Sunday, September 29, 2024 11:36 PM

1)

$f(x) = \sin(x)$

$$Df(x) \cdot v = \frac{d}{d\varepsilon} f(x + \varepsilon v) \Big|_{\varepsilon=0}$$

$$D\sin(x) \cdot v = \frac{d}{d\varepsilon} \sin(x + \varepsilon v) \Big|_{\varepsilon=0}$$

$$= v\cos(x + \varepsilon v) \Big|_{\varepsilon=0}$$

$$= v\cos(x)$$

2) Given a function

$$J(x(t)) = \int_0^{\frac{\pi}{2}} \frac{1}{2} x(t)^2 dt$$

compute the analytical solution when

$x = \cos(t)$

verify the answer by numerical integration.

$$J(x(t)) = \int_0^{\frac{\pi}{2}} \frac{1}{2} x(t)^2 dt \iff \int_0^{\frac{\pi}{2}} \frac{1}{2} \cos(t)^2 dt \iff \int_0^{\frac{\pi}{2}} \frac{c\text{i}}{}$$

3) $x(t) = \cos(t)$

$v(t) = \sin(t)$

$$Df(x) \cdot v = \frac{d}{d\varepsilon} f(x + \varepsilon v) \Big|_{\varepsilon=0}$$

$$J(x(t)) = \int_0^{\frac{\pi}{2}} \frac{1}{2} x(t)^2 \, dt$$

$$J(x(t)) = \int_0^{\frac{\pi}{2}} \frac{d}{d\varepsilon} \frac{1}{2}\left(x(t) + \varepsilon v(t)\right)^2 dt \Big|_{\varepsilon = 0}$$

$$= \int_0^{\pi/2} \left(x(t) + \varepsilon v(t)\right) v(t) \, dt \Big|_{\varepsilon = 0}$$

$$= \int_0^{\pi/2} x(t) \, v(t) \, dt$$

$$= \int_0^{\pi/2} \sin(t) \cos(t) \, dt$$

$$= \frac{1}{2} \int_0^{\pi/2} \sin(2t) \, dt$$

$$= \Big|_0^{\pi/2} -\frac{1}{4} \cos(2t)$$

$$= -\frac{1}{4} \cos(\pi) + \frac{1}{4} (\cos 0)$$

$$= \frac{1}{4}\left(\cos(0) - \cos(\pi)\right) = \frac{1}{2}$$

4)