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ASSIGNMENT#02

Cryptarithmetic Puzzle as CSP

① Define the Problem as a CSP:

⊙ Identify Variables

In;

CROSS + ROAD = DANGER

Consider the Distinct variables

$$V = \{C, R, O, S, A, D, N, G, E\}$$

⊙ Define Domain for each variables

As there are 9 distinct characters/variable so Domain will be (0-9), for each variable.

* As we are dealing with decimal number system in Cryptarithmetic Puzzle!

→ Each letter E {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

→ All letters will have different | distinct variables from the Domain.

→ The leading letters can't be zero
i.e # (C ≠ 0, R ≠ 0, D ≠ 0)

⊙ List the constraints that must be satisfied.
In the cryptarithmetic Puzzle as CSP :,
we normally add column by column from right to left including the carry values (when >9)

$C_4\ C_3\ C_2\ C_1$
C R O S S
C_5 + R O A D
D A N G E R

Constraint 1 → $S + D = R + 10 * C_1$
Constraint 2 → $S + A + C_1 = E + 10 * C_2$
Constraint 3 → $O + O + C_2 = G + 10 * C_3$
Constraint 4 → $R + R + C_3 = N + 10 * C_4$
Constraint 5 → $C + C_4 = A + 10 * C_5$
Constraint 6 → $C_5 = D$

∴ $C_1, C_2, C_3, C_4, C_5, E$ {0, 1}

② Solve the Problem Using a Systematic Approach :

⊙ Assign unique value to each variable Show step by-step Calculation :

The Systematic Approach Used here will be "Backtracking Approach"

- From Constraint 6 :

As; $D = C_5$ and $C_5 \in \{0,1\}$ and we know D being the leading letter can't be zero

Hence

$$\boxed{D = C_5 = 1}$$

- From Constraint 5 :

$$C + C_4 = A + C_5$$

$(\because C_5 = 1)$

$$C + C_4 = A + C_5 * 10$$

$$C + C_4 = A + 10$$

$$\boxed{C = A + 10 - C_4}$$

$\therefore C_4 \in \{0,1\}$

Choose $C_4 = 1$

Because $(C \leq 9)$

$$\boxed{\therefore C_4 = 1}$$

$$\boxed{C = A + 9}$$ → Comparing constants at both sides

$C = 9$; $A = 0$

- From Constraint 4 :

$$R + R + C_3 = N + 10C_4$$

$$2R + C_3 = N + 10C_4$$

$$(\because C_4 = 1)$$

$$2R = N + 10 - C_3$$

$$\because C_3 = 1$$

$$2R = N + 9$$

$2R > 9$ → not possible when choose from $\{2,3,4,5,6,7,8\}$

So:

No assignment of digits satisfying all constraints was found.

Hence;

CSP unsatisfiable : no solution !!

③ Discuss which heuristics could optimize the solution

⊙ Minimum Remaining Values (MRV):

Choose the letter with the fewest possible digits left. This helps focus on the most constrained variables first, preventing wasted assignments.

⊙ **Forward Checking:**
Each time a digit is assigned, eliminate it from other letters' domain.
This avoids conflicts earlier and proves invalid paths quickly.

⊙ **Arc Consistency (AC-3):**
Continuously enforce consistency b/w variables (e.g. if assigning one digit makes another variable's domain empty, backtrack immediately).
This reduces dead-ends in search.

* **Explain the efficiency of solving this problem using CSP techniques.**
Solving the cryptarithmetic puzzle as a Constraint Satisfaction Problem (CSP) is efficient because:

→ It reduces the search space by enforcing constraints

→ Instead of blindly trying all 10! possible assignments of digits to letter, CSP techniques prune impossible branches early.

→ Logical Deductions further shrink the domain before systematic search.