ξ - с. в. $\qquad 0 \le \mathbb{P} \le 1$

$$\mathbb{P}(\xi < x) = \int_{-\infty}^{x} f_\xi(t)\,dt$$

1. $f_\xi(x) \ge 0$

2. $\mathbb{P}(\xi < +\infty) = \underbrace{\int_{-\infty}^{+\infty} f_\xi(t)\,dt = 1}_{\text{нормировки плотности}}$

3. $\mathbb{P}(a < \xi < b) = \int_{a}^{b} f_\xi(t)\,dt$

4. $\mathbb{P}(\xi = a) = \int_{a}^{a} \ldots dt = 0$

$F_\xi(x)$ - функция распределения

$F_\xi(x) = \mathbb{P}(\xi < x)$

№1 Кубик - честный

$\xi \in \{1 \dots 6\}$

$\mathbb{E}\xi = \sum_{k=1}^{6} k \cdot p_k = 3.5$

$\longrightarrow \infty$ бросать кубик 3 4

ξ , $\underline{f_\xi(x)}$ $\underline{\mathbb{E} e^{\xi^2 + \frac{1}{\xi}} - ?}$

$$\mathbb{E} e^{\xi^2 + \frac{1}{\xi}} = \int_{-\infty}^{+\infty} e^{x^2 + \frac{1}{x}} f_\xi(x)\, dx$$

$\sin(\xi)$

$\cos(\xi)$

$$\mathbb{D}\xi = \mathbb{E}\left[(\xi - \mathbb{E}\xi)^2\right] =$$

$$= \mathbb{E}\left(\xi^2 - 2\xi \cdot \mathbb{E}\xi + (\mathbb{E}\xi)^2\right) =$$

$$= \mathbb{E}(\xi^2) - 2\,\mathbb{E}(\xi\,\mathbb{E}\xi) + \mathbb{E}(\mathbb{E}\xi)^2 =$$

$$= \mathbb{E}(\xi)^2 - 2\,\mathbb{E}\xi \cdot \mathbb{E}\xi + (\mathbb{E}\xi)^2 =$$

$$= \mathbb{E}\xi^2 - (\mathbb{E}\xi)^2$$

$$\mathbb{E}\xi^2 = \int_{-\infty}^{+\infty} x^2 f_\xi(x)\,dx$$

$D\xi \geq 0$

$D(\xi \pm \eta) - ?$

Если ξ, η - независимы

$D(\xi \pm \eta) = D\xi + D\eta$

Если ξ, η - зависимы

$D(\xi \pm \eta) = D\xi + D\eta \pm 2\operatorname{cov}(\xi, \eta)$

$\xi = \eta$

$\operatorname{cov}(\xi, \xi) = D\xi$

Теорема Коши-Буняковского

$|\operatorname{cov}(\xi, \eta)| \leq \sqrt{D\xi \cdot D\eta}$

1. $D\xi = 0$ или $D\eta = 0 \Rightarrow$

$\Rightarrow \xi = E\xi \Rightarrow \operatorname{cov}(\xi, \eta) = 0 \quad 0 \leq 0$

2. $\sqrt{D\xi \cdot D\eta} \neq 0$

$x\xi - \eta$

$0 \leq D(x\xi - \eta) =$

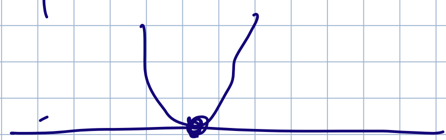
$\{ D(a\xi) = a^2 D\xi \} =$

$= x^2 D\xi + D\eta - 2x\,\mathrm{cov}(\xi, \eta)$

$f = ax^2 + bx + c \quad \forall x \quad f \geq 0 \Rightarrow$

$\Rightarrow D$ - дискриминант

$D \leq 0$



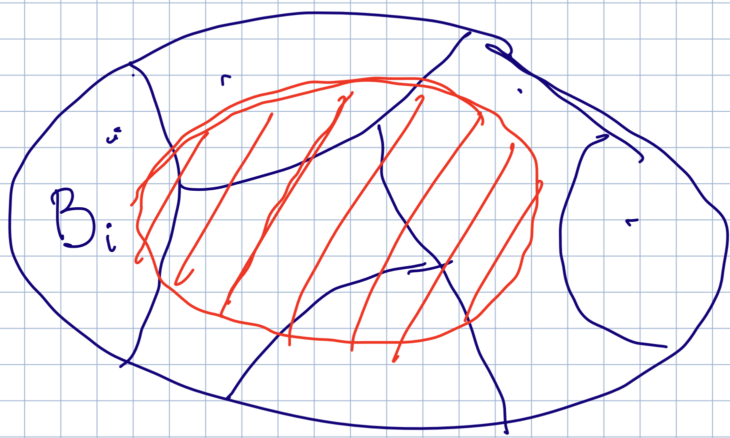
$D = (\mathrm{cov}(\xi, \eta))^2 - D\xi \cdot D\eta \leq 0$

$|\mathrm{cov}(\xi, \eta)| \leq \sqrt{D\xi \cdot D\eta}$

$\mathrm{corr}(\xi, \eta) \in [-1; 1]$

corr = нормированная степень лин. зав.

1. Полная группа событий



$B_i \cap B_j = \varnothing$, $i \neq j$

$\bigcup_i B_i = \Omega$

$\mathbb{P}(\xi = 5)$

1. $\xi \sim Be(p)$, $p > 0$

$\mathbb{P}(\xi = 1) = p$ $\quad$ $\mathbb{P}(\xi = 0) = 1 - p$

$\mathbb{E}\xi$ – ? $\quad$ $\mathbb{E}\xi = p$

$\mathbb{D}\xi = p(1-p)$

2. $\xi \sim Pois(\lambda)$, $\lambda > 0$

$$\mathbb{P}(\xi = k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$$

$$k \in \mathbb{N} \cup \{0\} = \mathbb{Z}_+$$

$$\mathbb{E}\xi = \sum_{k=1}^{\infty} k \cdot e^{-\lambda} \cdot \frac{\lambda^k}{k!} =$$

$$= \sum_{k=1}^{\infty} e^{-\lambda} \cdot \frac{\lambda^k}{(k-1)!} = \lambda \cdot e^{-\lambda} \underbrace{\sum_{k=1}^{\infty} \frac{\lambda^{k-1}}{(k-1)!}}_{\text{ряд Тейлора } e^{\lambda}} =$$

$$= \lambda \cdot e^{-\lambda} \cdot e^{\lambda} = \lambda$$

$$\mathbb{D}\xi = \mathbb{E}\xi^2 - (\mathbb{E}\xi)^2$$

второй момент

$$\mathbb{E}\xi^2 = \mathbb{E}\xi(\xi - 1) + \mathbb{E}\xi =$$

$$= \mathbb{E}(\xi^2 - \xi) + \mathbb{E}\xi = \mathbb{E}\xi^2 - \mathbb{E}\xi + \mathbb{E}\xi =$$

$$= \mathbb{E}\xi(\xi - 1) + \mathbb{E}\xi =$$

$$= \sum_{k=1}^{+\infty} k(k-1)e^{-\lambda} \cdot \frac{\lambda^k}{k!} + \lambda =$$

$$= e^{-\lambda} \cdot \lambda^2 \sum_{k=2}^{+\infty} \frac{\lambda^{k-2}}{(k-2)!} + \lambda = \lambda^2 + \lambda$$

$$\mathbb{D}\xi = \lambda^2 + \lambda - \lambda^2 = \lambda$$

$$\mathbb{D}\xi = \mathbb{E}\xi = \lambda$$

$$\longrightarrow \mathcal{N}$$

$$Bi(n,p) \sim Pois(\lambda)$$

$$Bi(n,p) \xrightarrow[n \cdot p = \bar{o}(1)]{} Pois(\lambda)$$

$$\mathbb{P}(\xi = k) = C_n^k p^k (1-p)^{n-k}$$

n – велико

p – мало ?

N 4 $\xi \sim \mathcal{N}(a, \sigma^2)$

$\mathbb{E}\xi = a$ — ?

$$f_\xi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-a)^2}{2\sigma^2}\right\}$$