



COMPUTER VISION

Le Thanh Ha, Ph.D

Assoc. Prof. at University of Engineering and Technology,
Vietnam National University

ltha@vnu.edu.vn; lthavnu@gmail.com; 0983 692 592

Hough Transform

- The Hough transform is a technique which can be used to isolate features of a particular shape: line, circle, ellipse
- It uses a voting mechanism in parameter space for detecting these shapes.

Parameter function

- Object in image can be defined by a function of parameter (parameter function):

$$f(x, y, a_1, \dots, a_n) = 0$$

In which (x, y) is pixel position and $\{a_1, \dots, a_n\}$ is a set of n parameters specifying the shape.

Hough transform

- Transform to parameter space:
 - Every pixel (x,y) in a 2D image is transformed into a surface of n -dimension space.
- Transform back to image spatial space:
 - Every point (a_1, \dots, a_n) in the n -dimension space is transformed to a shape in the 2D image.

Line detection

- Parameter function to be used:

$$y = ax + b$$

- It is a line in Image
- But, it is only a point (a,b) in parameter space.

Line Detection

- A point $(x_0, y_0) \in I$ corresponds to the line $a = y_0/x_0 - b/x_0$ in the parameter space P .
- And, a point $(a_0, b_0) \in P$ corresponds to the line $y = a_0x + b_0$ in image space I .

Line detection

- Construct the line function of following image by hough transform:

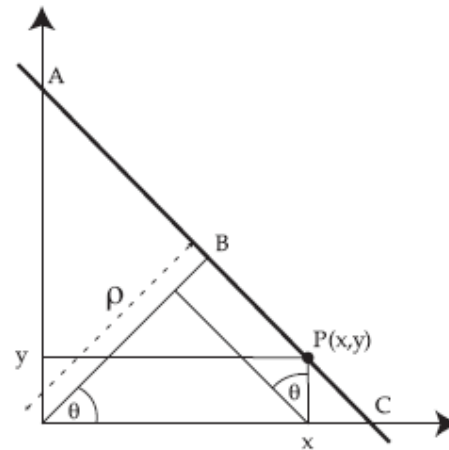
0	0	1	0
0	1	0	0
1	0	0	0
0	0	0	0

Line detection

- Another line function can be used:

$$f(x, y, \rho_0, \theta_0) = x \cos \theta_0 + y \sin \theta_0 - \rho_0 = 0$$

in which ρ_0 là is the distance between line and the origin, and θ_0 là is the angle between the distance vector and Ox axis.



Line detection algorithm

- Create a 2D matrix $H(\rho, \theta)$ for the parameter space. Initialize the matrix with 0 values.
- Find gradient $G(x,y)$ of $I(x,y)$
- For pixel (x,y) satisfies $|G(x,y)| > T_s$, increase $H(\rho, \theta)$ such that:

$$\forall \theta \quad | \quad \rho = x \cos \theta + y \sin \theta$$

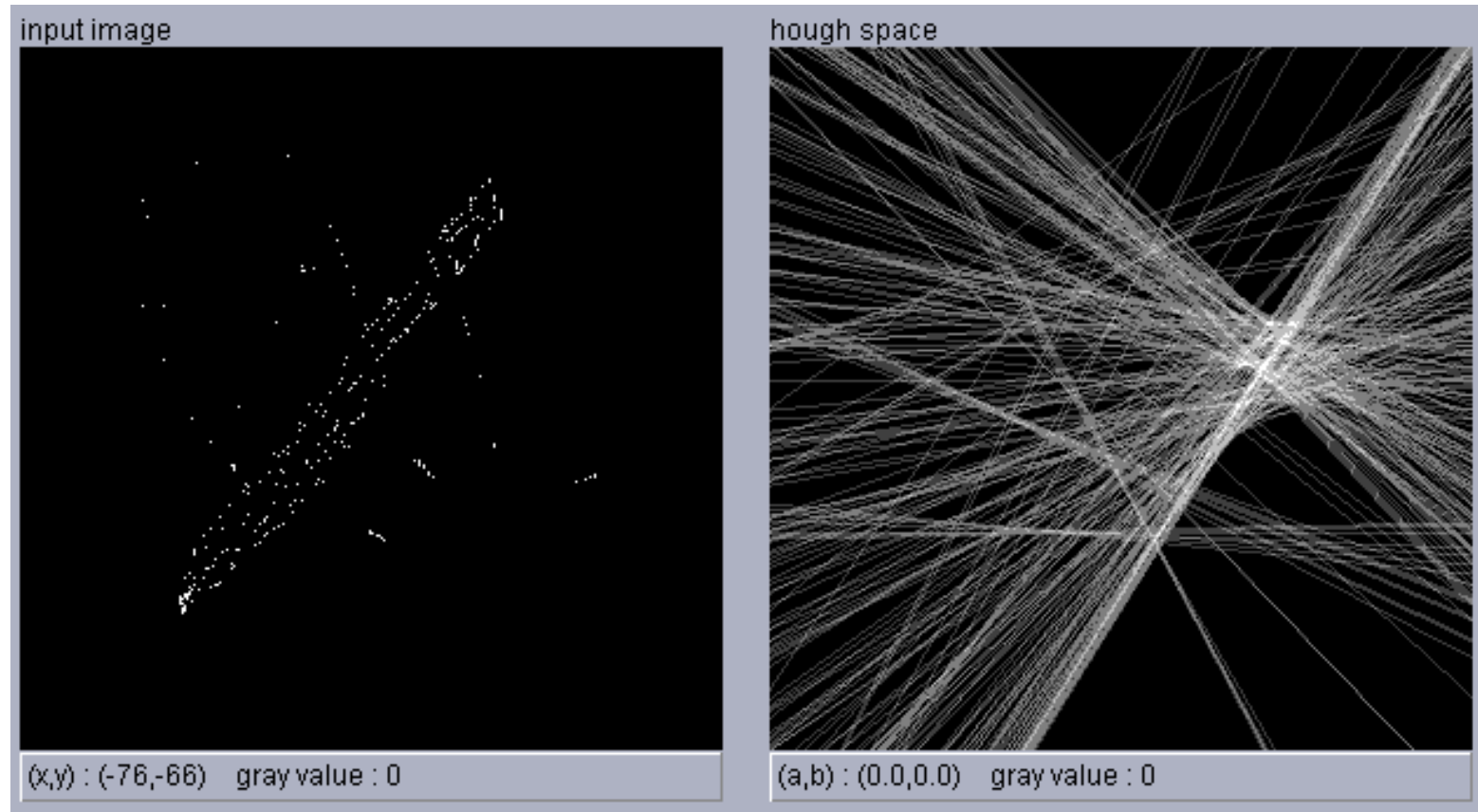
$$H(\rho, \theta) = H(\rho, \theta) + 1;$$

Line detection algorithm

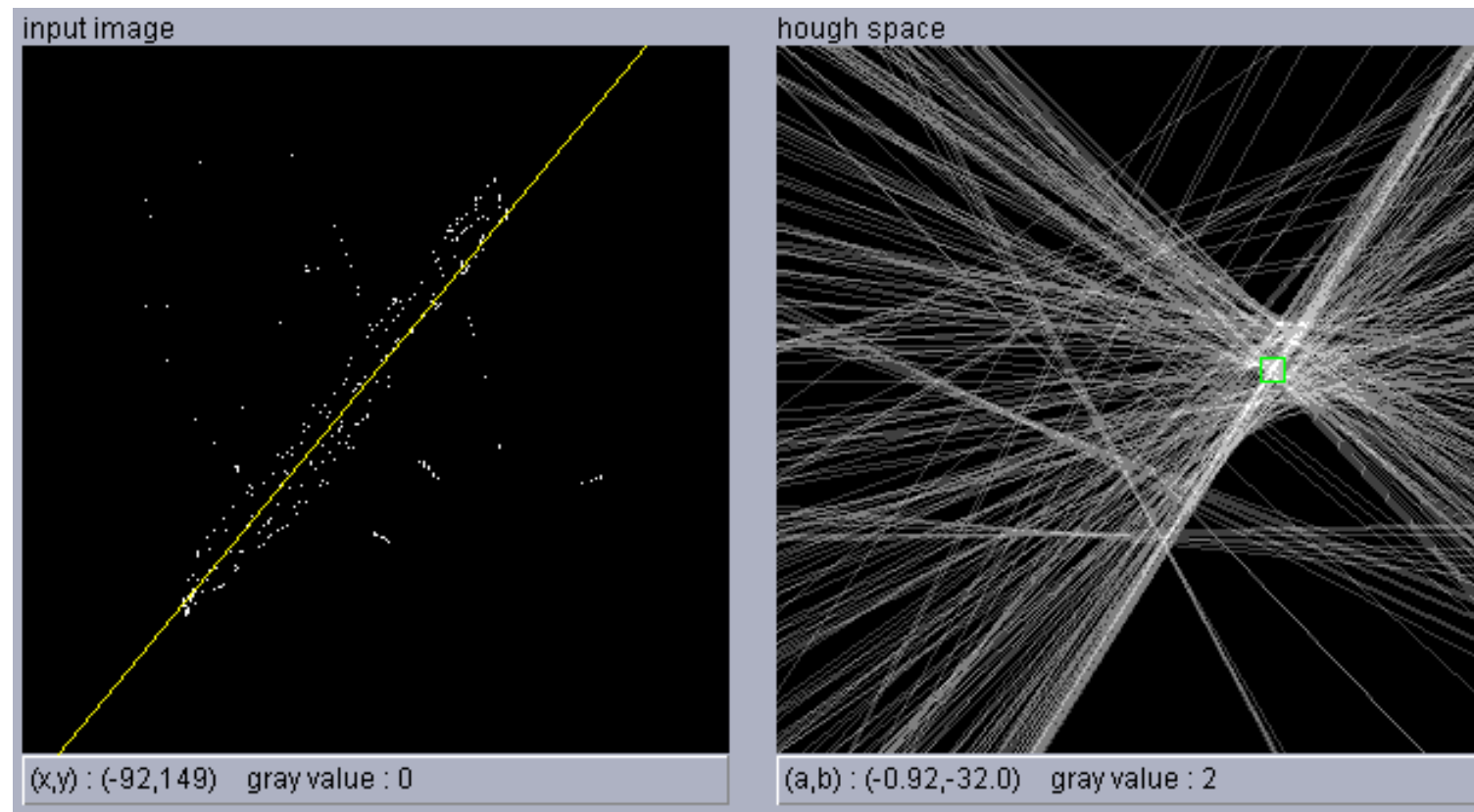
- This algorithm can be improved by using the gradient angle $\angle G(x,y)$

$$\begin{aligned}\forall \theta \quad & | \quad \angle G(x,y) - \Delta\theta \leq \theta \leq \angle G(x,y) + \Delta\theta \\ & \rho = x \cos \theta + y \sin \theta \\ & H(\rho, \theta) = H(\rho, \theta) + 1;\end{aligned}$$

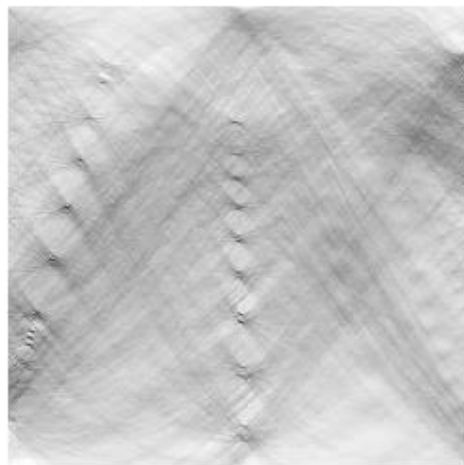
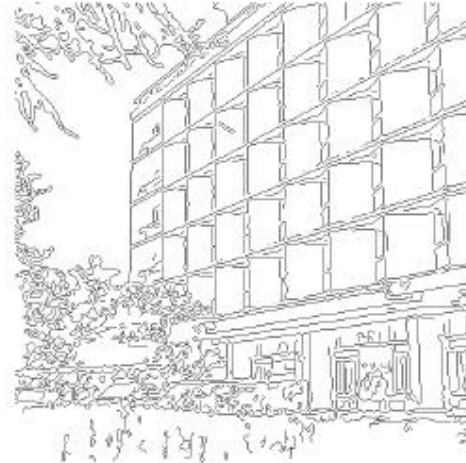
Line detection



Line detection



Line detection



Circle detection

- A circle with radius R at position (a,b) can be specified by a function:

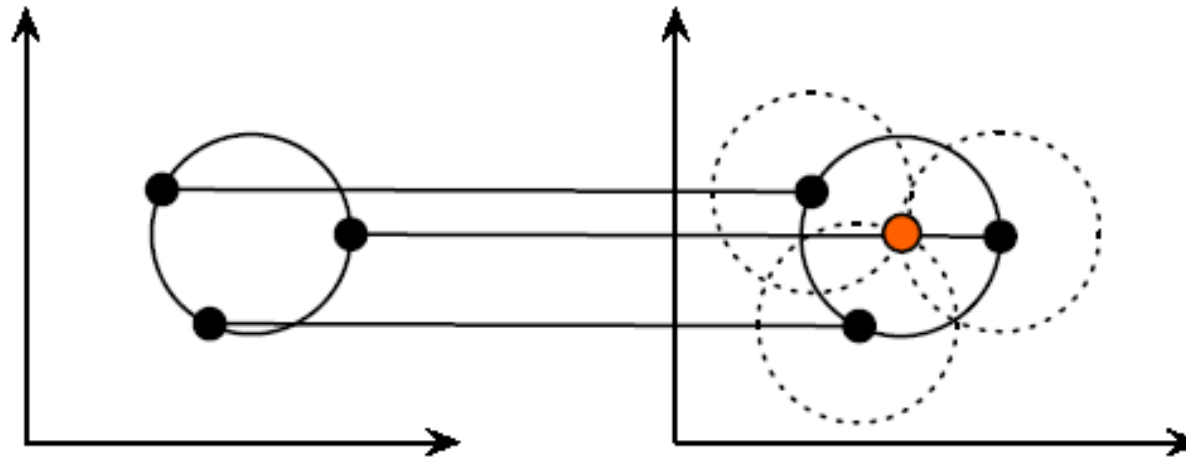
$$x = a + R \cos(\theta)$$

$$y = b + R \sin(\theta)$$

- θ scans from 0 to 360 degree
- A pixel belonging to a circle corresponds to a set of (a,b,θ)

Circle detection

- With a fixed radius R , (a,b) has a circle orbit centered at (x,y) and radius R .
- The coordinate of the real circle can be found by a cumulative table



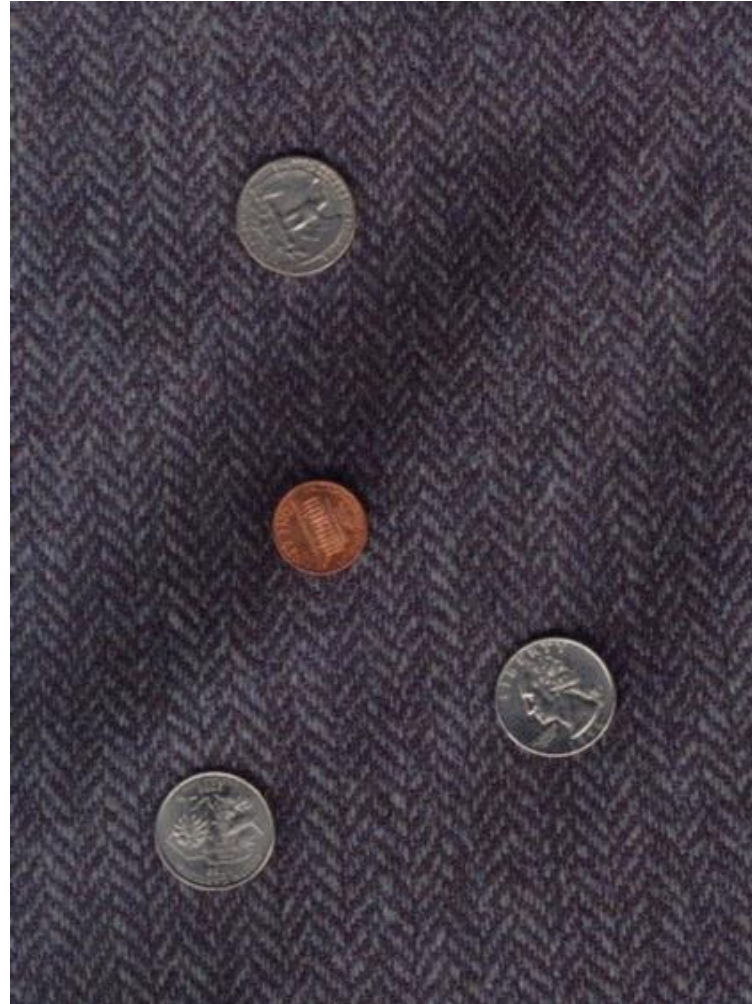
Circle Detection

- For pixel (x,y) satisfies $|G(x,y)| > T_s$, increase the cumulative table in parameter space such that:

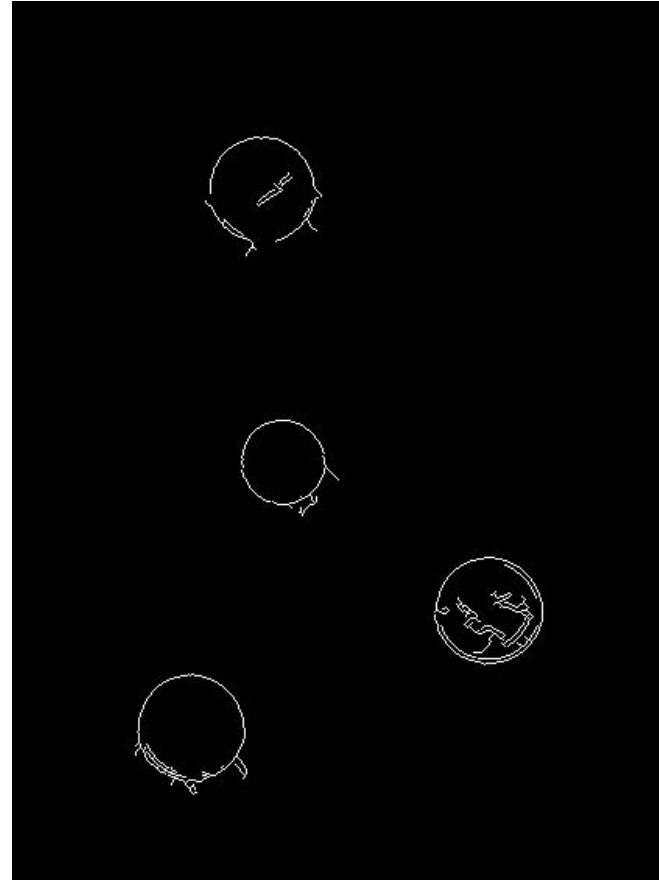
$$\forall R, \quad \begin{cases} a = x \pm R \cos \angle G \\ b = y \pm R \sin \angle G \end{cases}$$

$$H(a,b,R) = H(a,b,R) + 1$$

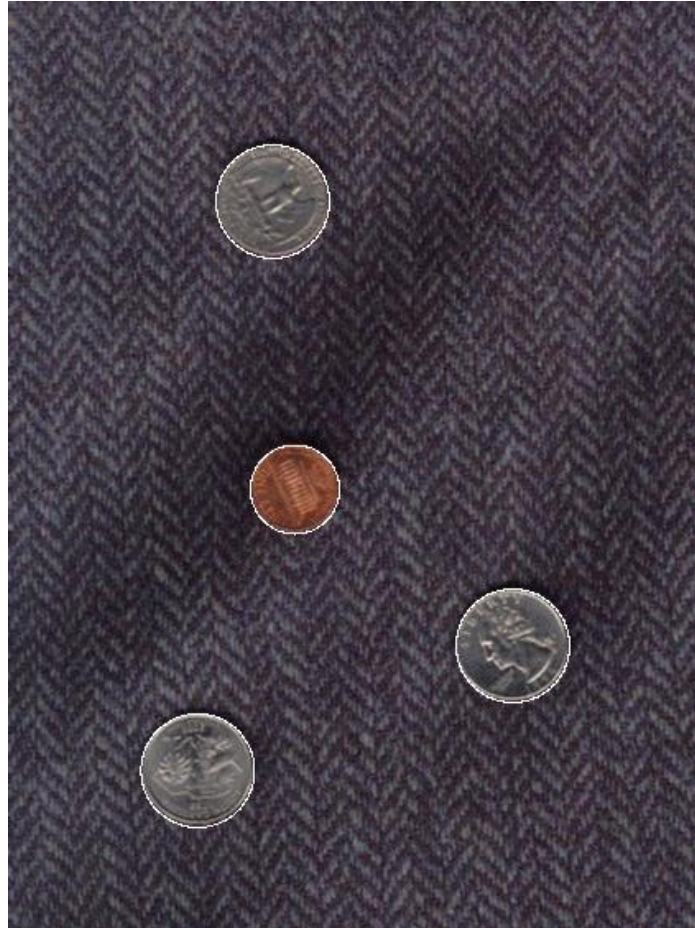
Circle Detection



Circle Detection



Circle Detection



Summary

- Hough transform
- It can be used to detect some image features having a particular shape properties: line, circle, ...