ML- Dr Shaneela Naz

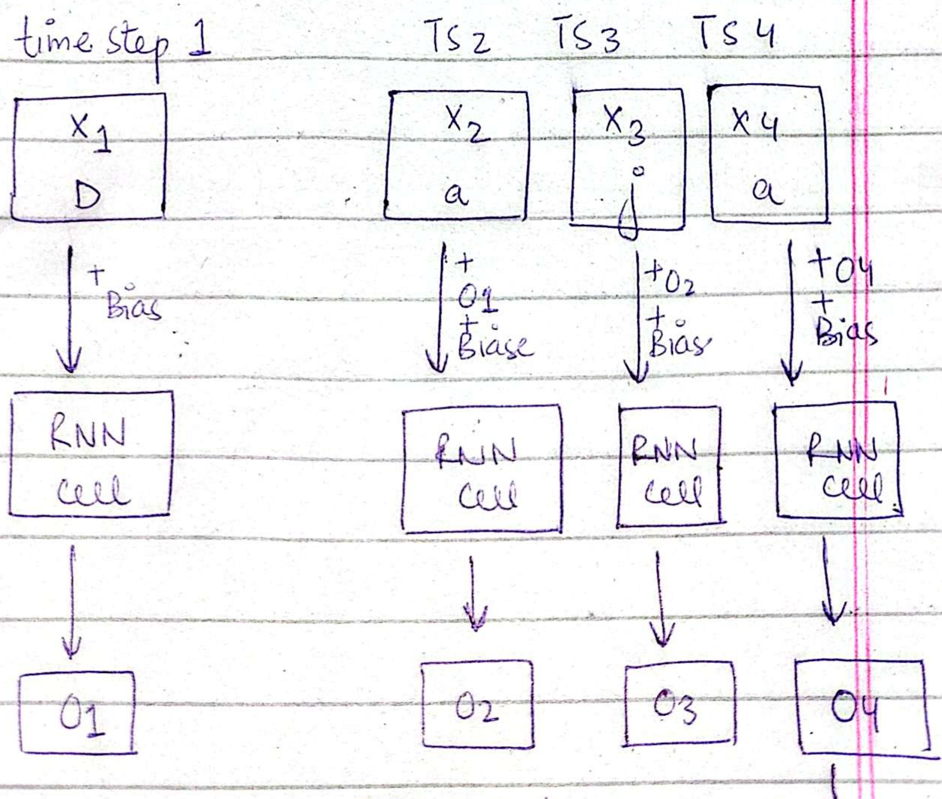
ASSIGNMENT # 04

Khadija Jumani
SP23-BDS-023

Question # 01

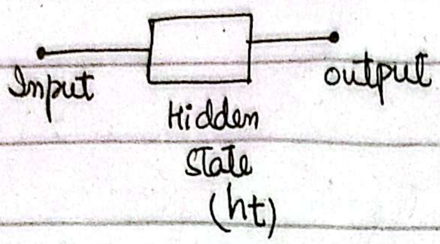
- → Execute the forward pass of RNN for a problem where 4 characters of a word are entered and RNN predicts 5th character.
- → Draw the architecture of RNN as well.

Solution :



* Fifth character "a"
Prediction : a

ⓒ Forward Pass Execution :-



1. **Input Representation :-**

→ Each character is converted into it's vector (hot coded)

→ Suppose we have 27 characters (26 letters + spare)

→ Each character becomes a vector of size 27.

2. RNN cell operations :-

x_1 → input at time step ($t=1$)

h_t → hidden state at time step (t).

y_t → output at time step (t).

w_i, w_h → weight metrices

b_i, b_h → Bias Vectors.

3. Equations & Forward Pass Computation

→ $h_t = \tanh(w_i x_i + b_h)$] At hidden Layer

$y_t = \text{softmax}(w_i x_i + b_h)$

→ $\boxed{\text{Time } (t=1)}$:-

* Input = x_1

* Initial hidden state (h_0) = $\vec{0}$ or random #

* Compute → $h_1 = \tanh(w_1 x_1 + b_i)$; $y_1 = \text{softmax}(w_1 x_1 + b_i)$

→ We use h_1 but ignore y_1 for prediction.

→ $\boxed{\text{Time } (t=2)}$:-

Input → x_2

Compute

$$h_2 = \tanh(w_i x_2 + \underbrace{y(w_h h_1)}_{\text{output of } h_1} + b)$$

$$y_2 = \text{softmax}(w_i x_2 + b_i)$$

→ $\boxed{\text{Time } (t=3)}$:-

Input → x_3

Compute :

$$h_3 = \tanh(w_i x_3 + O_2 + b_i)$$

$$y_3 = \text{softmax}(w_i x_3 + b_i)$$

→ $\boxed{\text{Time } (t=4)}$:-

Input → x_4

Compute :

$$h_4 = \tanh(w_i x_4 + O_3 + b_i)$$

$$y_4 = \text{softmax}(w_i x_4 + b_i)$$

→ Time (Prediction)

→ y_4 is a probability vector of size 27

→ Example output "a" has highest probability say (0.85) other characters have lower probabilities

→ Final prediction → $\arg\max(y_4) =$ "a"

Question #02

Find Hidden State Sequence
for observation [ok, fail, perfect]

→ π = [tired, Happy] = [0.1, 0.9]

→

$$\begin{array}{c|ccc} & \text{Fail} & \text{ok} & \text{Perfect} \\ \hline \text{Tired} & 0.3 & 0.5 & 0.2 \\ \text{happy} & 0.1 & 0.5 & 0.4 \end{array}$$

Emission Matrix

→ Transition Probabilities :- (moving from state to state)

$$\begin{array}{c|cc} & \text{tired} & \text{Happy} \\ \hline \text{Tired} & 0.4 & 0.6 \\ \text{Happy} & 0.2 & 0.8 \end{array}$$

→ Step 1 → Observation 1 (ok)

P(ok | tired) = 0.1 x 0.5 = 0.05

P(ok | happy) = 0.9 x 0.5 = $\boxed{0.45}$ ✓

* (Choose happy - higher Prob)

→ Step 2 → Observation 2 (fail) tired (fail) happy

P(fail | happy → tired) = 0.45 x 0.3 x 0.2

= 0.027

P(fail | happy → happy) = 0.45 x 0.8 x 0.1

= $\boxed{0.036}$ ✓ choose happy

→ Step #03 → Observation (perfect):-

$P(\text{Perfect} \mid \text{happy} \rightarrow \text{tired}) = 0.036 \times 0.2 \times 0.2$

$= 0.00144$

$P(\text{Perfect} \mid \text{happy} \rightarrow \text{happy}) = 0.036 \times 0.8 \times 0.4$

$= \boxed{0.01152}$ ✓

Choose happy!

Hence;

$\boxed{\text{Hidden State Sequence} = [\text{Happy}, \text{Happy}, \text{Happy}]}$

Ans:-