Machine Learning Overview and Neural Network

Deep Learning (DSE316/616)

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Lecture overview

- What is learning?
- Types of machine learning problems
- Image classification
- Perceptron
- Multilayer Neural Network
- Disclaimer: Much of the material and slides for this lecture were borrowed from —Bernhard Schölkopf's MLSS 2017 lecture, —Tommi Jaakkola's 6.867 class, —Fei-Fei Li, Andrej Karpathy and Justin Johnson's CS231n class —Hongsheng Li's ELEG5491 class –CMP784: Deep Learning Fall 2021 Erkut Erdem Hacettepe University

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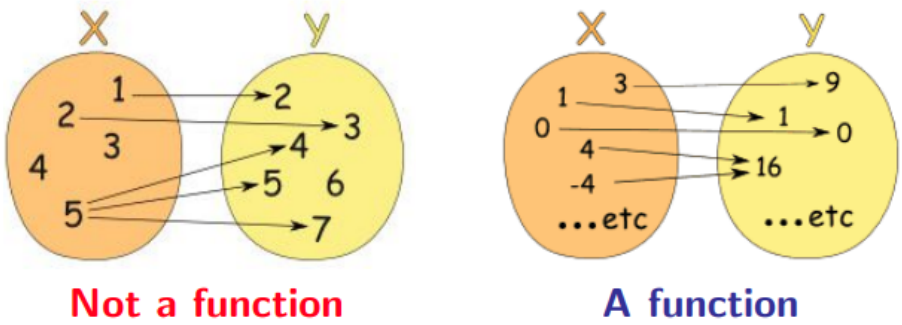
Machine learning aims at learning a function

- Given an input value or vector, a function assigns it with a value or vector.
- "One-to-many" mapping is not a function. "Many-to-one" mapping is a function.



- Note that a function can have a vector output or matrix output. For instance, the following formula is still a function

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_1 x_2 \end{bmatrix} \tag{1}$$

Function estimation

- We are interested in predicting y from input x and assume there exists a function that describes the relationship between y and x, e.g., $y = f(x)$.
- If the function f's parametric form is fixed, prediction function f can be parametrized by a parameter vector θ
- Estimating f from a training set $D = \{(x_1^{train}, y_1), (x_2^{train}, y_2), ...(x_n^{train}, y_n)\}$.
- With a better design of the parametric form of the function, the learner could achieve better performance.
- This design process typical involves domain knowledge.

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Types of machine learning problems

- Supervised learning
- Unsupervised learning
- Semi-supervised learning
- Reinforcement learning

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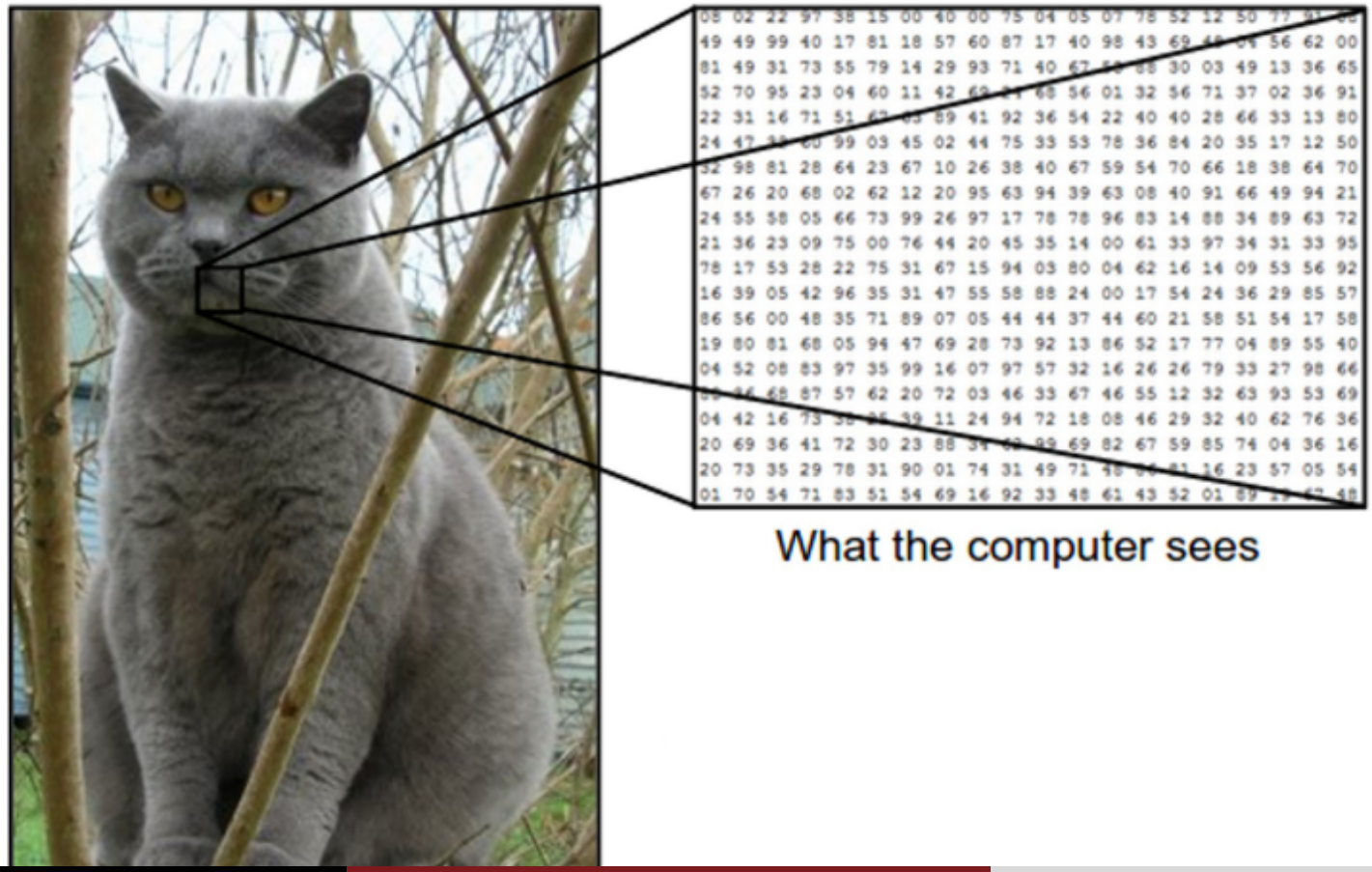
Image Classification: a core task in Computer Vision

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

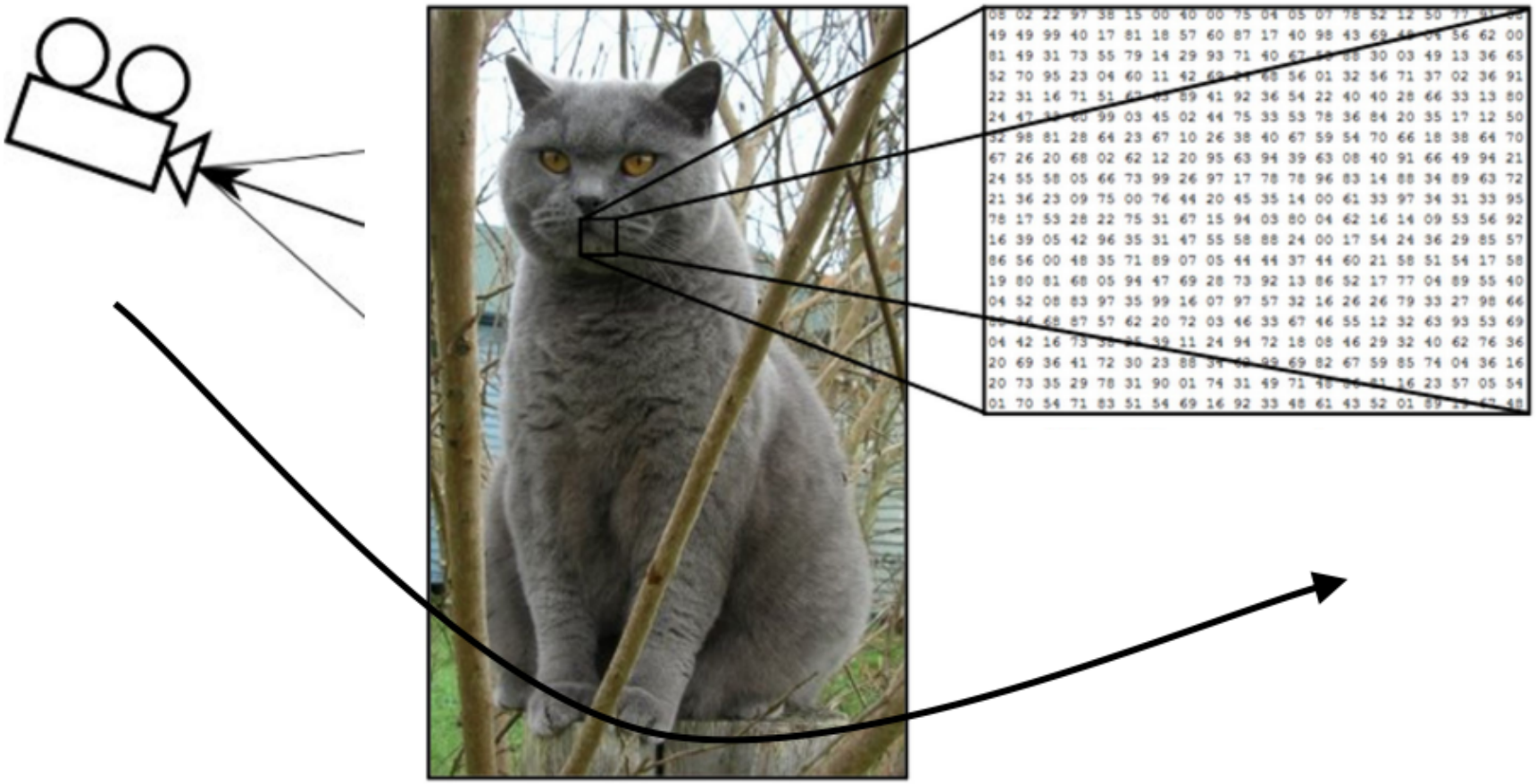
cat

The problem: semantic gap

- Images are represented as 3D arrays of numbers, with integers between [0, 255].
- e.g. 300 x 100 x 3 (3 for 3 color channels RGB)

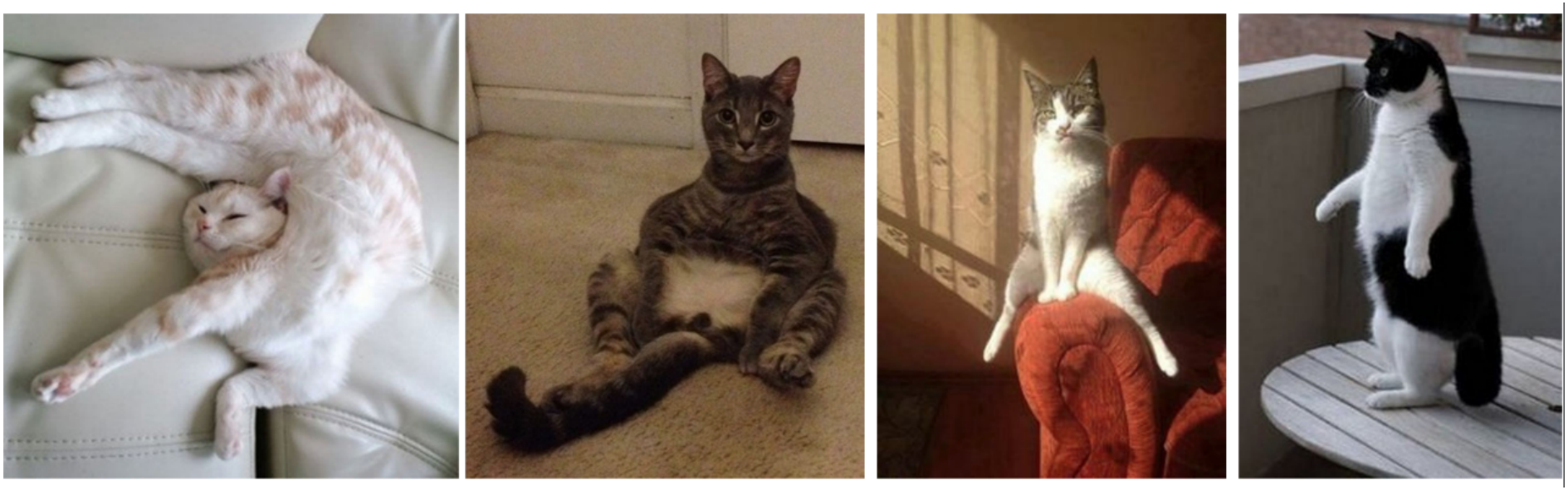


Challenges: Viewpoint Variation

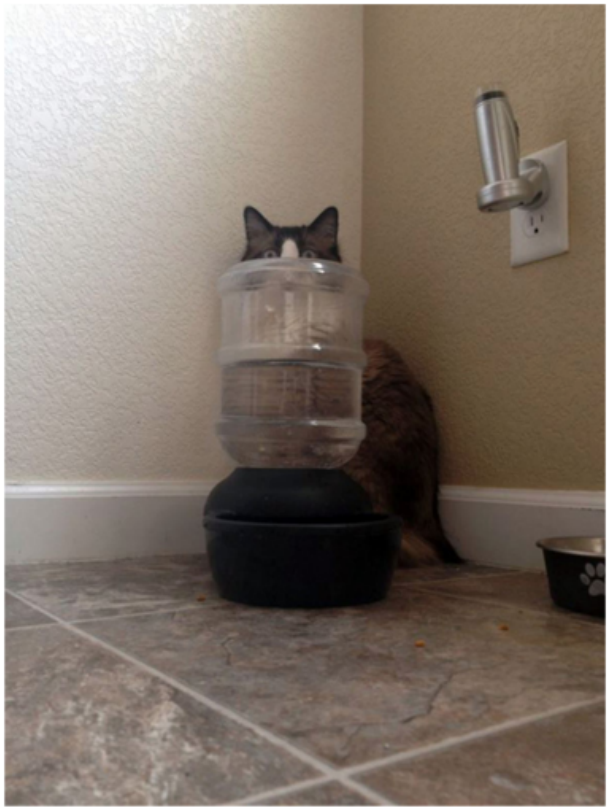


Challenges: Illumination

Challenges: Deformation



Challenges: Occlusion



Challenges: Background clutter

Linear classifier: image classification

[32x32x3]
array of numbers 0...1
(3072 numbers total)

image parameters

$$f(x, W)$$

10 numbers, indicating class scores

Linear classifier: image classification

$$f(x, W) = Wx$$

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10x3072

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parameters, or "weights"

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$f(x, W) = Wx$

3072x1

(+b) 10x1

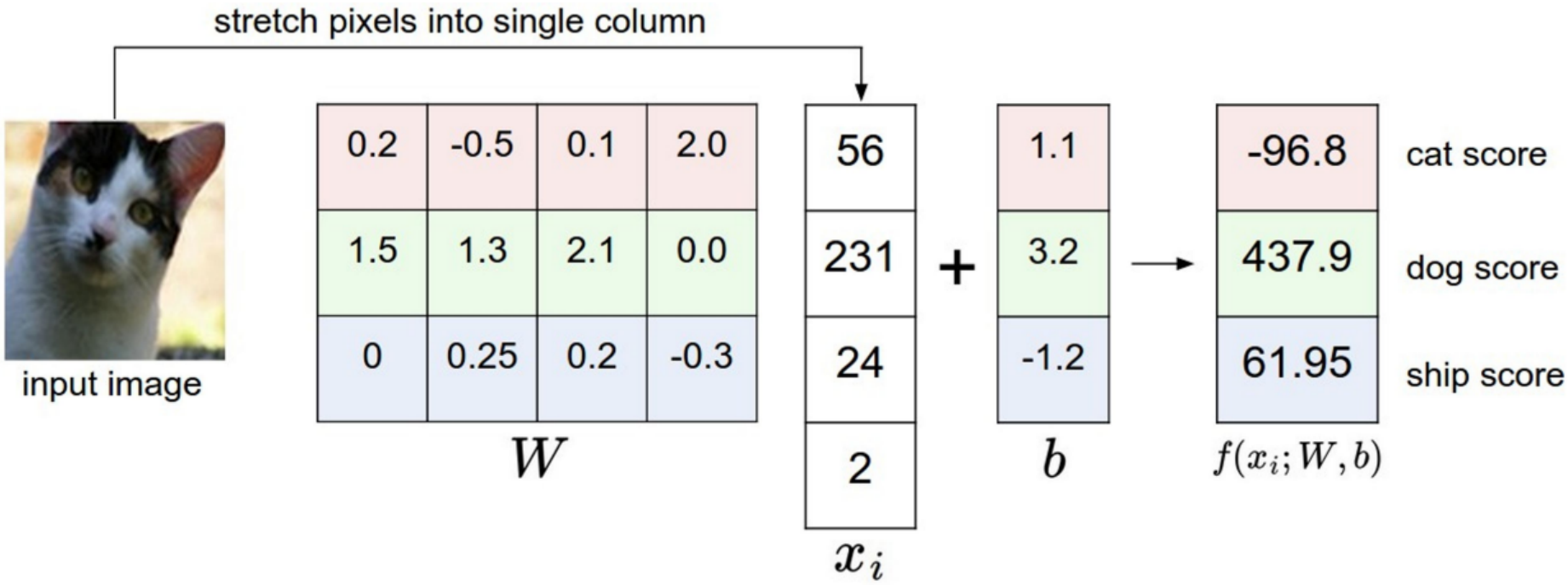
10x1

10x3072

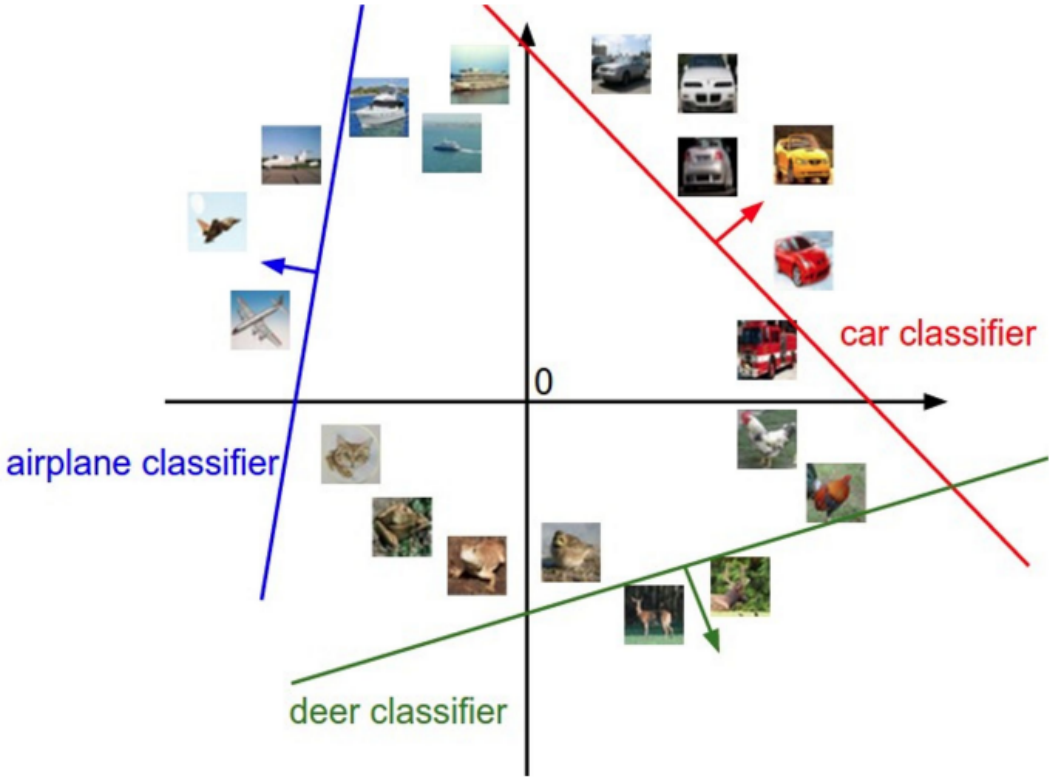
10 numbers, indicating class scores

parameters, or "weights"

Linear classifier: image classification



Interpreting a Linear Classifier



$$f(x_i, W, b) = Wx_i + b$$

[32x32x3]
array of numbers 0...1
(3072 numbers total)

Review: The learning problem

- **Hypothesis class:** we consider some **restricted** set $\mathcal{F}$ of mappings $\mathcal{F} : \mathcal{X} \rightarrow \mathcal{L}$ from images to labels
- **Estimation:** on the basis of a training set of examples and labels,$\{(x_1, y_1), (x_2, y_2), ...(x_n, y_n)\}$ we find an estimate $f \in \mathcal{F}$
- **Evaluation:** we measure how well f to yet unseen examples i.e whether $f(x_{new})$ agrees with y_{new}

Estimation criterion

- We can formulate the binary classification problem more explicitly by defining a zero-one loss:

$$\text{Loss}(y, \hat{y}) = \begin{cases} 0, y = \hat{y} \\ 1, y \neq \hat{y} \end{cases} \tag{2}$$

$$\frac{1}{n}\sum_{i=1}^{n} \text{Loss}\left(y_i, \hat{y}_i\right) = \frac{1}{n}\sum_{i=1}^{n} \text{Loss}\left(y_i, f\left(\mathbf{x}_i; \theta\right)\right) \tag{3}$$

  give the fractions of prediction errors on the training set
- This is a function of the parameters θ and we can try to minimize it directly

Suppose: 3 training examples, 3 classes. With some W the scores are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Multiclass SVM loss:

- Given an example $\{(x_i, y_i)\}$ where x_i is the image and where $y_i$is the (integer) label, and using the shorthand for the scores vector:$s_i = f(x_i, W)$

the SVM loss has the form:

$$\mathcal{L}_i = \sum_{i \neq j} max(0, s_j - s_{y_i} + 1)$$

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$$= max(0, 5.1 - 3.2 + 1)$$

$$+ max(0, -1.7 - 3.2 + 1)$$

$$= 2.9$$

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$$= max(0, 1.3 - 4.9 + 1)$$

$$+ max(0, 2.0 - 4.9 + 1)$$

$$= 0$$

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$$= max(0, 2.2 - (-3.1) + 1)$$

$$+ max(0, 2.5 - (-3.1) + 1)$$

$$= 10.9$$

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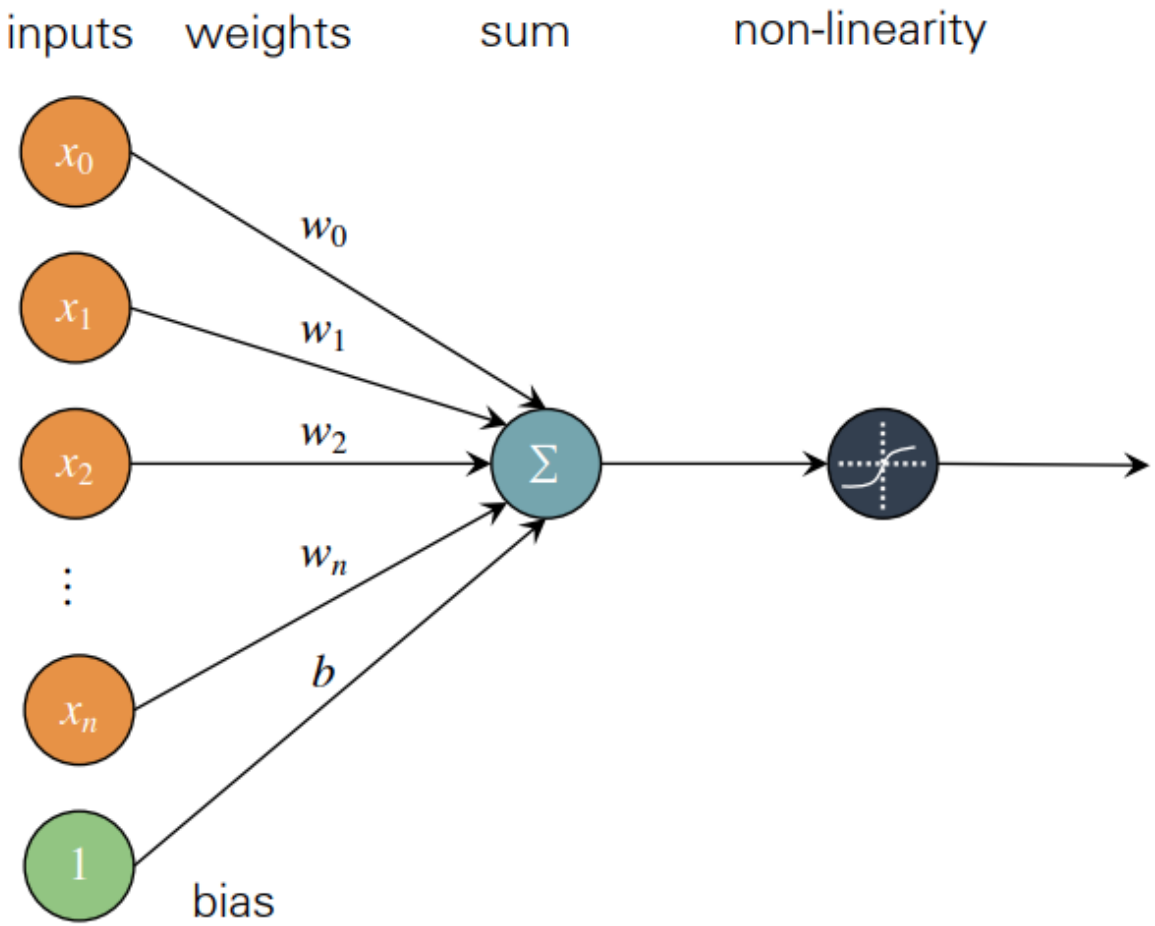
$$\mathcal{L}_i = \sum_{i \neq j} max(0, s_j - s_{y_i} + 1) \quad (4)$$

$$\mathcal{L} = \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}_i$$

$$\mathbf{L} = (2.9 + 0 + 10.9)/3$$

$$= 4.6$$

The Perceptron



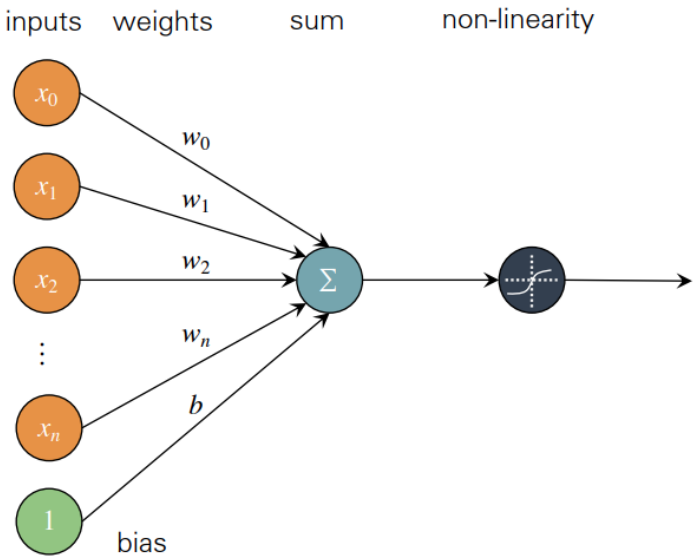
Perceptron

- Neuron preactivations (or input activation)

$$a(x) = b + \sum_i w_i x_i = w^T X$$

- Neuron output activation:

$$h(x) = g(a(x)) = g(b + \sum_i w_i x_i)$$



where
w are the weights (parameters)
b is ht bias term
$g(.)$ is the activation function

Output Activation of The Neuron

$$h(x) = g(a(x)) = g(b + \sum_i w_i x_i)$$

Range is determined by $g(\cdot)$

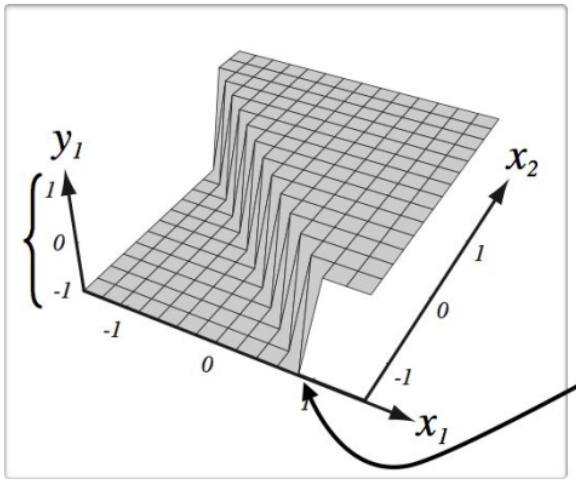
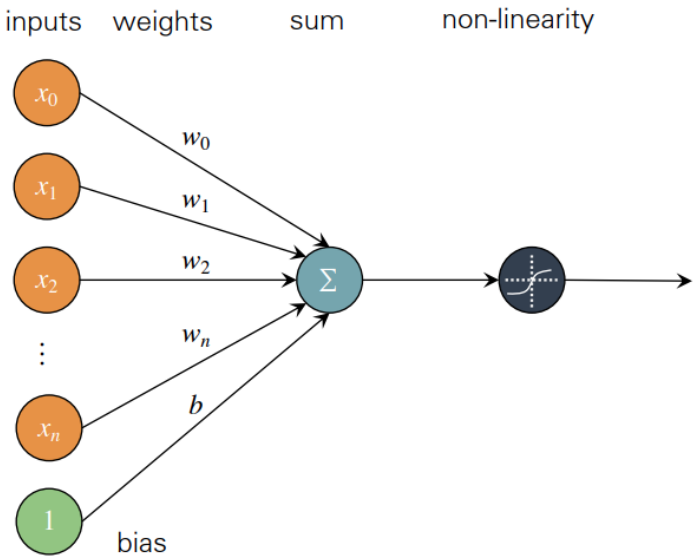


Image credit: Pascal Vincent

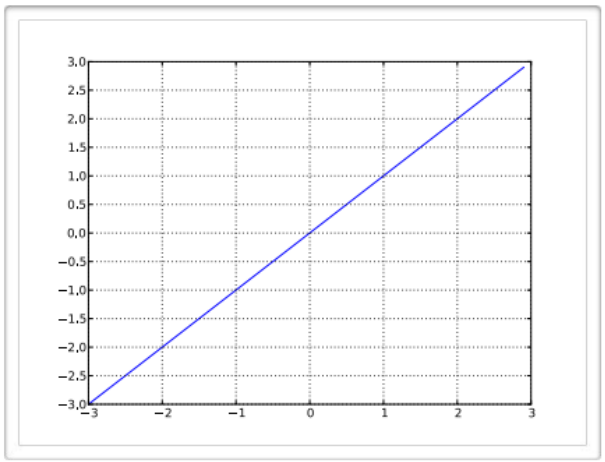
Bias only changes the position of the riff

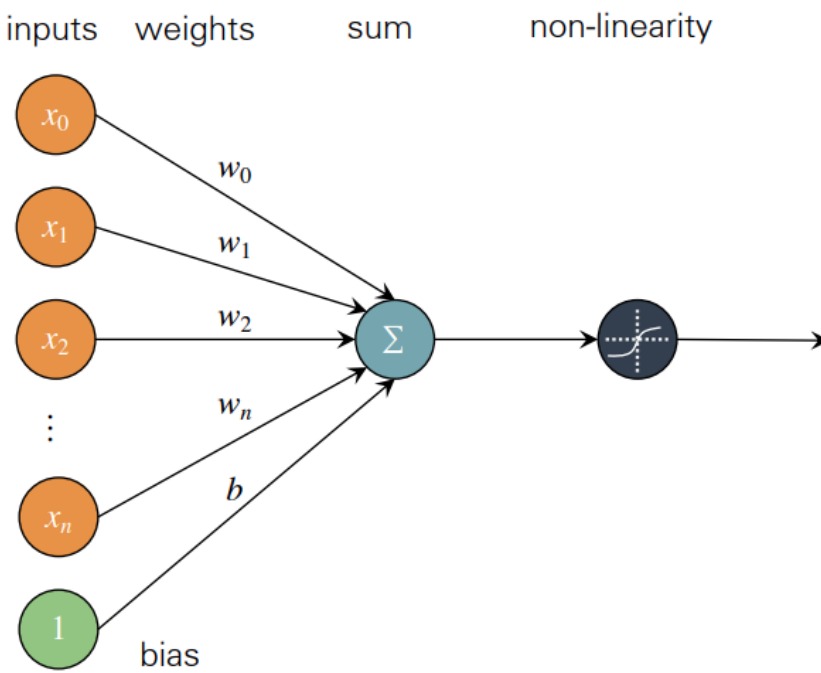


Linear Activation Function

$$h(x) = g(a(x)) = g(b + \sum_i w_i x_i)$$

$$g(a) = a$$





- No non-linear transform
- No squashing

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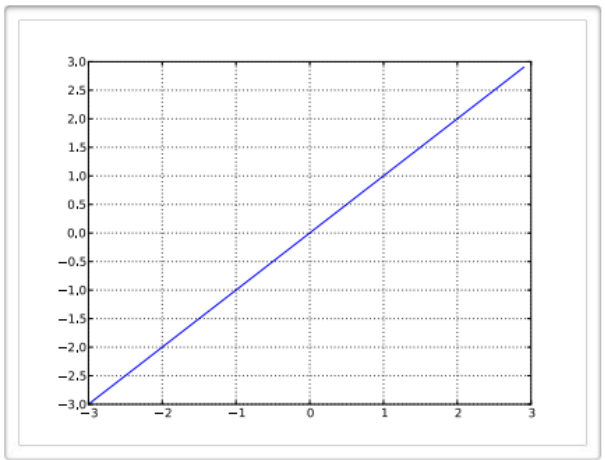

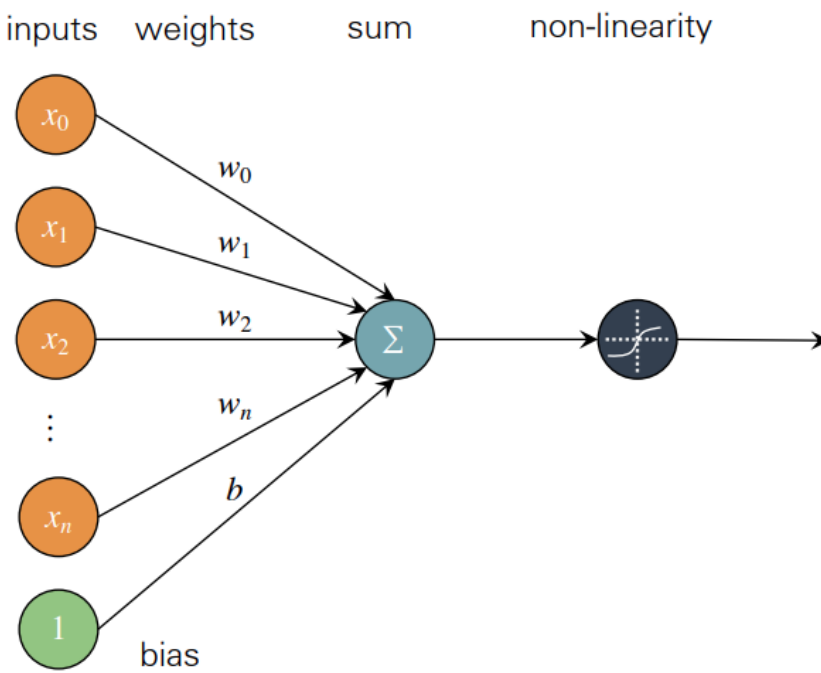

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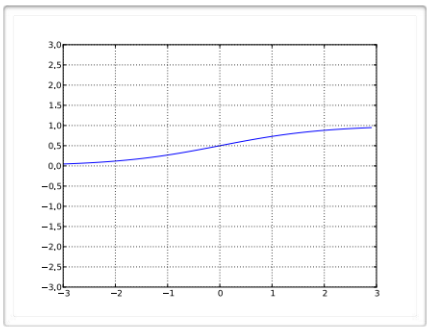
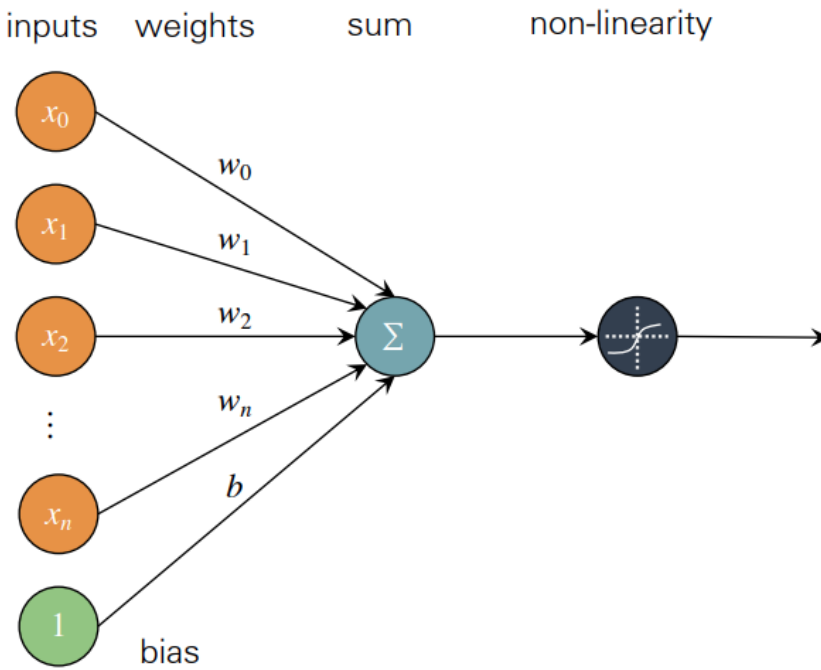
Sigmoid Activation Function

$$h(x) = g(a(x)) = g(b + \sum_i w_i x_i)$$

$$g(a) = sigm(a) = \frac{1}{1 + exp(-a)}$$





- Squash neuron output between 0 and 1
- Always positive
- Bounded

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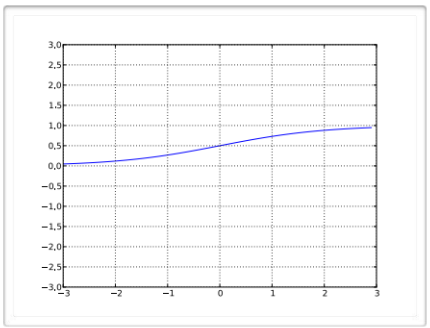

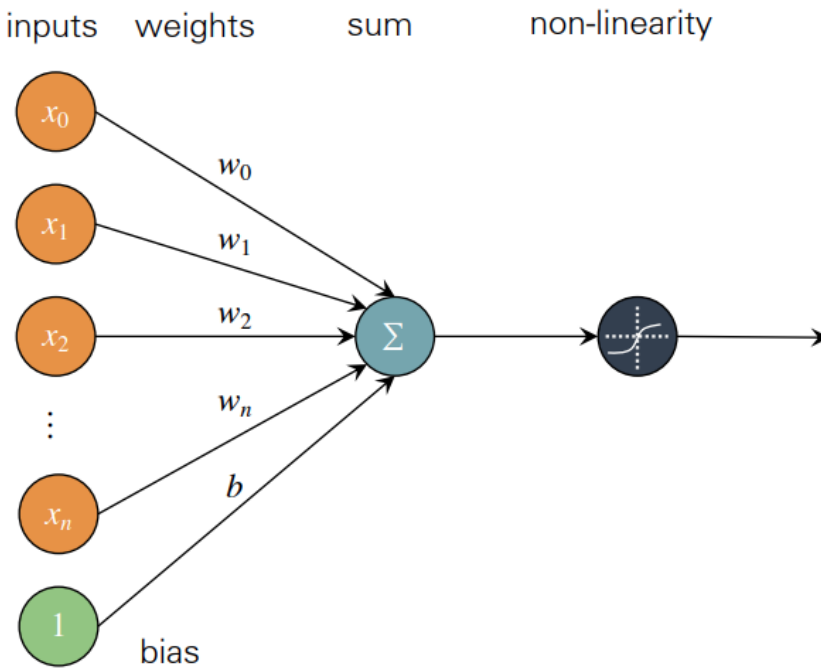

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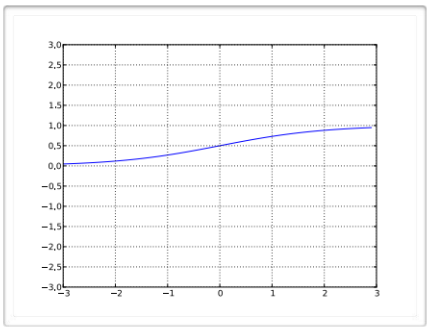

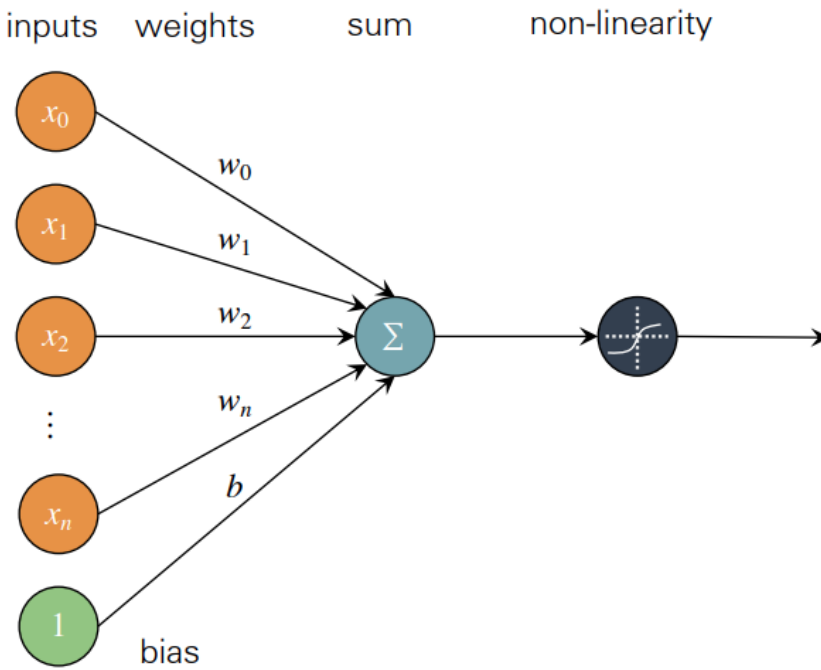

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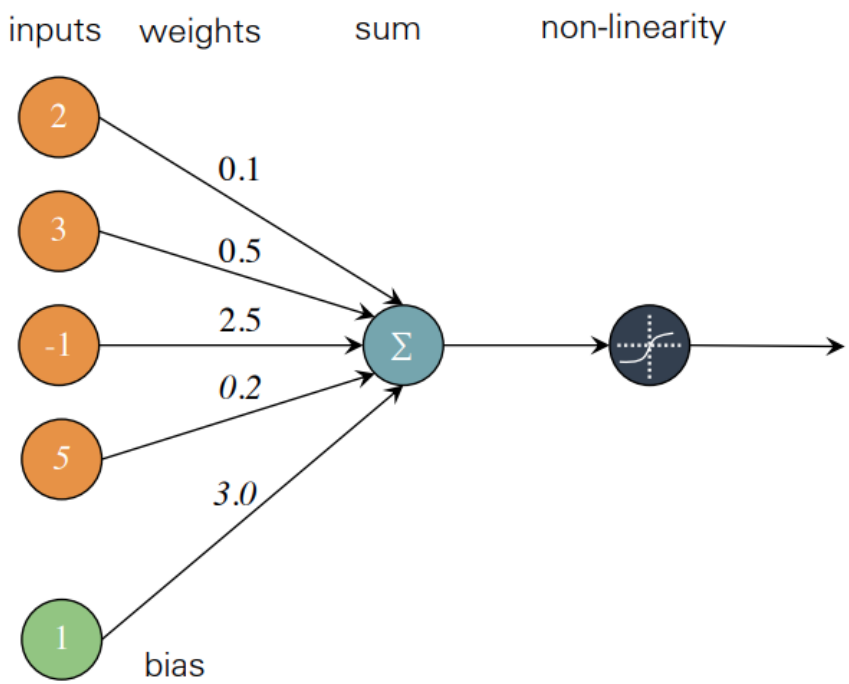
Perceptron Forward Pass

$$h(x) = g(a(x)) = g(b + \sum_i w_i x_i)$$

$$h(x) = g(3.2) = \sigma(3.2)$$

$$\frac{1}{1 + e^{-3.2}} = 0.96$$

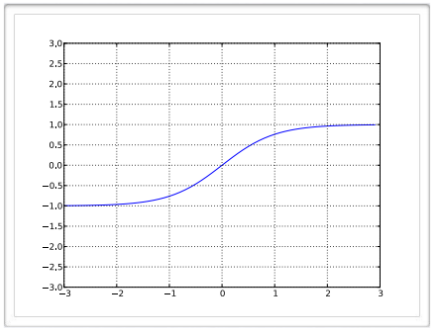


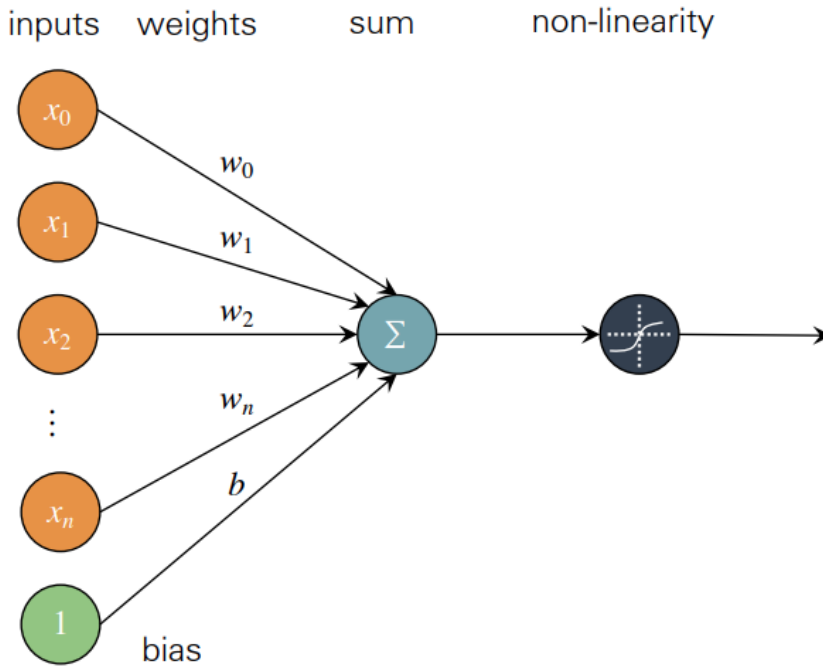
Hyperbolic Tangent (tanh) Activation Function

$$h(x) = g(a(x)) = g(b + \sum_i w_i x_i)$$

$$g(a) = tanh(a) =$$

$$= \frac{exp(a) - exp(-a)}{exp(a) + exp(-a)} = \frac{exp(2a) - 1}{exp(2a) + 1}$$





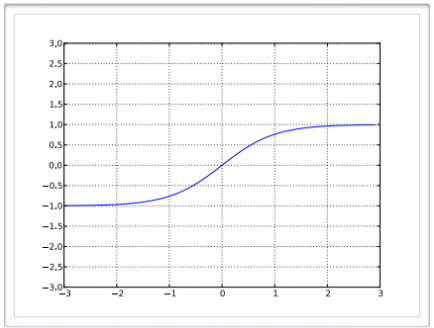
- Squashes the neuron output between 1 an -1
- Bounded

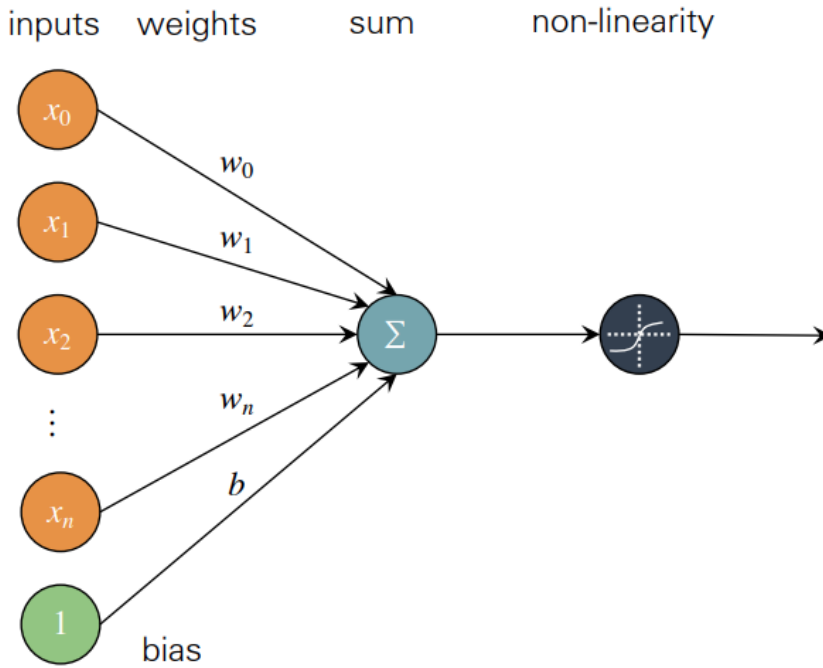
Hyperbolic Tangent (tanh) Activation Function

$$h(x) = g(a(x)) = g(b + \sum_i w_i x_i)$$

$$g(a) = tanh(a) =$$

$$= \frac{exp(a) - exp(-a)}{exp(a) + exp(-a)} = \frac{exp(2a) - 1}{exp(2a) + 1}$$



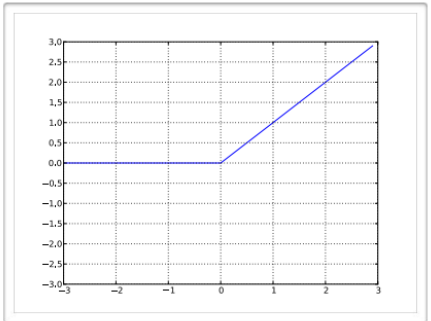


- Squashes the neuron output between 1 an -1
- Bounded

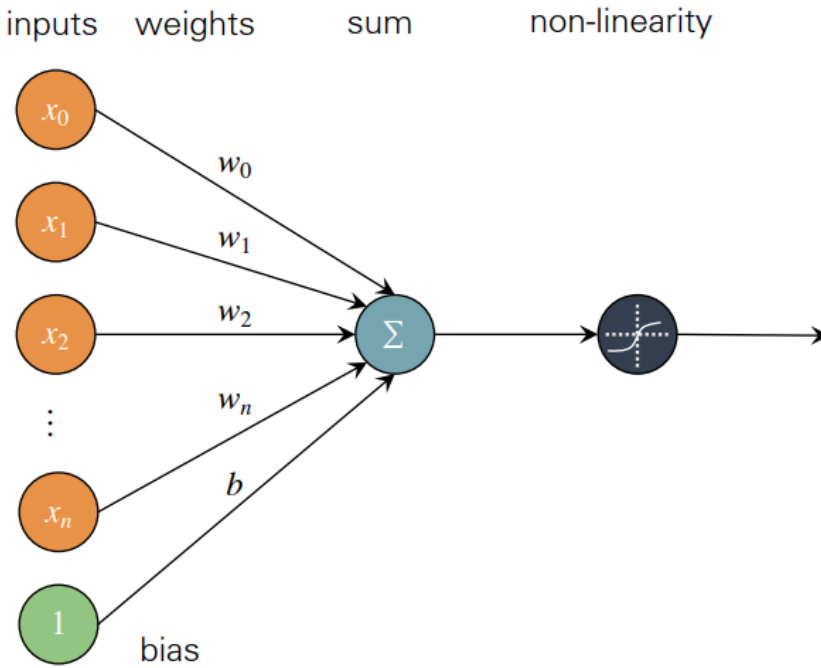
Rectified Linear (ReLU) Activation Function

$$h(x) = g(a(x)) = g(b + \sum_i w_i x_i)$$

$$g(a) = Relu(a) == \max(0, a)$$



- Bounded by 0
- Not upper bound



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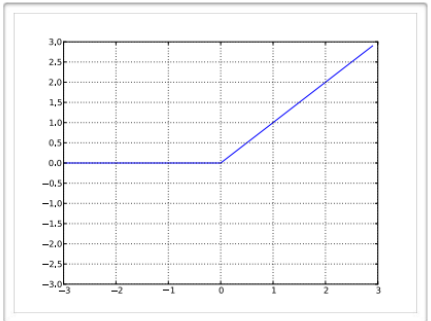
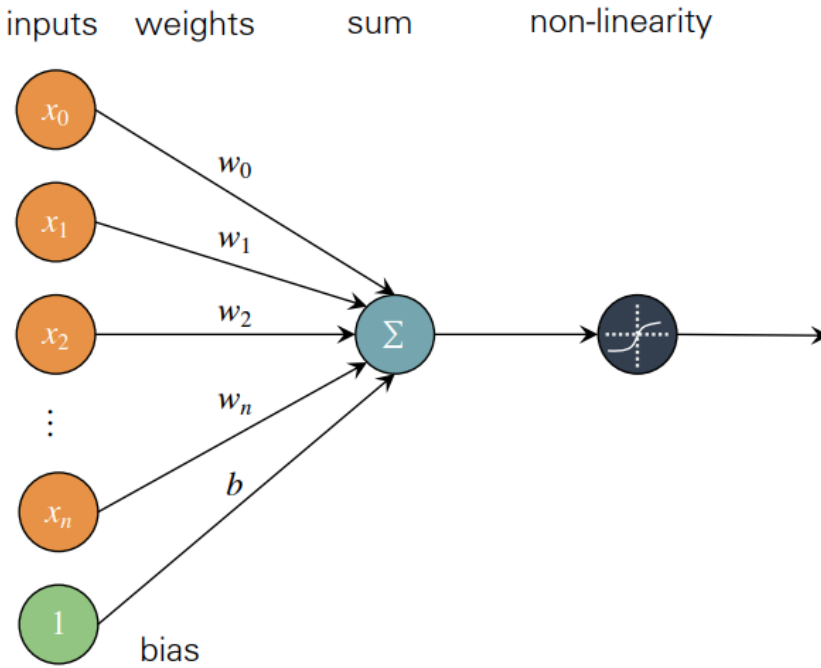
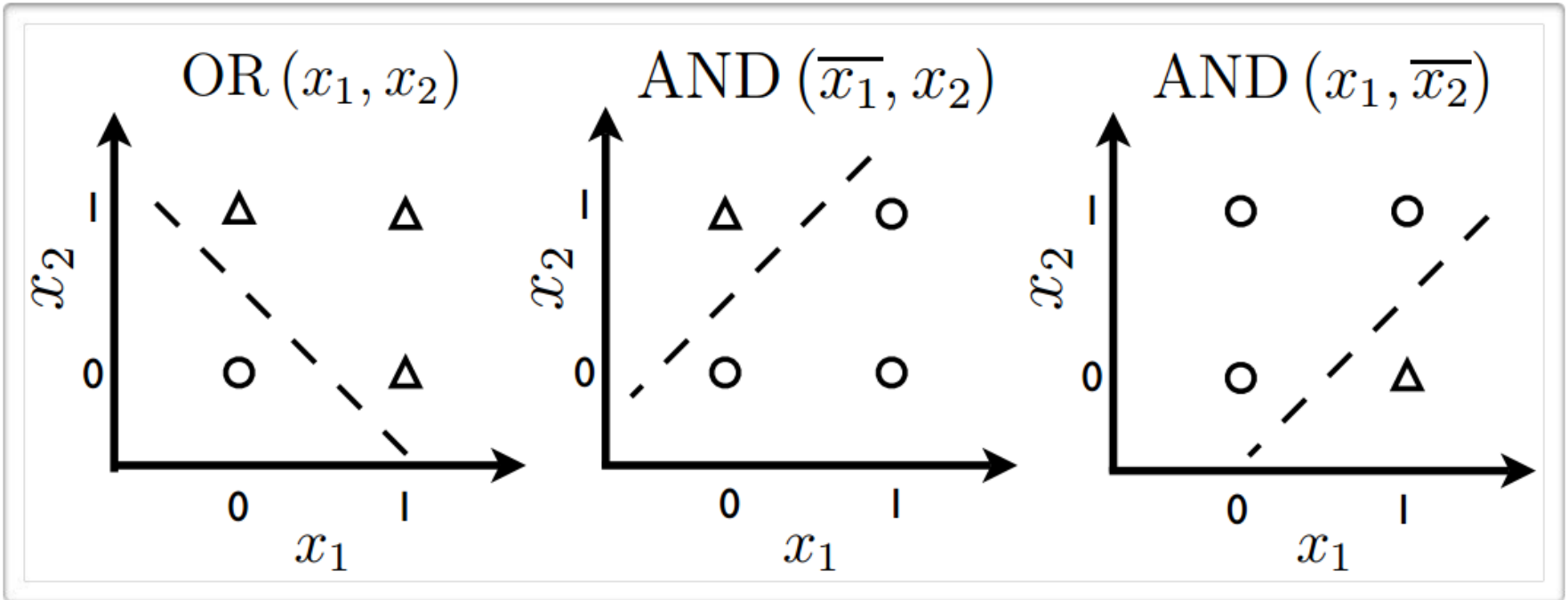


- Bounded by 0
- Not upper bound



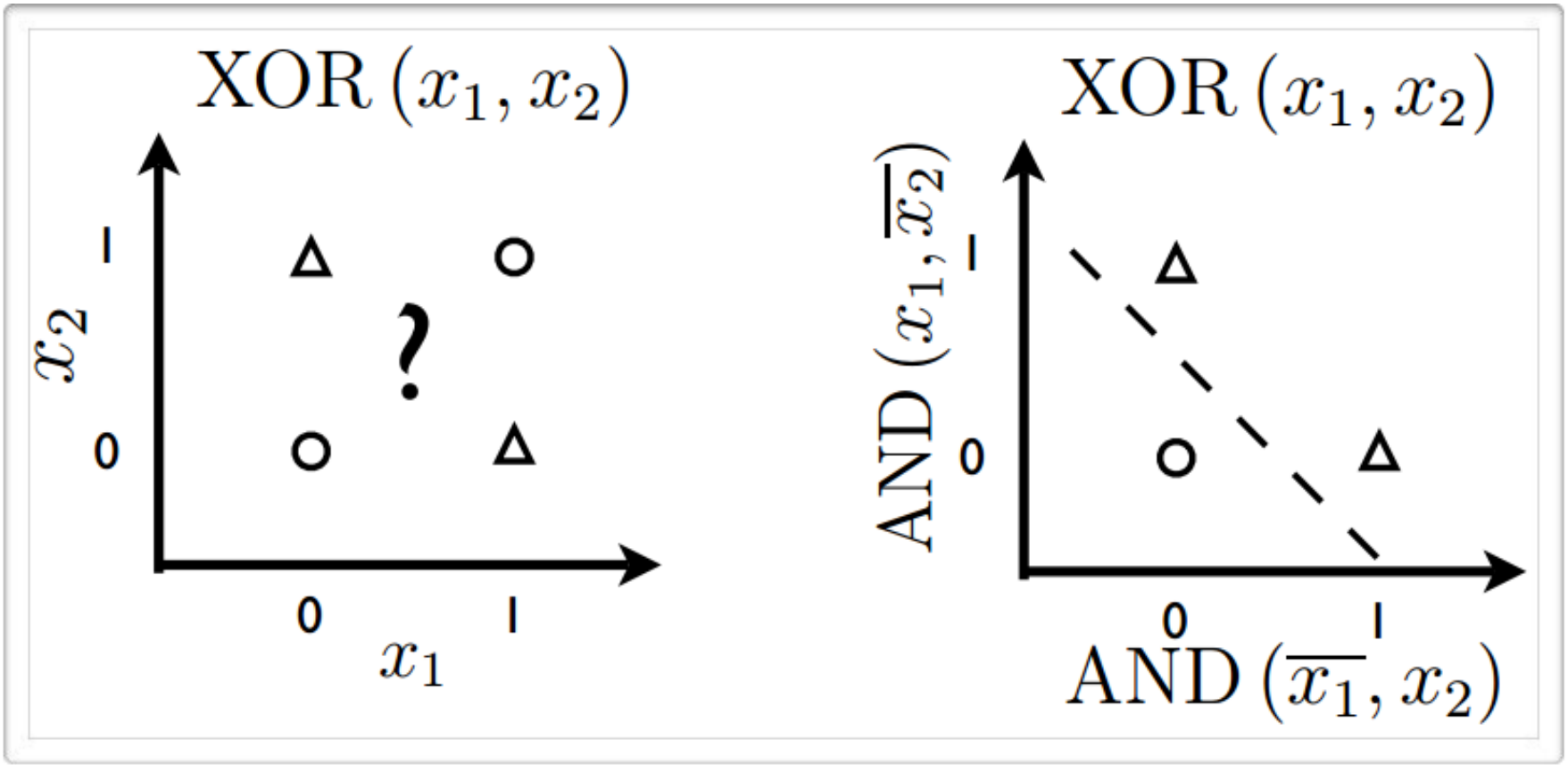
Capacity of Single Neuron

- Can solve linearly separable problems



Capacity of Single Neuron

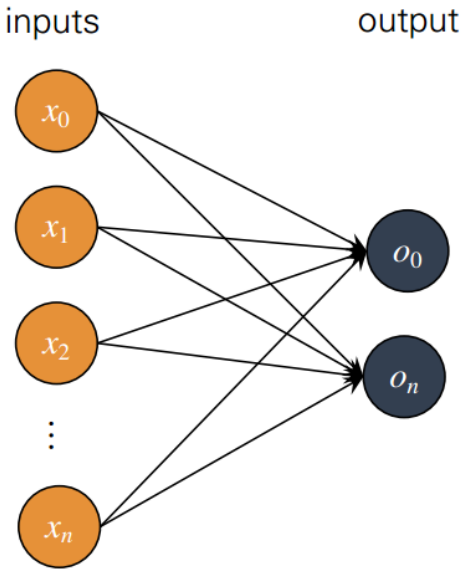
- Can not solve non-linearly separable problems



- Need to transform the input into a better representation
- Remember the Basis Functions!!

Multi-Output Perceptron

- Remember multi-way classification
  - We need multiple outputs (1 output per class)
  - We need to estimate conditional probability p(y = c | x)
  - Discriminative Learning
- Softmax activation function at the output



$$\mathbf{o}(\mathbf{a}) = \text{softmax}(\mathbf{a}) = \left[\frac{\exp(a_1)}{\sum_c \exp(a_c)} \cdots \frac{\exp(a_C)}{\sum_c \exp(a_c)} \right]$$

  - strictly positive
  - sums to one
- Predict class with the highest estimated class conditional probability

Multi-Layer Perceptron

Multi-Layer Perceptron

Single Hidden Layer Neural Network

- Hidden layer pre-activation

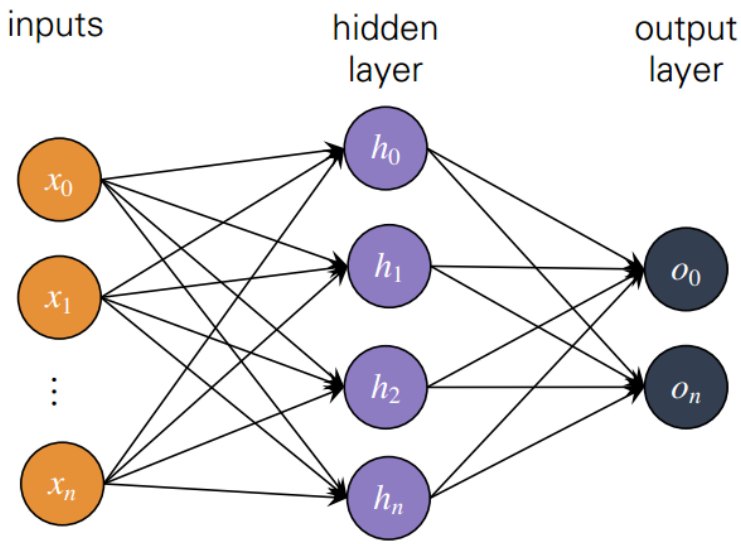
$$a(x) = b^{(1)} + W^{(1)}x$$

- Hidden layer activation:

$$h(x) = g(a(x))$$

- Output layer activation

$$o(x) = o(b^{(2)} + w^{(2)}h^{(1)}x)$$



Multi-Layer Perceptron (MLP)

- Consider a network with L hidden layers
  - layer pre-activation for k>1

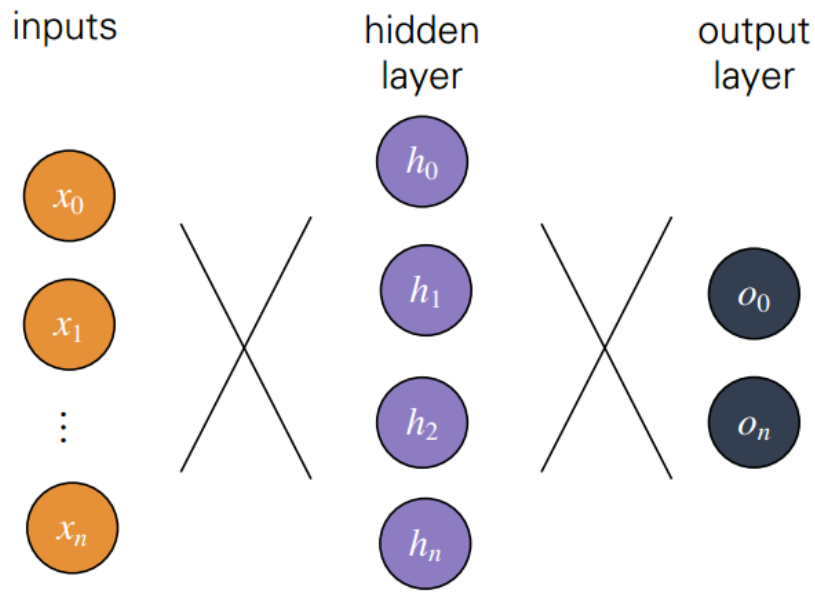
$$a^{(k)}(x) = b^{(k)} + W^{(k)} h^{(k-1)}(x)$$

  - hidden layer activation from 1 to L

$$h^{(k)}(x) = g(a^{(k)}(x))$$

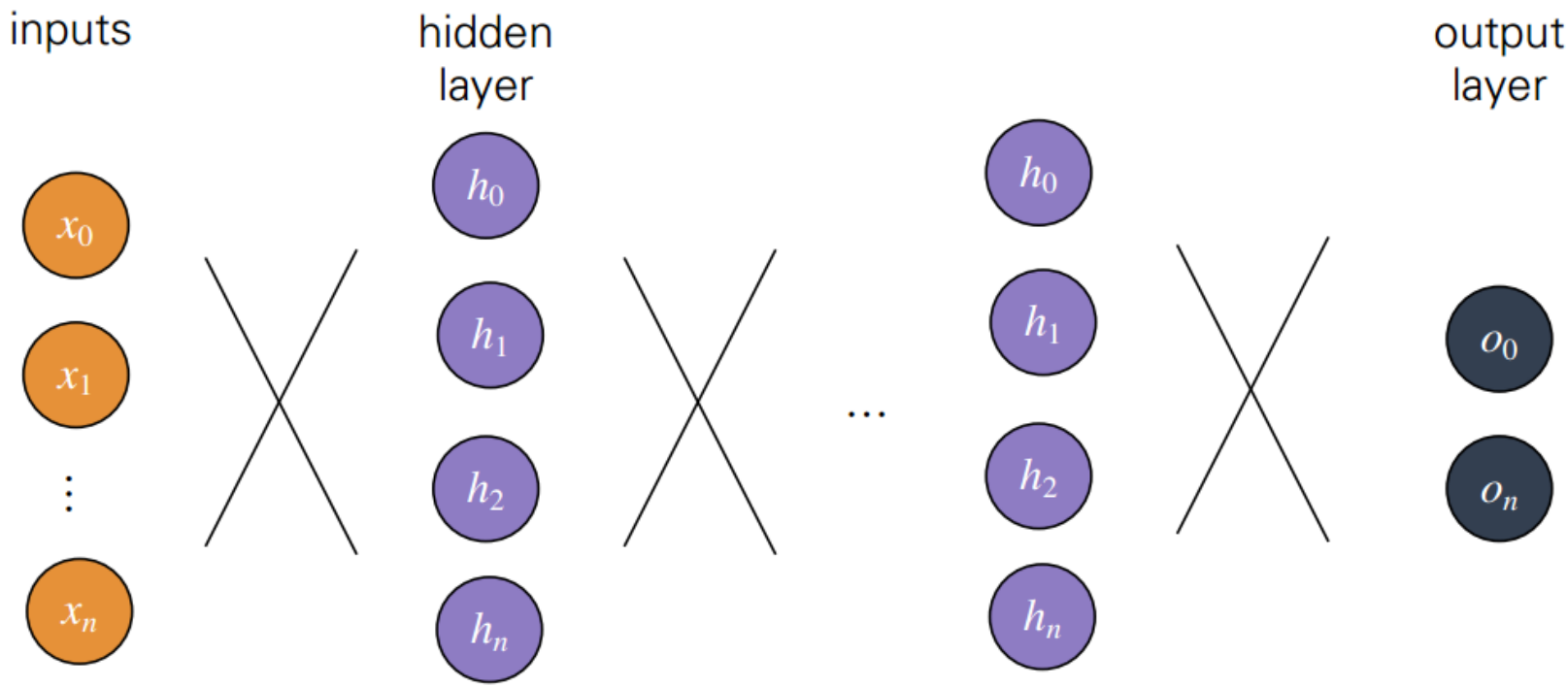
  - output layer activation (k=L+1)

$$h^{(L+1)}(x) = o(a^{(L+1)}(x)) = f(x)$$



$$(\mathbf{h}^{(0)}(\mathbf{x}) = \mathbf{x})$$

Deep Neural Network



Deep Learning Training

Next Class..