⑥ $f_1(y) = \frac{1}{\sqrt{2\pi D_x}} e^{-\frac{(x-m_x)^2}{2D}}$, $f_2(y) = \frac{1}{\sqrt{2\pi D_y}} e^{-\frac{(y-m_y)^2}{2D}}$

Формула свёртки

$$f_3(z) = \int_{-\infty}^{\infty} f_2(z-x) f_1(x)\, dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi D_y}} \cdot e^{-\frac{(z-x-m_y)^2}{2D_y}} \cdot \frac{1}{\sqrt{2\pi D_x}} \cdot e^{-\frac{(x-m_x)^2}{2D_x}} dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \cdot 2\pi \cdot D_x \cdot D_y}} \cdot$$

$$\exp\left[-\frac{D_x(z-x-m_y)^2 + D_y(x-m_x)^2}{2D_yD_x}\right] dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \cdot 2\pi \cdot D_x \cdot D_y}} \cdot \exp\left[-\frac{D_x(z^2+x^2+m_y^2-2xz-2zm_y}{}\right.$$

$$\left.\frac{+2xm_y) + D_y(x^2+m_x^2-2xm_x)}{2D_xD_y}\right] dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \cdot 2\pi \cdot D_x \cdot D_y}} \cdot \exp\left[-\frac{x^2(D_x+D_y) - 2x(D_x(z-m_y)+D_y m_x) \oplus}{}\right.$$

$$\left.\frac{D_x(z^2+m_y^2-2zm_y) + D_y m_x^2}{2D_xD_y}\right] dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \cdot D_z} \cdot \frac{1}{\sqrt{2\pi} \cdot \sqrt{\frac{D_x \cdot D_y}{D_z}}} \exp\left[-\frac{x^2 - 2x\frac{D_x(z-m_y)+D_y \cdot m_x}{D_z} \oplus}{\frac{2\,D_xD_y}{D_z}}\right]$$

$D_z = D_x + D_y$

$$\oplus \frac{D_x(z^2+m_y^2-2zm_y)+D_y\cdot m_x^2}{\quad}\Big]dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi D_z}} \cdot \frac{1}{\sqrt{2\pi}\cdot\sqrt{\frac{D_x\cdot D_y}{D_z}}} \exp\Big[-\frac{\left(x-\frac{D_x(z-m_y)+D_y m_x}{D_z}\right)^2}{\quad}\ominus$$

$$\ominus\frac{\left(\frac{D_x(z-m_y)+D_y m_x}{D_z}\right)^2+\frac{D_x(z-m_y)+D_y m_x^2}{D_z}}{\frac{2D_xD_y}{D_z}}\Big]dx = \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi D_z}}\exp\Big[-\frac{D_z(D_x(z-m_y)^2+D_y m_x^2)\ominus}{2D_zD_xD_y}$$

$$\ominus\frac{(D_x(z-m_y)+D_y m_x)^2}{\quad}\Big]\frac{1}{\sqrt{2\pi\frac{D_xD_y}{D_z}}}\exp\Big[-\frac{\left(x-\frac{D_x(z-m_y)+D_y m_x}{D_z}\right)^2}{\frac{2D_xD_y}{D_z}}\Big]dx\ominus$$

$$\ominus\frac{1}{\sqrt{2\pi D_z}}\exp\Big[-\frac{(z-(m_x+m_y))^2}{2D_z}\Big]\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi\frac{D_xD_y}{D_z}}}\exp\Big[-\frac{\left(x-\frac{D_x(z-m_y)^2+D_y m_x}{D_z}\right)^2}{\frac{2D_xD_y}{D_z}}\Big]dx \to 1$$

$$f_3(z)=\frac{1}{\sqrt{2\pi D_z}}\exp\Big[-\frac{(z-(m_x+m_y))^2}{2D_z}\Big]$$