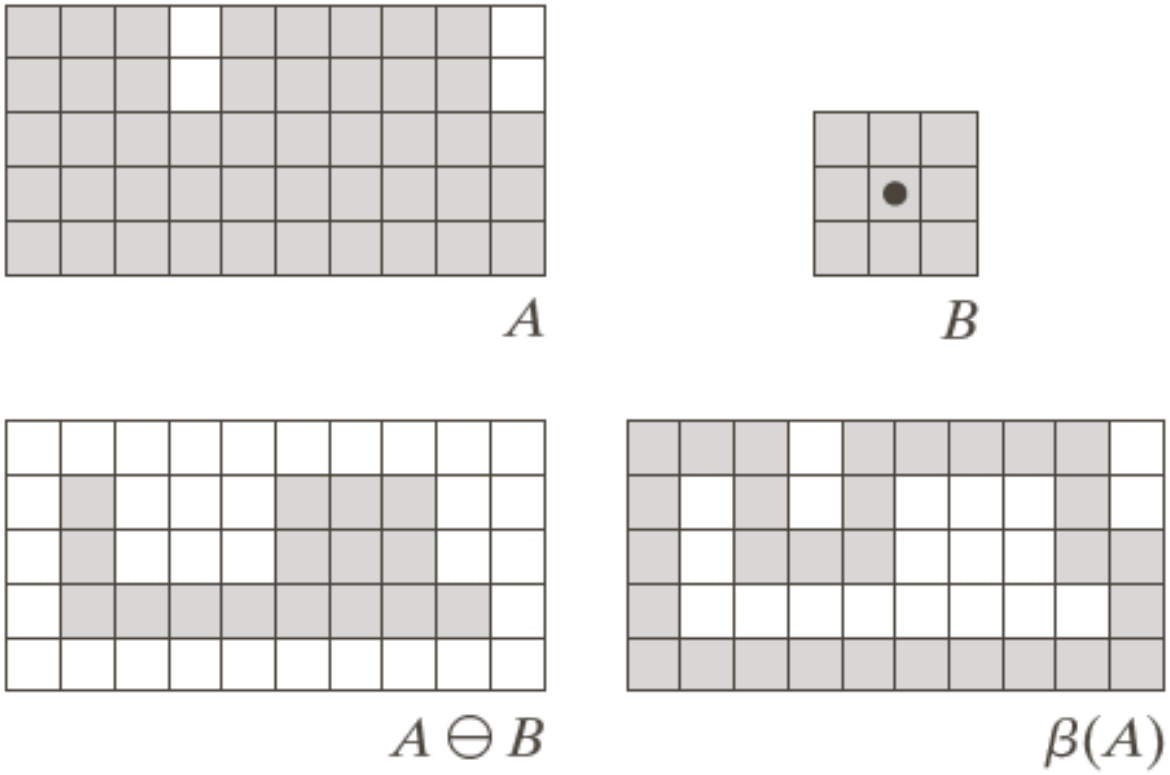
Morphological Image Processing

a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Morphological Image Processing

- Boundary



a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

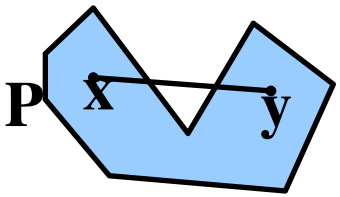
Morphological Image Processing

a b

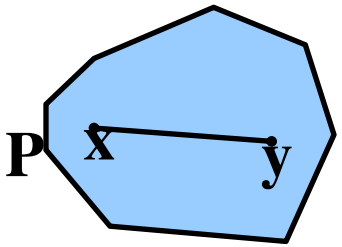
FIGURE 9.14
(a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Morphological Image Processing

- Convex hull
  - A polygon P is convex if for every pair of points x and y in P, the line xy is also in P; otherwise, it is called concave.



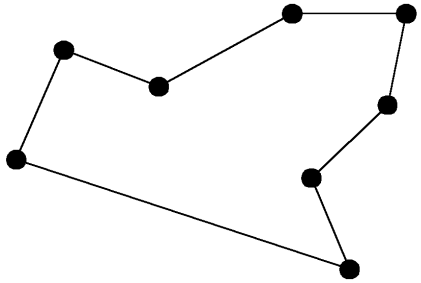
concave



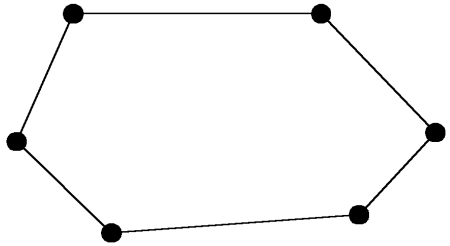
convex

Convex hull

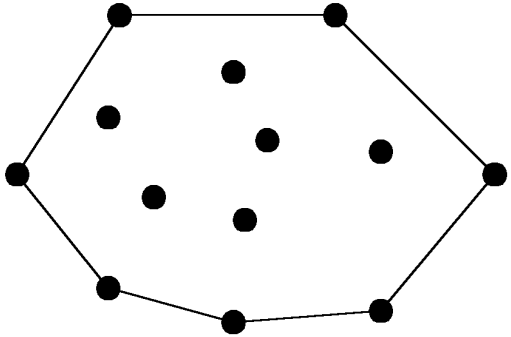
concave polygon:



convex polygon:



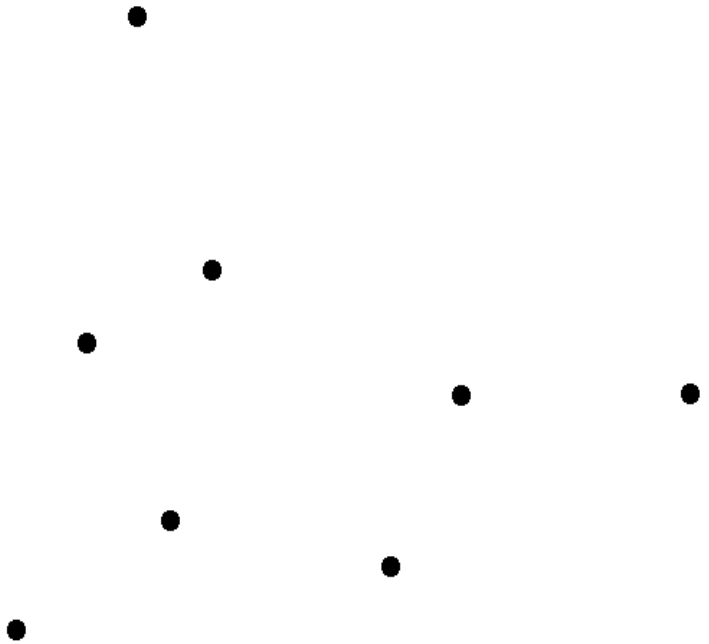
- The convex hull of a set of planar points is the smallest convex polygon containing all of the points.



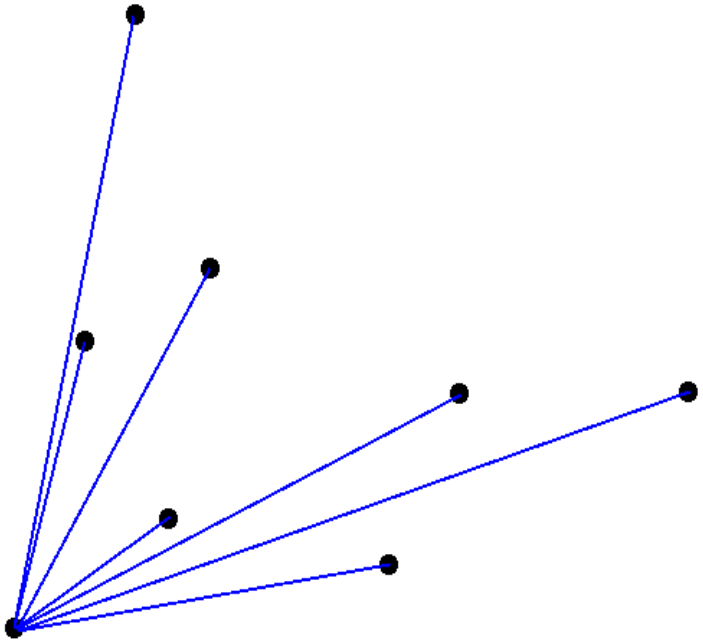
Graham's Scan

- Start at point guaranteed to be on the hull. (the point with the minimum y value)
- Sort remaining points by polar angles of vertices relative to the first point.
- Go through sorted points, keeping vertices of points that have left turns and dropping points that have right turns.

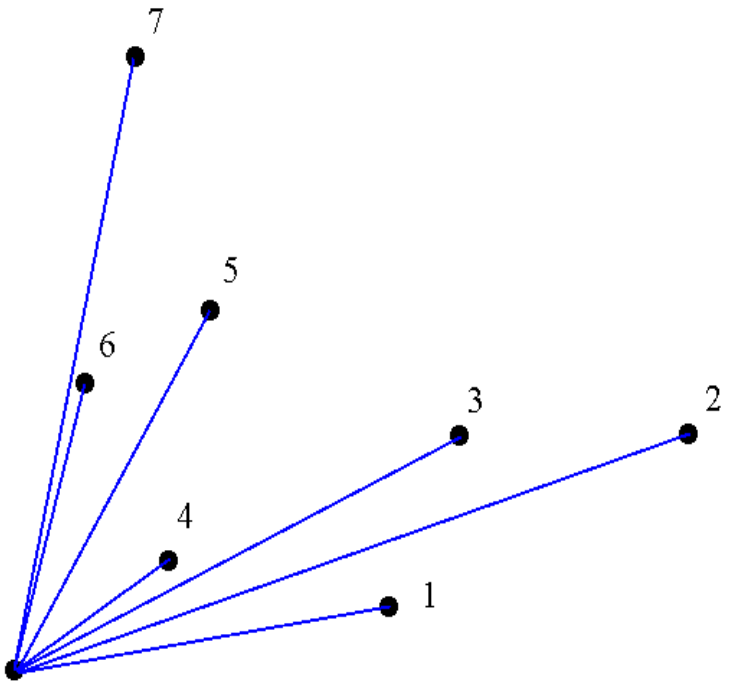
Graham's Scan



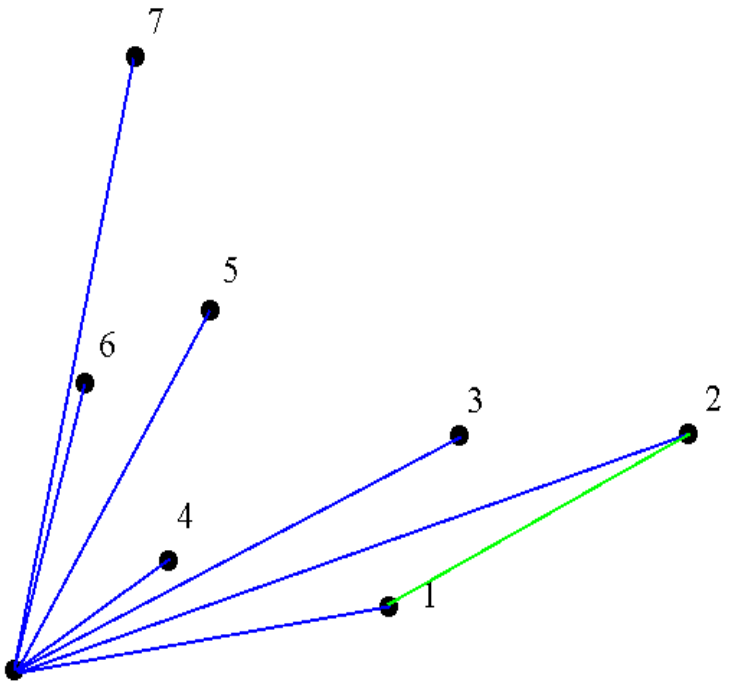
Graham's Scan



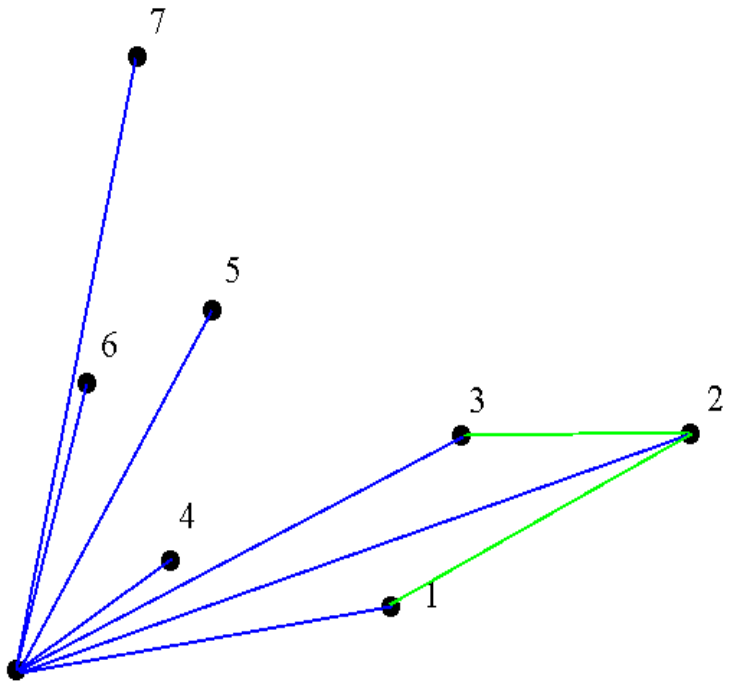
Graham's Scan



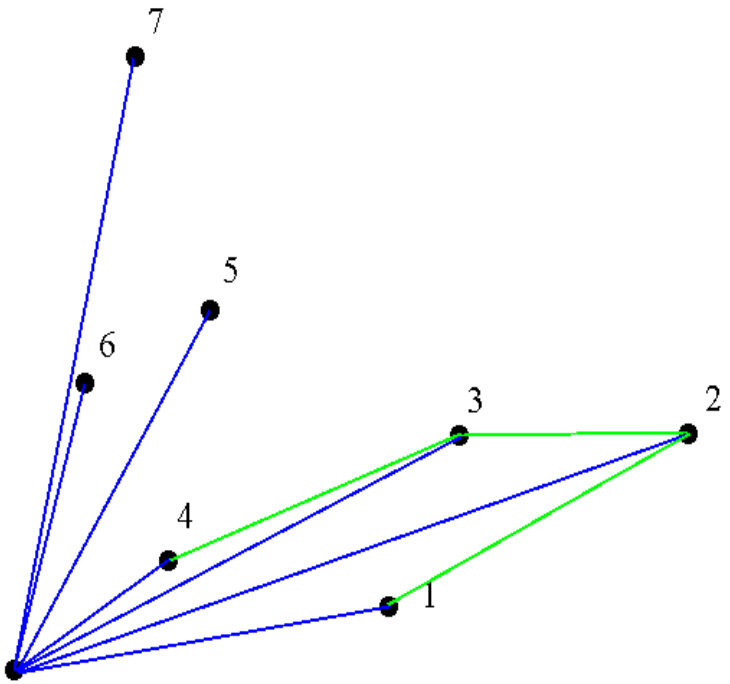
Graham's Scan



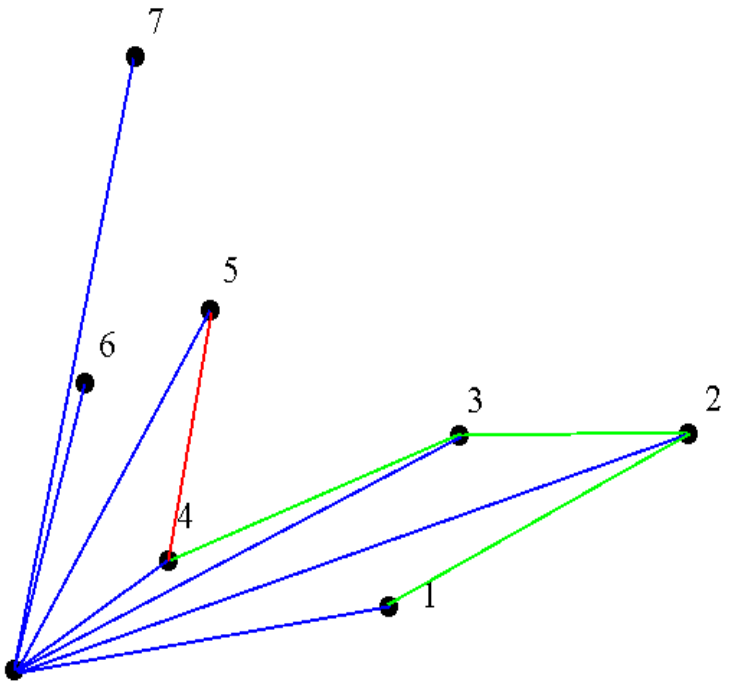
Graham's Scan



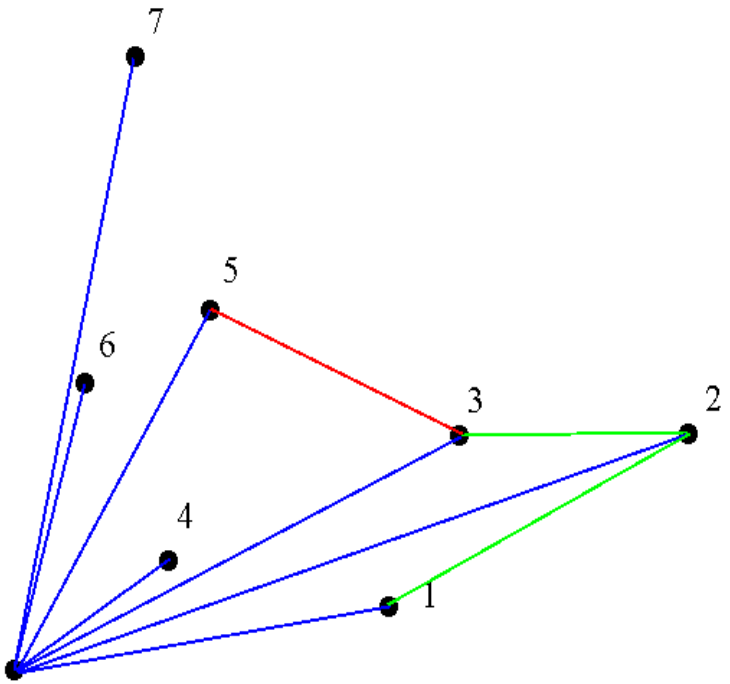
Graham's Scan



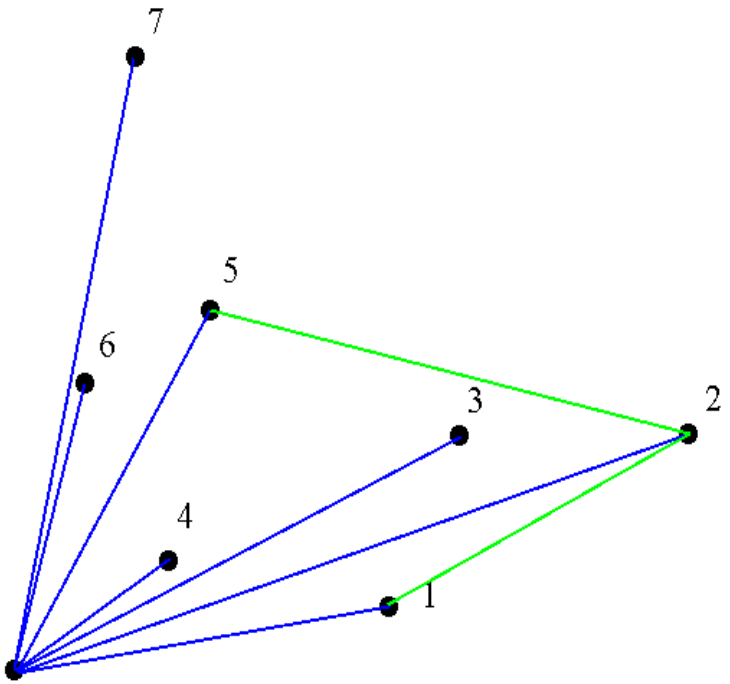
Graham's Scan



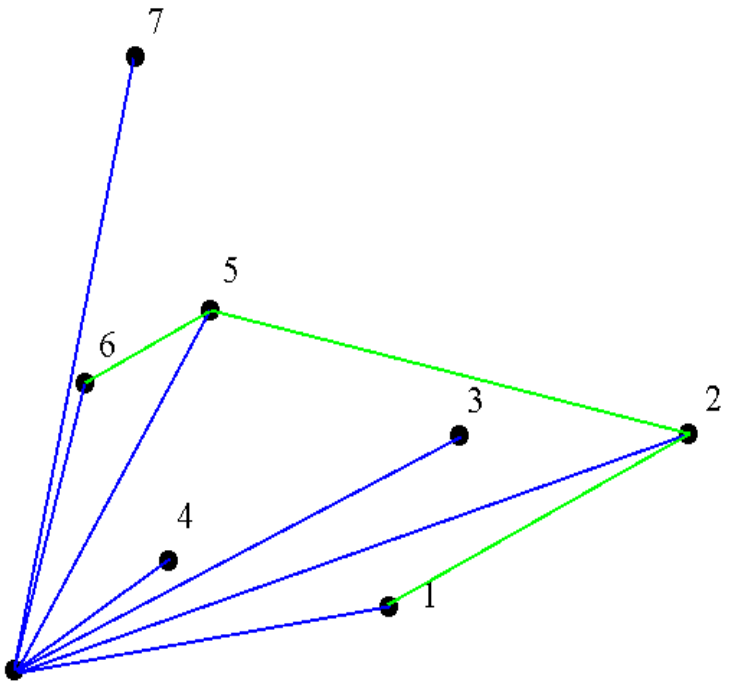
Graham's Scan



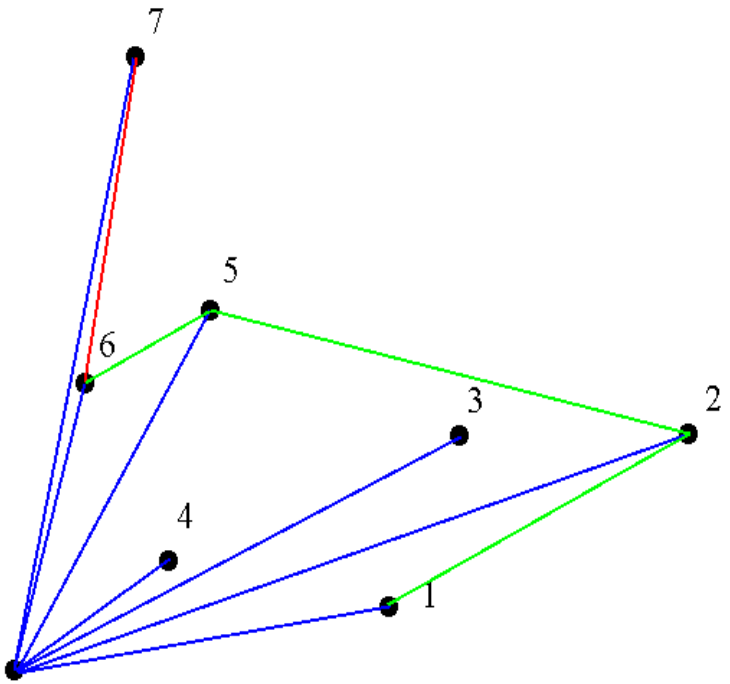
Graham's Scan



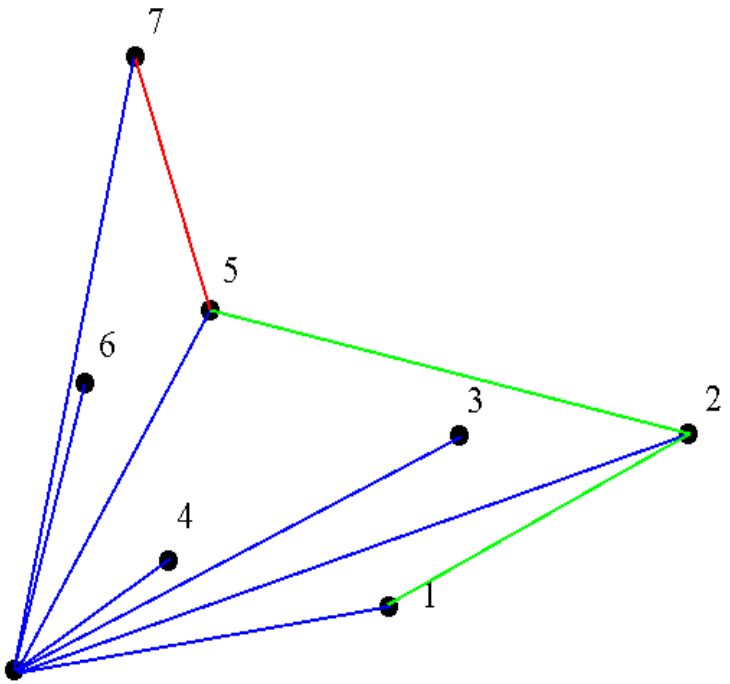
Graham's Scan



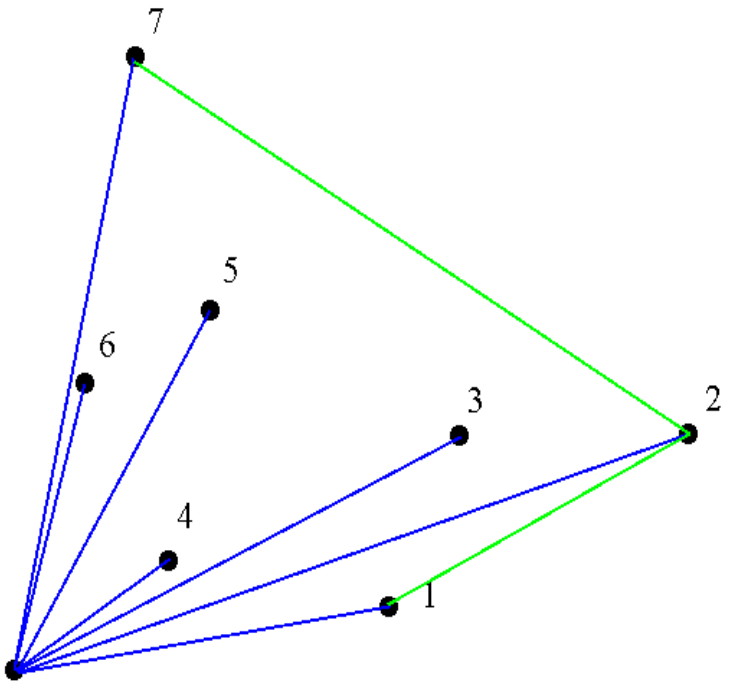
Graham's Scan



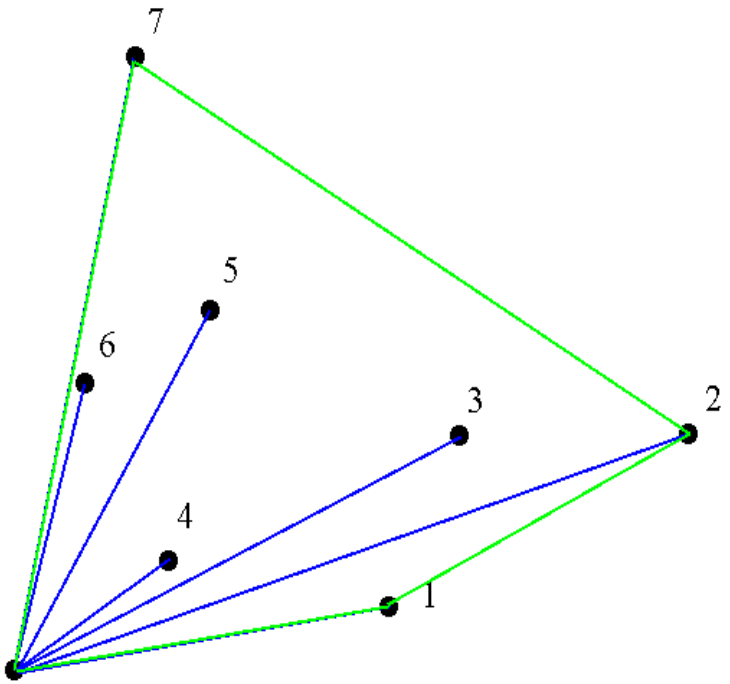
Graham's Scan



Graham's Scan



Graham's Scan



Graham's Runtime

- Graham's scan is O(n log n) due to initial sort of angles.

A more detailed algorithm

GRAHAM-SCAN(Q)

```
1   let p_0 be the point in Q with the minimum y-coordinate,
          or the leftmost such point in case of a tie
2   let ⟨p_1, p_2, ..., p_m⟩ be the remaining points in Q,
          sorted by polar angle in counterclockwise order around p_0
          (if more than one point has the same angle, remove all but
          the one that is farthest from p_0)
3   PUSH(p_0, S)
4   PUSH(p_1, S)
5   PUSH(p_2, S)
6   for i ← 3 to m
7       do while the angle formed by points NEXT-TO-TOP(S), TOP(S),
                    and p_i makes a nonleft turn
8              do POP(S)
9          PUSH(p_i, S)
10  return S
```

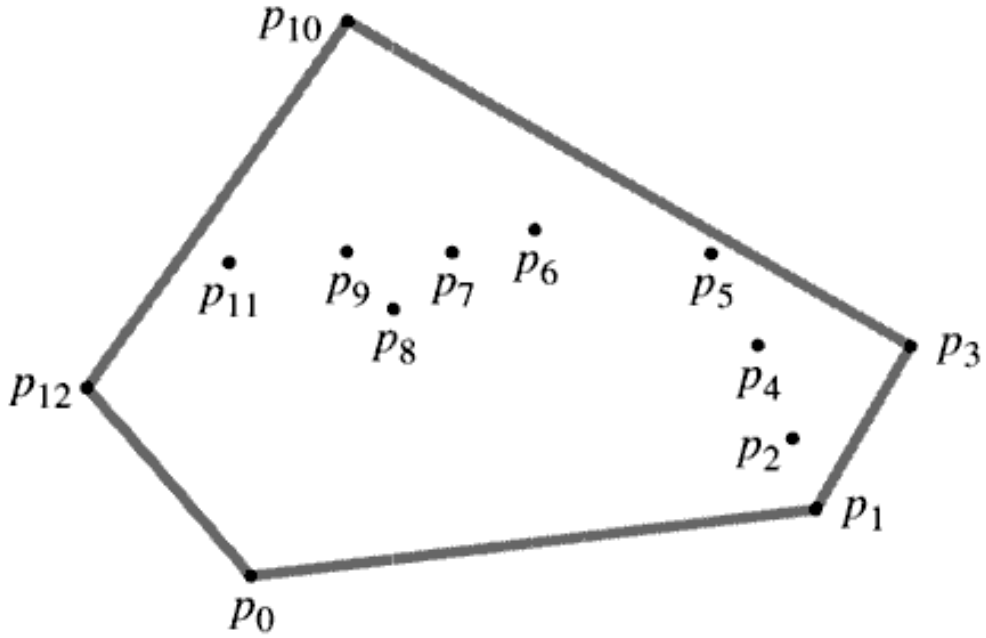


Figure 33.6 A set of points $Q = \{p_0, p_1, \ldots, p_{12}\}$ with its convex hull $\mathrm{CH}(Q)$ in gray.

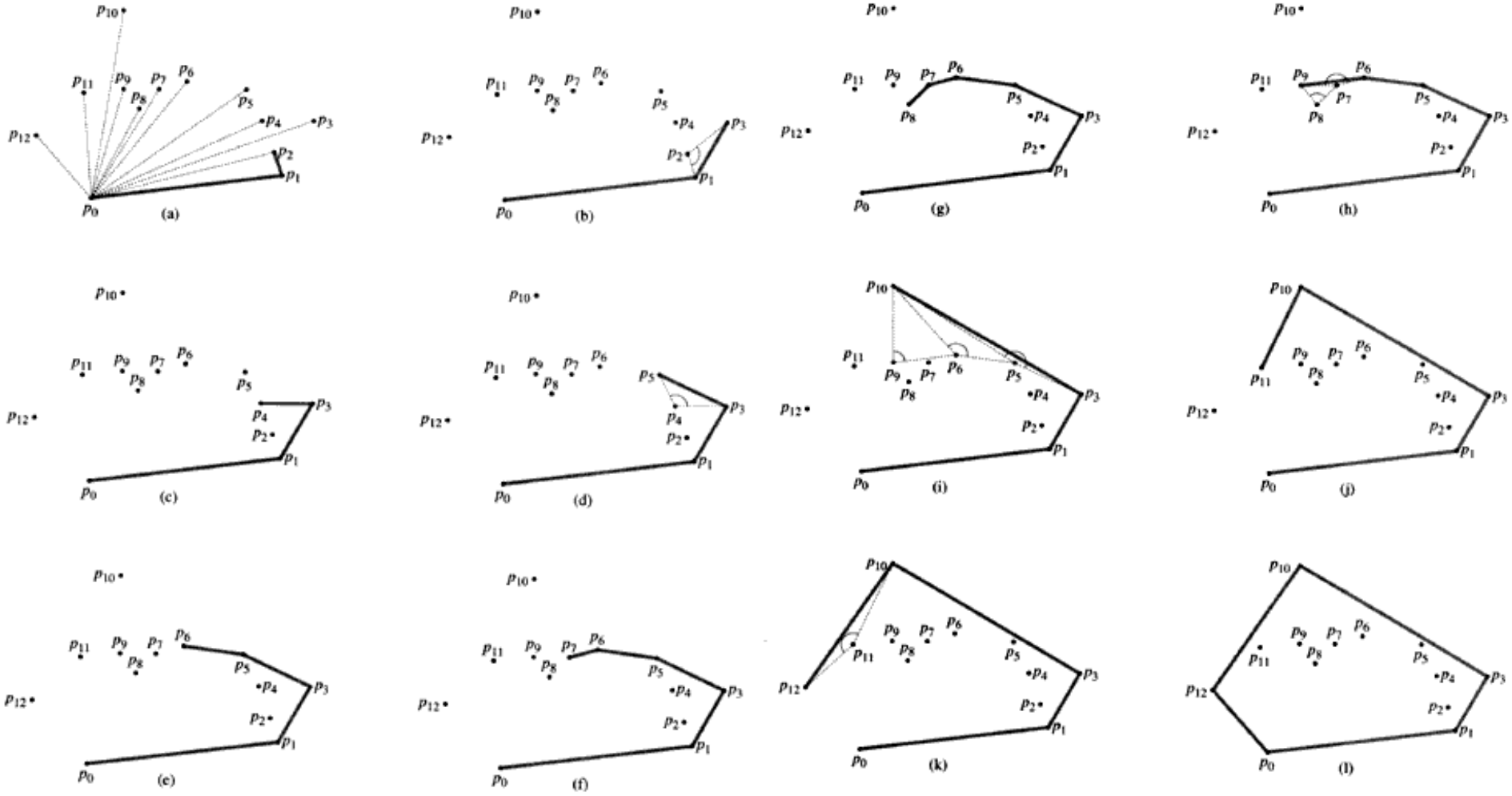
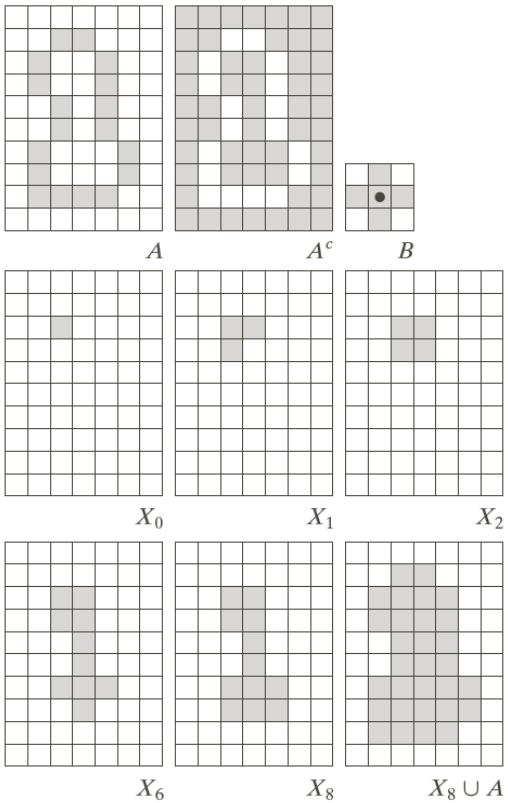


Figure 33.7 The execution of GRAHAM-SCAN on the set Q of Figure 33.6. The current convex hull contained in stack S is shown in gray at each step. **(a)** The sequence $\langle p_1, p_2, \ldots, p_{12} \rangle$ of points numbered in order of increasing polar angle relative to p_0, and the initial stack S containing p_0, p_1, and p_2. **(b)–(k)** Stack S after each iteration of the **for** loop of lines 6–9. Dashed lines show nonleft turns, which cause points to be popped from the stack. In part (h), for example, the right turn at angle $\angle p_7 p_8 p_9$ causes p_8 to be popped, and then the right turn at angle $\angle p_6 p_7 p_9$ causes p_7 to be popped. **(l)** The convex hull returned by the procedure, which matches that of Figure 33.6.

Morphological Image Processing

- Hole
- (background pixels enclosed by the foreground pixels)
- 1s or 0s

Morphological Image Processing



a b c
d e f
g h i

FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Morphological Image Processing

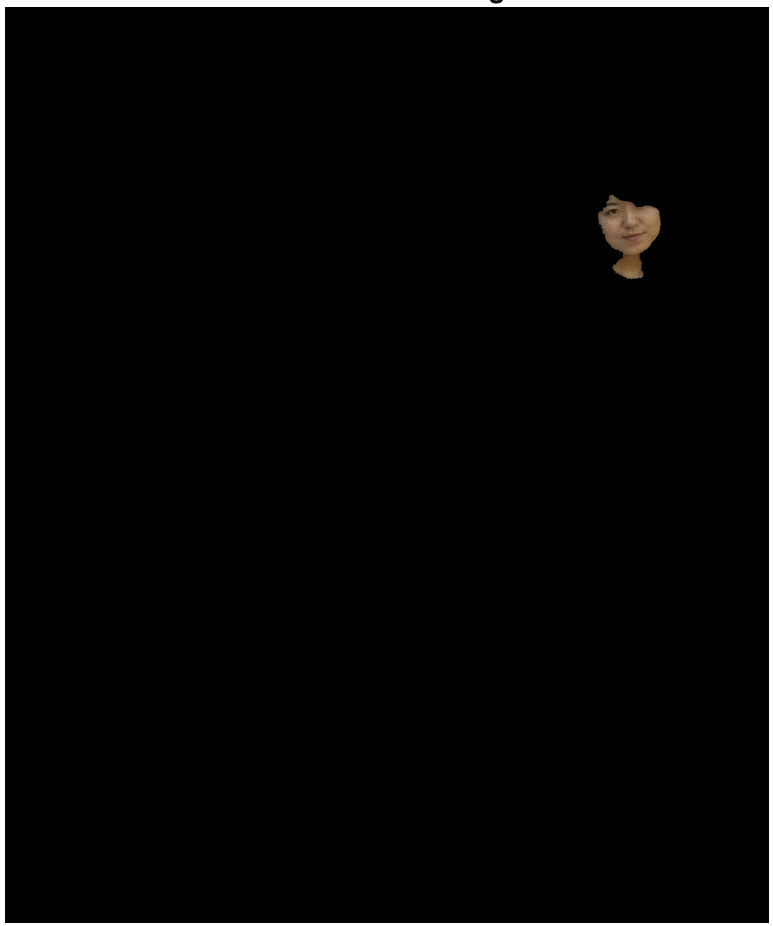
a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

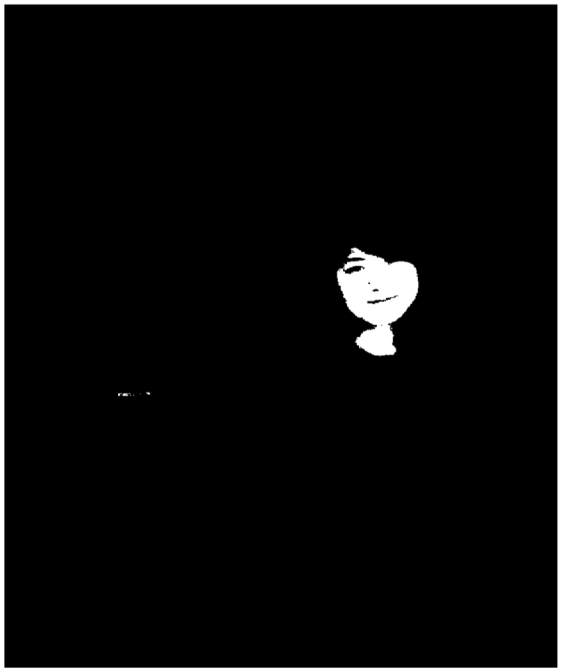
Exercise *face_detect_open_fill_ED.m*

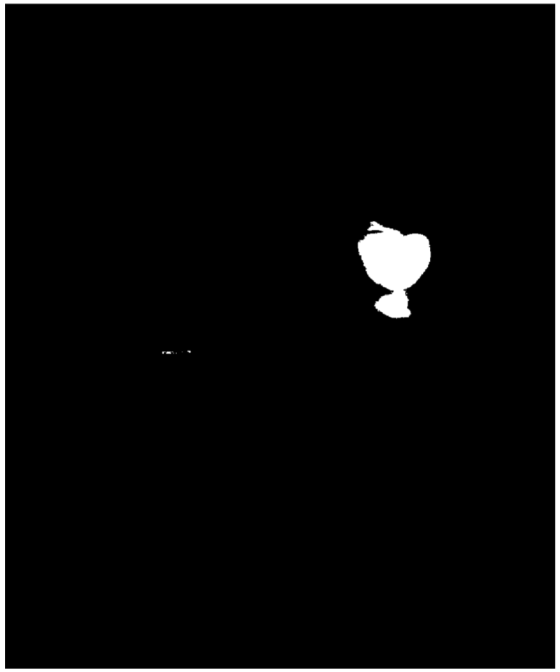
- **Strel()**
- **Imerode()**
- **Imdilate()**
- **Bwareaopen()**
- **Imfill(im,'holes')**
  - **help; e.g. FillingHoles.m**

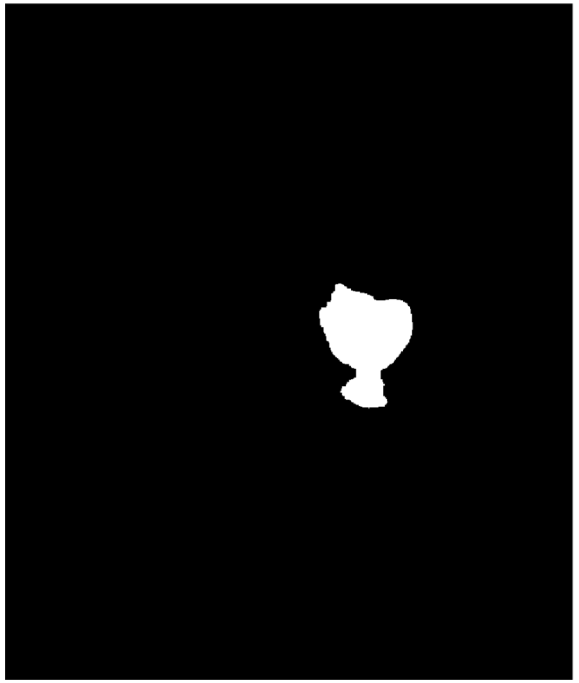
Detected Face Image

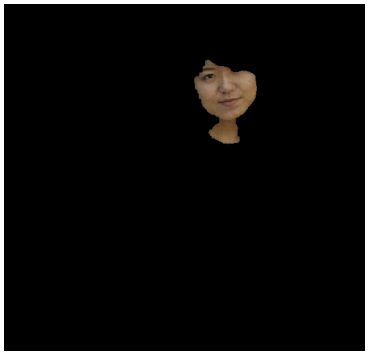


Exercise *face_detect_open_fill_ED.m*







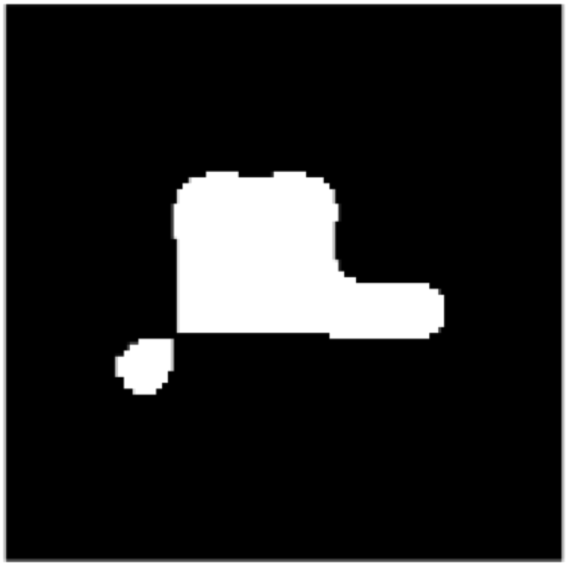


Exercise

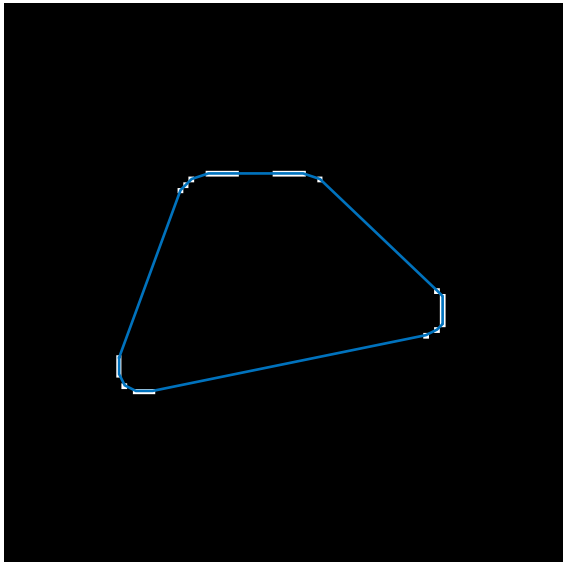
- **Face detection**

- Select one of your own image
- Perform skin detection
- Optimize your skin detection result by morphological methods we learned

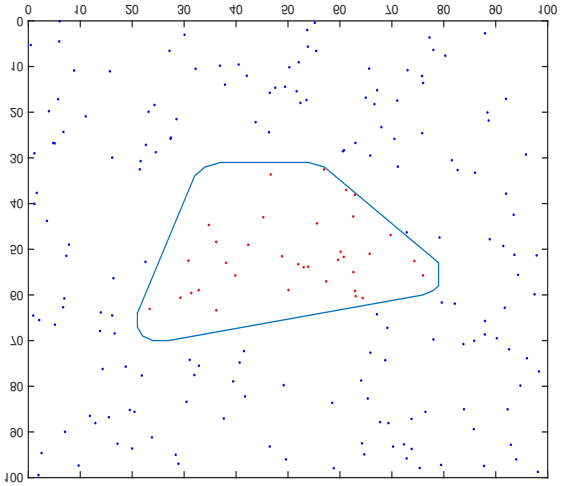
Exercise-convex hull



A test matrix



Convex hull of the test matrix



Classification result of random points

Exercise-convex hull

- **Test image generation**
  - **“convexhull_input.m”**
- **Convex hull calculation “convhull()” function**
  - **e.g. k=convhull(xt,yt)**
- **Show pixels on the convex hull**
  - **Imshow() and plot()**
- **Generate random points and check if they are inside or outside of the hull**
  - **Inpolygon() function and example code “PolygonTest.m”**