Training Neural Network

Deep Learning (DSE316/616)

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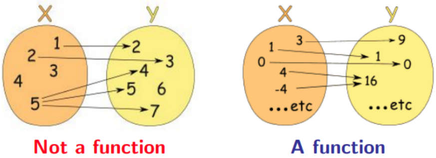
Aug 08, 2022

Disclaimer

- Much of the material and slides for this lecture were borrowed from
  - Bernhard Schölkopf's MLSS 2017 lecture,
  - Tommi Jaakkola's 6.867 class,
  - CMP784: Deep Learning Fall 2021 Erkut Erdem Hacettepe University
  - Fei-Fei Li, Andrej Karpathy and Justin Johnson's CS231n class
  - Hongsheng Li's ELEG5491 class

Previous Class: Recap

- Machine learning is '*Function Estimation*' .
- Assumptions: $y = f(x)$.
- Given: $D = \{(x_1^{train}, y_1), (x_2^{train}, y_2), ...(x_n^{train}, y_n)\}$.



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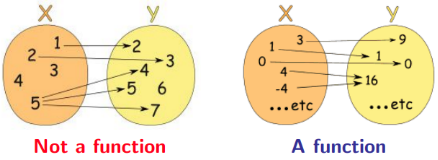

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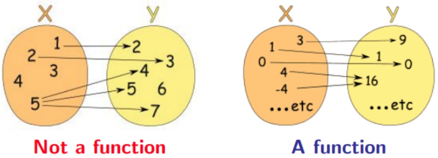
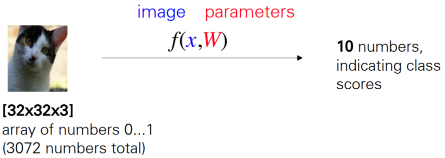
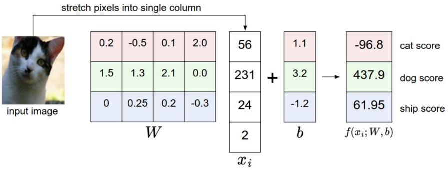


Previous Class: Recap

- Example of Image classification problem and challenges i.e : viewpoint variation, illumination, occlusion etc.





Previous Class: Recap

Suppose: 3 training examples, 3 classes. With some W the scores are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	10.9

Multiclass SVM loss:

- Given an example $\{(x_i, y_i)\}$ where x_i is the image and where $y_i$is the (integer) label, and using the shorthand for the scores vector:$s_i = f(x_i, W)$

the SVM loss has the form:

$$\boxed{\mathcal{L}_i = \sum_{i \neq j} max(0, s_j - s_{y_i} + 1)}$$

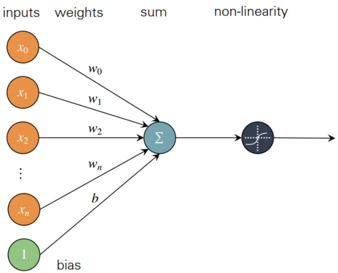
$$\mathcal{L} = \frac{1}{N}\sum_{i=1}^{N}\mathcal{L}_i$$

$$\mathbf{L} = (2.9 + 0 + 10.9)/3$$

Previous Class: Recap of Perceptron

- Neuron preactivations (or input activation)

$$a(x) = b + \sum_i w_i x_i = w^T X$$



- Neuron output activation:

$$h(x) = g(a(x)) = g(b + \sum_i w_i x_i)$$

where
w are the weights (parameters)
b is ht bias term
$g(.)$ is the activation function

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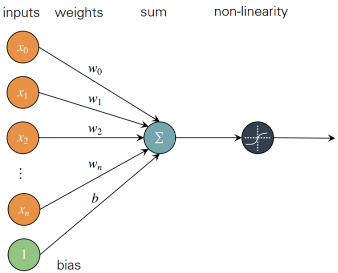

Previous Class: Recap of activation functions

- Linear Activation Function
- Sigmoid
- Hyperbolic Tangent (tanh) Activation Function
- ReLu

Multi-Output Perceptron

- Softmax activation function at the output

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Previous Class:Multi-Layer Perceptron (MLP)

- Consider a network with L hidden layers
  - layer pre-activation for k>1

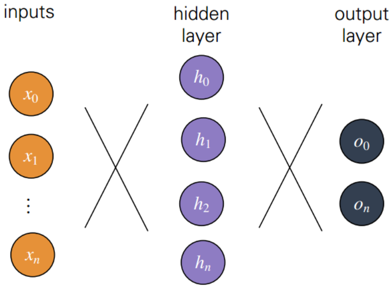
    $$a^{(k)}(x) = b^{(k)} + W^{(k)} h^{(k-1)}(x)$$

  - hidden layer activation from 1 to L

    $$h^{(k)}(x) = g(a^{(k)}(x))$$

  - output layer activation (k=L+1)

    $$h^{(L+1)}(x) = o(a^{(L+1)}(x)) = f(x)$$



$$(\mathbf{h}^{(0)}(\mathbf{x}) = \mathbf{x})$$

Example Problem: Will my flight be delayed?

- Temperature: -20 F
- Wind Speed: 45 mph

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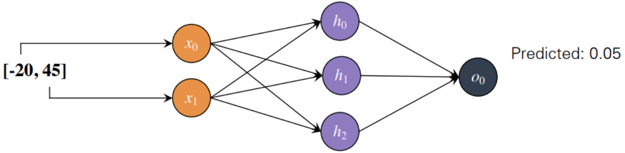

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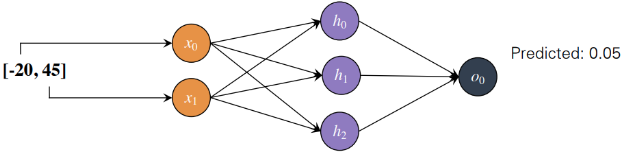

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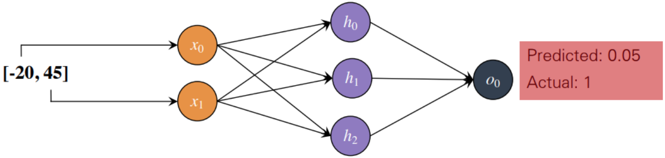

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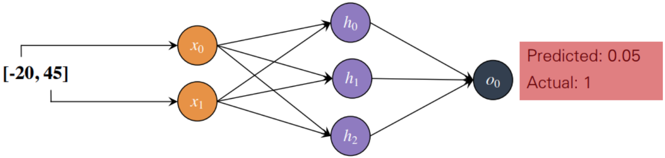
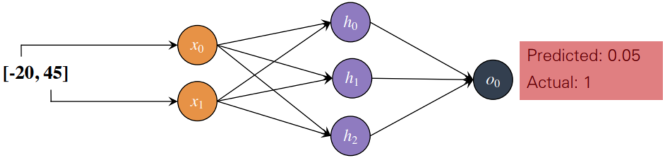


Quantifying Loss



$$\ell(\underbrace{f(\mathbf{x}^{(i)};\theta)}, \underbrace{y^{(i)}}_{\textit{Actual}})$$

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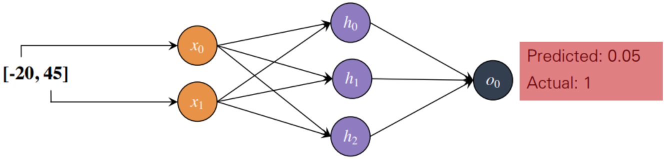

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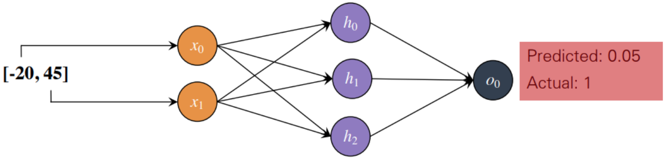
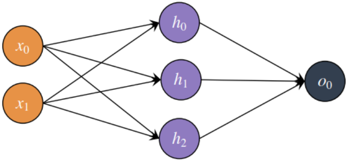


$$\ell(\underbrace{f(\mathbf{x}^{(i)};\theta)}_{\text{Predicted}}, \underbrace{y^{(i)}}_{\text{Actual}})$$

Total Loss

Input

[
[-20, 45],
[80, 0],
[4, 15],
[45, 60],
]



Predicted	Actual
[	[
0.05	1
0.02	0
0.96	1
0.35	1
]	]

$$J(\theta) = \frac{1}{N} \sum_{i} \ell(\underbrace{f(\mathbf{x}^{(i)}; \theta)}_{\text{Predicted}}, \underbrace{y^{(i)}}_{\text{Actual}})$$

Total Loss

Input

[
[-20, 45],
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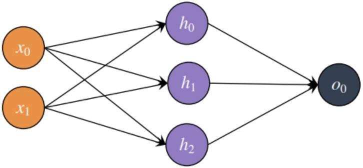

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0.05	1
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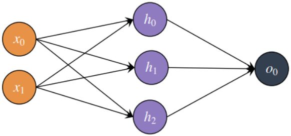
$$J(\theta) = \frac{1}{N} \sum_i \ell\big(\underbrace{f(\mathbf{x}^{(i)}; \theta)}_{\text{Predicted}}, \underbrace{y^{(i)}}_{\text{Actual}}\big)$$

Binary Cross Entropy Loss

Input

[
[-20, 45],
[80, 0],
[4, 15],
[45, 60],
]



Predicted	Actual
[	[
0.05	1
0.02	0
0.96	1
0.35	1
]	]

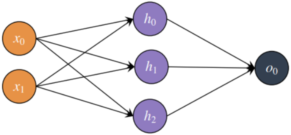
$$\mathcal{J}_{cross-entropy}(\theta) = \frac{1}{N}\sum_i y^{(i)} log(f(\mathbf{x}^{(i)};\theta) + (1 - y^{(i)}) log(1 - f(\mathbf{x}^{(i)};\theta))$$

- For classification problems with a softmax output layer.
- Maximize log-probability of the correct class given an input

Mean Square Error Loss

Input

[
[-20, 45],
[80, 0],
[4, 15],
[45, 60],
]



Predicted	Actual
[	[
0.05	1
0.02	0
0.96	1
0.35	1
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$$\mathcal{J}_{MSE}(\theta) = \frac{1}{N}\sum_{i}(f(\mathbf{x}^{(i)};\theta) - y^{(i)})^2$$

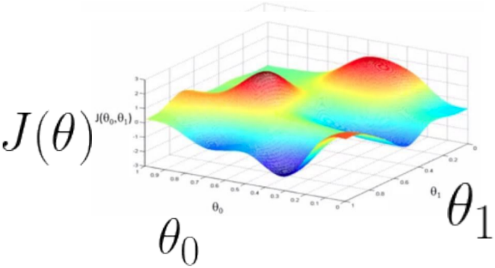
Training

$$\mathcal{J}(\theta) = \frac{1}{N}\sum_i l(\underbrace{f(\mathbf{x}^{(i)};\theta)}_{\text{Predicted}}, \underbrace{y^{(i)}}_{\text{Actual}})$$

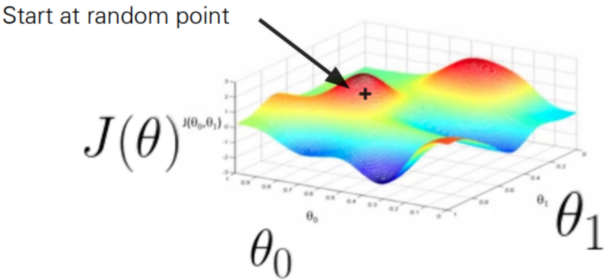
$$\theta = W1, W2, W3...$$

- Learning is cast as optimization.
    - For classification problems, we would like to minimize classification error
    - Loss function can sometimes be viewed as a surrogate for what we want to optimize (e.g. upper bound)

Loss is a function of the model's parameters



How to minimize loss?



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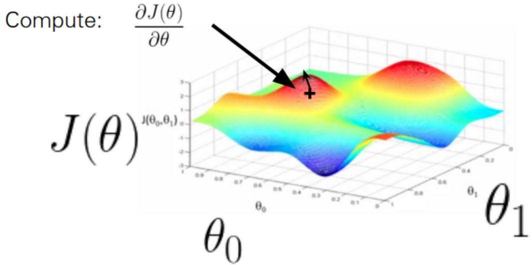
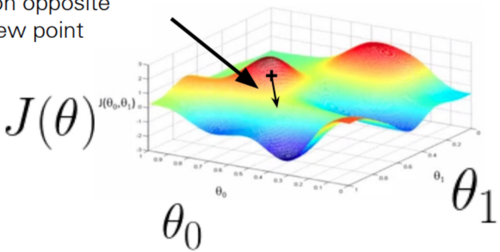


How to minimize loss?

Move in direction opposite
of gradient to new point



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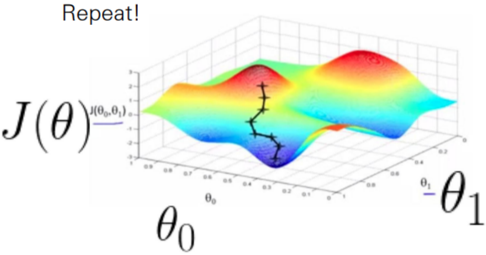
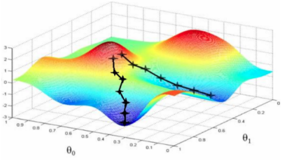


Stochastic Gradient Descent (SGD)

- Initialize θ randomly
- For N Epochs
  - For each training example (x, y):
    - Compute Loss Gradient: $\frac{\partial \mathcal{J}(\theta)}{\partial \theta}$
    - Update θ with update rule:
      $\theta = \theta - \eta \frac{\partial \mathcal{J}(\theta)}{\partial \theta}$



Gradient Descent (SGD)

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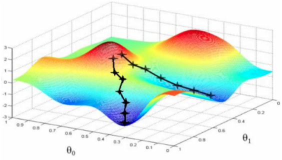

Stochastic Gradient Descent?

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Advantages

- More accurate estimation of gradient
  - Smoother convergence
  - Allows for larger learning rates
- Minibatches lead to fast training!
  - Can parallelize computation

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Stochastic Gradient Descent (SGD)

- Algorithm that performs updates after each example
  - initialize $\theta = \{W^{(1)}, b^{(1)} ... W^{(L+1)}, b^{(L+1)}\}$
  - for N iterations
    - for each training example $(x^{(i)}, y^{(i)})$ or batch $\Delta = \nabla_\theta \ell(f(x^{(i)}; \theta), y^{(i)})$
- To apply this algorithm to neural network training, we need
  - the loss function $\ell(f(x^{(i)}; \theta), y^{(i)})$
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What is a neural network again?

- A family of parametric, non-linear and hierarchical representation learning functions
- $a_L(x;\theta_1,...,\theta_L) = h_L(h_{L-1}(...h_1(x;\theta_1),\theta_{L-1});\theta_L)$
  - x: input
  - θ_l: parameter of layer l
  - $a_l = h_l(x;\theta_l)$: (non)-linear function
- Given training corpus X, Y find optimal parameters

$$\theta^* = \underset{\theta}{\operatorname{argmin}} \sum_{(x,y)\in(X,Y)} \ell(y, a_L(x;\theta_1,...,\theta_L))$$

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- A family of parametric, non-linear and hierarchical representation learning functions
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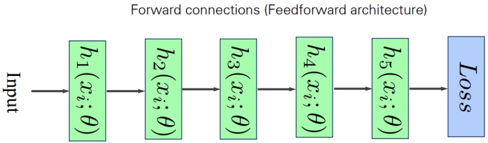
Neural network models

- A neural network model is a series of hierarchically connected functions
- The hierarchy can be very, very complex



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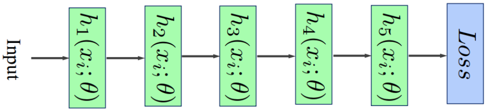
Forward connections (Feedforward architecture)



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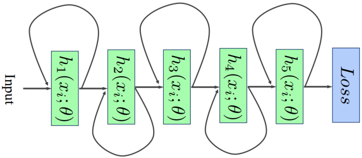

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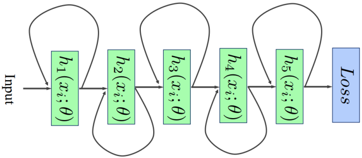

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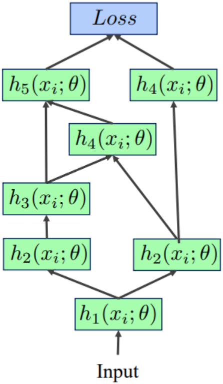

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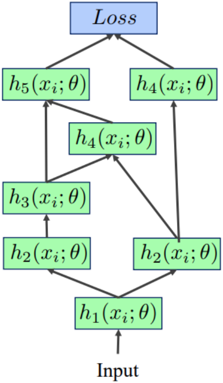

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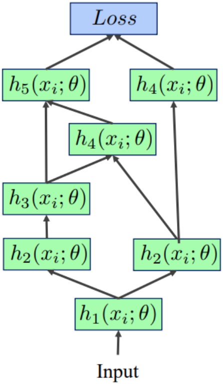

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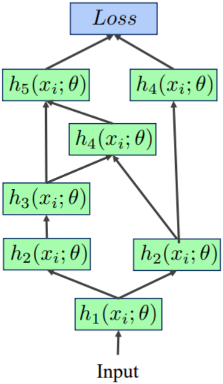

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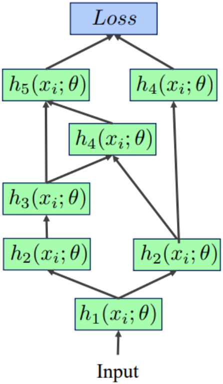

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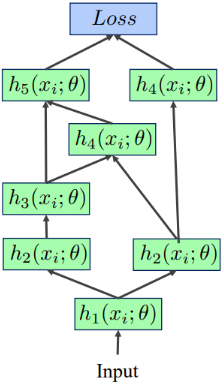

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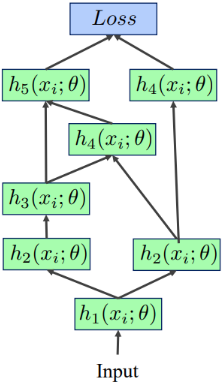

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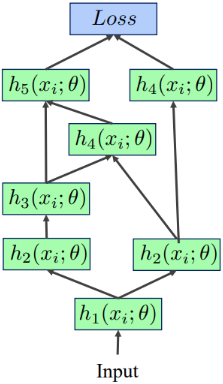

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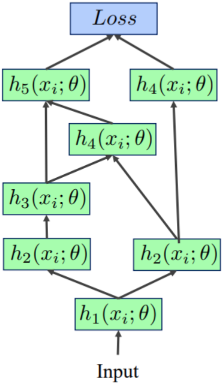

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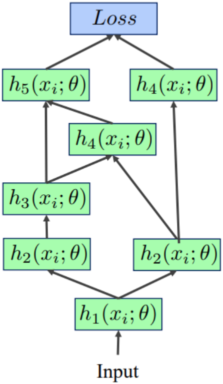

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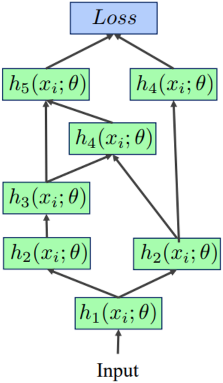

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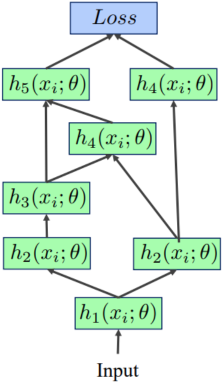

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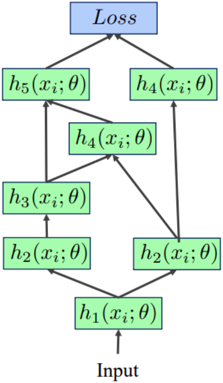

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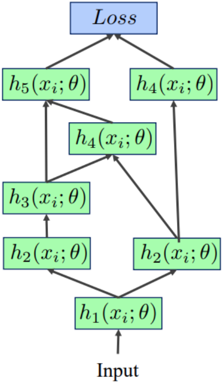

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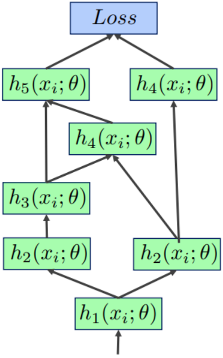

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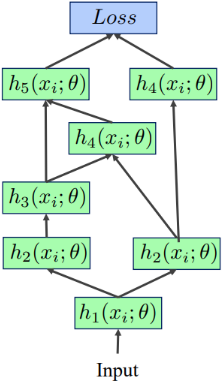

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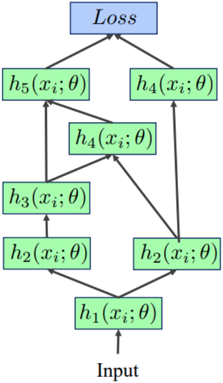

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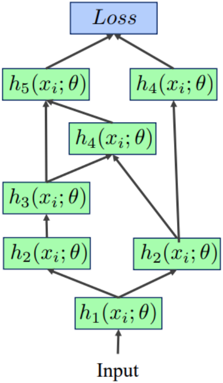

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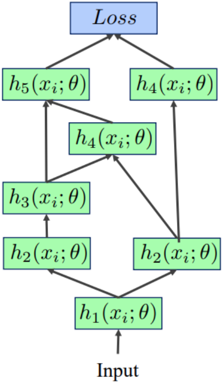

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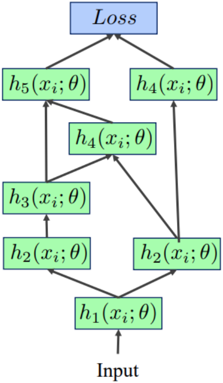

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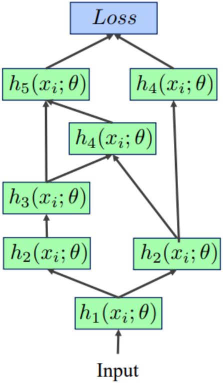

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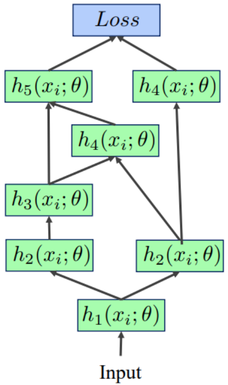

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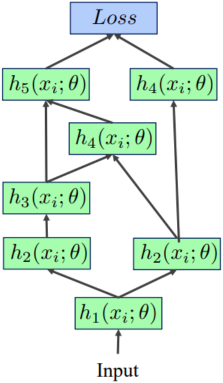

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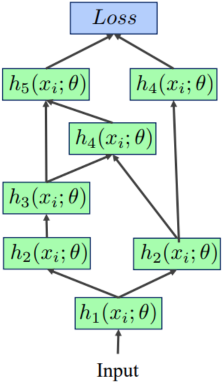

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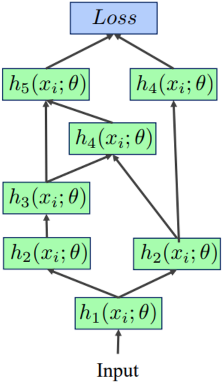

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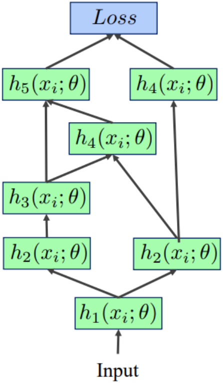

Again, what is a neural network again?

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- To use any gradient descent based optimization

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- we need the gradients $\frac{\partial \mathcal{L}}{\partial \theta_l}$; $l = 1,2,...L$
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backpropagation examples and optimizers

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