Torques and Rotational Equilibrium of a Rigid Body

OBJECTIVES

- ❑ Apply the conditions for equilibrium of a rigid body to a meter stick pivoted on a knife edge.
- ❑ Determine the center of gravity of the meter stick, mass of the meter stick, and mass of an unknown object by applying known torques.
- ❑ For a given applied force needed to produce equilibrium, compare the theoretically predicted location of the force to an experimentally determined location.

EQUIPMENT LIST

- Meter stick with adjustable knife-edge clamp and support stand
- Laboratory balance and calibrated hooked masses
- Thin nylon thread and unknown mass with hook

THEORY

If a force F acts on a rigid body that is pivoted about some axis, the body tends to rotate about that axis. The tendency of a force to cause a body to rotate about some axis is measured by a quantity called **torque** τ. It is defined by

$$\tau = Fd_{\perp} \qquad \text{(Eq. 1)}$$

with F the magnitude of the force, and $d_{\perp}$ the lever arm of the force. The units of torque are N–m. Torque caused by a given force must be defined relative to a specific axis of rotation. Figure 10-1 shows two forces F_1 and F_2 acting on an arbitrarily shaped body.

The axis of rotation is along a line through O perpendicular to the page. The direction of the line of action of each force is shown as a dotted line extended in either direction along the force vector. The lever arm for each force is shown as the perpendicular distance from O to the line of action of the force. In this case there are two torques τ_1 and τ_2 acting on the body given by

$$\tau_1 = F_1 d_1 \quad \text{and} \quad \tau_2 = F_2 d_2 \qquad \text{(Eq. 2)}$$

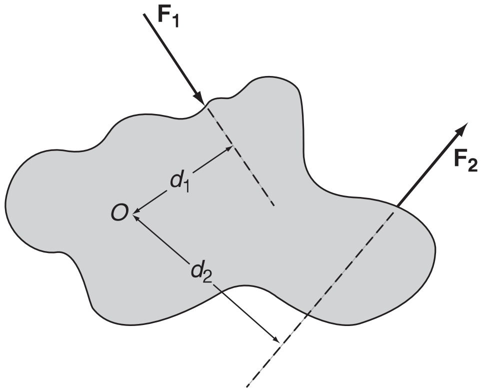

Figure 10-1 Lever arms about the point O for two forces acting on a body.

Torques tend either to rotate the body clockwise or counterclockwise about the axis. The convention in this laboratory will be to consider counterclockwise torques positive and clockwise torques negative. That convention gives net torque due to F_1 and F_2 about an axis through O as

$$\tau_{net} = F_2 d_2 - F_1 d_1 \tag{Eq. 3}$$

The net torque τ_{net} will be either counterclockwise, clockwise, or zero depending upon the magnitudes of $F_2 d_2$ and $F_1 d_1$. A meter stick will be the rigid body to which forces will be applied to produce mechanical equilibrium. The two conditions that must be satisfied for complete equilibrium of a rigid body are:

(1) Translational equilibrium is achieved if the vector sum of all the forces acting on the body is zero.

(2) Rotational equilibrium is achieved if the magnitude of $\sum \tau_{ccw}$ (sum of counterclockwise torques) is equal to the magnitude of $\sum \tau_{cw}$ (sum of clockwise torques).

The **center of gravity** of a body is defined as that point through which the sum of all the torques due to all the differential elements of mass of the body is zero. If the gravitational field is uniform throughout the body, the center of gravity and the **center of mass** are the same point. A uniform and symmetric meter stick has its center of gravity at the 0.500 m (50.0 cm) mark. Any meter stick will probably be close to uniform and symmetric, and its center of gravity will be close to the 0.500 m mark.

A meter stick with a knife-edge clamp on a support stand is shown in Figure 10-2. The mass of the meter stick is m_o, and three other masses m_1, m_2, and m_3 are shown hung from the meter stick. The masses produce forces where they are placed equal to the weight of the masses $m_1 g$, $m_2 g$, and $m_3 g$. The support exerts a force F_S directed upward at the point of the support. The weight of the meter stick $m_o g$ is exerted at the center of gravity of the meter stick x_g.

The meter stick in Figure 10-2 is in equilibrium. Forces in the upward direction are positive, and forces in the downward direction are negative. Take torques about an axis perpendicular to the page

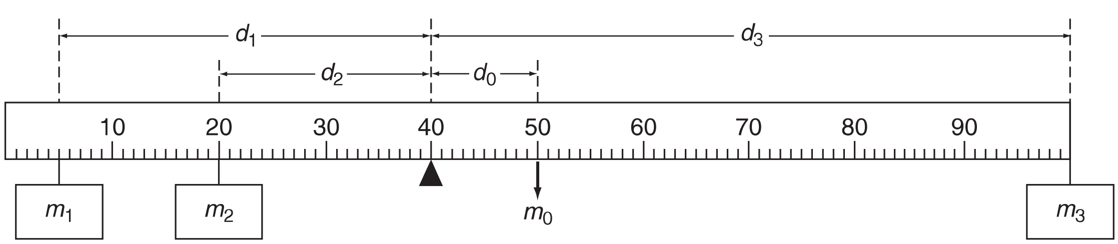

Figure 10-2 Meter stick balanced at point not the center of gravity. There are torques from the three applied masses and from the meter stick mass at the center of gravity.

through the point of support x_o. The lever arm for each mass is $d_i = |x_i - x_o|$ with x_i the position of the ith mass. The two conditions for equilibrium give

$$\sum F = 0 \quad \text{leads to} \quad F_S - m_1 g - m_2 g - m_3 g - m_o g = 0 \qquad \text{(Eq. 4)}$$

$$\sum \tau_{\text{ccw}} - \sum \tau_{\text{cw}} = 0 \quad \text{leads to} \quad m_1 g d_1 + m_2 g d_2 - m_o g d_o - m_3 g d_3 = 0 \qquad \text{(Eq. 5)}$$

The following is a numerical example of the arrangement in Figure 10-2. The mass of the meter stick is $m_o = 0.120$ kg, and masses $m_1 = 0.150$ kg and $m_2 = 0.200$ kg are placed as in the figure. The point of support is at the 0.400 m mark. What value of mass m_3 must be placed at the 1.000 m mark to put the system in equilibrium, and what is the resulting support force F_S? The solution is given by:

$$\sum \tau_{\text{ccw}} - \sum \tau_{\text{cw}} = 0 \text{ is } (0.150)g(0.350) + (0.200)g(0.200) - (0.120)g(0.100) - m_3 g(0.600) = 0$$

which reduces to $\quad 0.0525 + 0.0400 - 0.0120 = m_3\,(0.600)$

Solving the above equation gives $\quad m_3 = \dfrac{0.0805}{0.600} = 0.134\,\text{kg}$

$$\sum F = 0 \text{ gives} \quad F_S = (m_1 g + m_2 g + m_3 g + m_o g) = (0.150 + 0.200 + 0.134 + 0.120)(9.80) = 5.92\,\text{N}$$

In the numerical example, the value of m_3 was determined for torques about an axis through the point of support. *When the conditions of complete equilibrium have been met for some specific axis, the sum of torques about any axis is then ensured to be zero.* To confirm this for the numerical example given above, take torques about the axis perpendicular to the page through the left end of the meter stick. The masses m_1, m_2, m_3, and m_o exert clockwise torques, and the force F_S exerts the only counterclockwise torque. Calculating those values gives

$$\tau_{\text{cw}} = [(0.150)(0.050) + (0.200)(0.200) + (0.120)(0.500) + (0.134)(1.00)][9.80] = 2.37\,\text{Nm}$$

$$\tau_{\text{ccw}} = [(F_S)0.400] = (5.92)(0.400) = 2.37\,\text{Nm}$$

In this laboratory, attention will be directed to satisfying the conditions of rotational equilibrium. The support force F_S will always act through the support position. If the support position is chosen as the axis for torques, F_S will not contribute to the torque because it will have a zero lever arm. When the rotational equilibrium conditions are met, the value of the support force F_S will ensure translational equilibrium as well.

EXPERIMENTAL PROCEDURE

Part 1. Torque due to Two Known Forces

1. Remove the knife-edge clamp from the meter stick. Use the laboratory balance to determine the mass of the meter stick. Record it in the Meter Stick Data Table.
2. Place the knife-edge clamp on the meter stick and place it on the support. Adjust the position of the clamp until the best balance is achieved. Record the position of the knife-edge clamp as x_g in the Meter Stick Data Table.
3. With the meter stick supported at x_g, place a mass $m_1 = 0.100$ kg at the 0.100 m mark. Determine and record in Data and Calculations Table 1 the position x_2 at which a mass $m_2 = 0.200$ kg balances the meter stick. Use a small loop of nylon thread to hang the hooked masses at a given position. It may prove helpful to use a very small piece of tape to hold the thread at the desired position.

4. Calculate the lever arm for each force $d_i = |x_g - x_i|$where x_i is the position of the ith mass. With the support at the position x_g, the meter stick mass has zero lever arm and contributes no torque. Record the values of d_1 and d_2 in Data and Calculations Table 1.
5. Calculate and record in Data and Calculations Table 1 the value of the torques. The only counterclockwise torque is due to m_1 and $\sum \tau_{ccw} = m_1 g d_1$. The only clockwise torque is due to m_2 and $\sum \tau_{cw} = m_2 g d_2$. Use a value of 9.80 m/s^2 for g for these and all other calculations.
6. Calculate the percentage difference between $\sum \tau_{ccw}$ and $\sum \tau_{cw}$ and record it in Data and Calculations Table 1.

Part 2. Torque due to Three Known Forces

1. Support the meter stick at x_g. Place $m_1 = 0.100$ kg at 0.100 m, and $m_2 = 0.200$ kg at 0.750 m. Determine the position x_3 at which $m_3 = 0.050$ kg balances the system. Record the value of x_3 in Data and Calculations Table 2.
2. The meter stick mass m_o makes no contribution to the torque. Calculate the lever arm for each of the masses and record the values in Data and Calculations Table 2. ($d_i = |x_g - x_i|$)
3. Calculate the values of $\sum \tau_{ccw}$ and $\sum \tau_{cw}$ and record them in Data and Calculations Table 2.
4. Calculate the percentage difference between $\sum \tau_{ccw}$ and $\sum \tau_{cw}$ and record it in Data and Calculations Table 2.

Part 3. Determination of the Meter Stick Mass by Torques

1. Place a mass $m_1 = 0.200$ kg at the 0.100 m position. Loosen the knife-edge clamp and slide the meter stick in the clamp until the torque exerted by $m_1 g$ is balanced by the torque of the meter stick weight acting at x_g. When the best balance is achieved, tighten the clamp. The position at which the meter stick is supported is x_o. Record x_o in Data and Calculations Table 3.
2. The values of the lever arms are given by $d_1 = |x_1 - x_o|$ and $d_o = |x_g - x_o|$. Calculate and record the values of d_1 and d_o in Data and Calculations Table 3.
3. For these conditions, $\sum \tau_{ccw} = m_1 g d_1$ and $\sum \tau_{cw} = m_o g d_o$ where m_o stands for the assumed unknown mass of the meter stick. Equating the two torques gives $m_o = m_1(d_1/d_o)$. Calculate and record in Data and Calculations Table 3 this value as $(m_o)_{exp}$.
4. Calculate and record in Data and Calculations Table 3 the percentage error in $(m_o)_{exp}$ compared to the meter stick mass determined by the laboratory balance.

Part 4. Comparison of Experimental and Theoretical Determinations of the Location of an Applied Force

1. Adjust the knife-edge clamp to the 0.400 m mark. Place $m_1 = 0.050$ kg at 0.050 m, $m_2 = 0.300$ kg at 0.300 m, and $m_3 = 0.200$ kg at 0.700 m as shown in Figure 10-3. *With the meter stick supported at the 0.400 m mark*, determine the position at which mass $m_4 = 0.100$ kg balances the meter stick. Record this

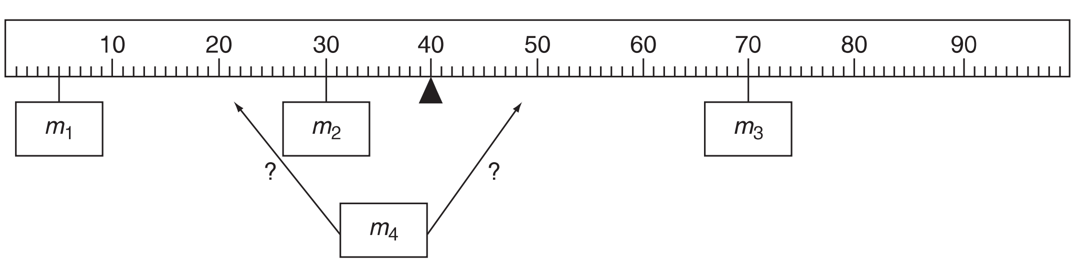

Figure 10-3 The location of mass m_4 needed to place the system in equilibrium is to be determined both experimentally and theoretically, and then compared.

position as x_4 in Data and Calculations Table 4 and use this value of x_4 to calculate the lever arm d_4 and record it in Data and Calculations Table 4 as $(d_4)_{exp}$.

2. In the space provided in Data and Calculations Table 4, write the equation for the rotational equilibrium with counterclockwise torques positive and clockwise torques negative. Use m_1, m_2, m_3, and m_4 as the symbols for the appropriate mass, and d_1, d_2, d_3, and d_4 as the symbols for the lever arms. Include the contribution from the meter stick mass m_o acting at the center of gravity of the meter stick x_g.
3. In the equation, treat the lever arm d_4 of mass m_4 as unknown and all the other quantities as known. Solve the equation to obtain a value for d_4 and record that result in Data and Calculations Table 4 as $(d_4)_{theo}$.
4. Calculate the percentage error in $(d_4)_{exp}$ compared to $(d_4)_{theo}$ and record it in Data and Calculations Table 4.

Part 5. Determination of an Unknown Mass by Torques

1. Use an experimental arrangement with the meter stick that is similar to those that we have used thus far to devise a method to determine the mass of an unknown mass. Describe carefully the procedure that is followed. Use at least one known mass and state its value and location on the meter stick. Write an equation that describes the equilibrium of the system treating the mass as unknown. Include a sketch of the experimental arrangement showing the position of all masses known and unknown. Construct your own Data and Calculations Table 5 listing all the relevant quantities.
2. Use the laboratory balance to determine the value of the unknown mass. Calculate the percentage error in your experimental value of the unknown mass compared to that obtained using the laboratory balance.

This page intentionally left blank

Name | Section | Date

10 LABORATORY 10 *Torques and Rotational Equilibrium of a Rigid Body*

PRE-LABORATORY ASSIGNMENT

1. State a definition of torque and give an equation for torque. Define the terms in the equation.

2. What are the conditions for equilibrium of a rigid body? State in words and equation form and define the terms of the equations.

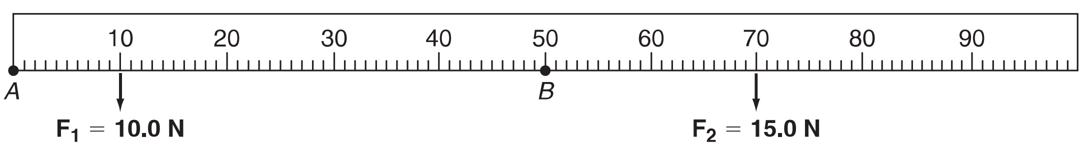

Figure 10-4 Meter stick with two forces F_1 and F_2 acting at points shown.

3. For the meter stick shown in Figure 10-4, the force F_1 10.0 N acts at 10.0 cm. What is the magnitude of the torque due to F_1 about an axis through point A perpendicular to the page? Is it clockwise, or is it counterclockwise? Show your work and give correct units.

4. In Figure 10-4 the force $F_2 = 15.0\,\text{N}$ acts at the point 70.0 cm. What is the magnitude of the torque due to F_2 about an axis through point B and perpendicular to the page? Is the torque clockwise, or is it counterclockwise? Show your work and give correct units.

5. For the meter stick in Figure 10-4, what is the magnitude of the *net* torque due to both forces F_1 and F_2 about an axis perpendicular to the page through point A? Is it clockwise or counterclockwise? Show your work.

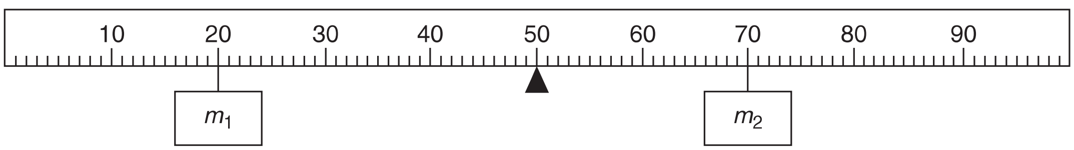

Figure 10-5 Meter stick with forces applied by hanging two masses m_1 and m_2.

6. In Figure 10-5 if mass $m_1 = 0.100\,\text{kg}$ acts at 20.0 cm, what is the value of mass m_2 that must be placed at the position 70.0 cm shown to put the system in equilibrium? Write the equation for $\sum \tau_{\text{ccw}} = \sum \tau_{\text{cw}}$ with the mass m_2 as unknown and solve for m_2. Assume that the meter stick is uniform and symmetric. Show your work.

Name | Section | Date

Lab Partners

10 LABORATORY 10 Torques and Rotational Equilibrium of a Rigid Body

LABORATORY REPORT

Meter Stick Data Table

$m_o =$ kg	$x_g =$ m

Data and Calculations Table 1

Mass (kg)	*Position* (m)	*Lever arm* (m)	*Torque* (N−m)	% *Difference*
$m_1 = 0.100$	$x_1 = 0.100$	$d_1 =$	$\tau_{ccw} =$	
$m_2 = 0.200$	$x_2 =$	$d_2 =$	$\tau_{cw} =$	

Data and Calculations Table 2

Mass (kg)	*Position* (m)	*Lever arm* (m)	*Torque* (N−m)	% *Difference*
$m_1 = 0.100$	$x_1 = 0.100$	$d_1 =$	$\tau_{ccw} =$	
$m_2 = 0.200$	$x_2 = 0.750$	$d_2 =$		
$m_3 = 0.050$	$x_3 =$	$d_3 =$	$\tau_{cw} =$	

Data and Calculations Table 3

Support Position $x_o =$ m				
Mass (kg)	*Position* (m)	*Lever arm* (m)	$(m_o)_{exp} =$ kg	% *Error*
$m_1 = 0.200$	$x_1 = 0.100$	$d_1 =$		
$m_o =$	x_g	$d_o =$		

Data and Calculations Table 4

Support Position $x_o = 0.400$ m			
Mass (kg)	*Position* (m)	*Lever Arm* (m)	*Equation for the Torque*
$m_1 = 0.050$	$x_1 = 0.050$	$d_1 =$	
$m_2 = 0.300$	$x_2 = 0.300$	$d_2 =$	
$m_3 = 0.200$	$x_3 = 0.700$	$d_3 =$	Solving equation for d_4 gives $(\mathrm{d}_4)_{\mathrm{theo}} =$ m
$m_4 = 0.100$	$x_4 =$	$(d_4)_{\mathrm{exp}} =$	
$m_o =$	$x_g =$	$d_o =$	

Data and Calculations Table 5

SAMPLE CALCULATIONS

1. $d_i = |x_i - x_o| =$

2. $\sum \tau_{\text{ccw}} =$

3. $\sum \tau_{\text{cw}} =$

4. % Diff torques $= |\tau_{\text{ccw}} - \tau_{\text{cw}}| / \left(0.5(\tau_{\text{ccw}} + \tau_{\text{cw}})\right) \times 100\% =$

5. $(m_o)_{\text{exp}} = m_1(d_1/d_o) =$

6. % Error for $(m_o)_{\text{exp}} = |\text{E} - \text{K}|/\text{K} \times 100\% =$

7. $(d_4)_{\text{theo}} =$

8. % Error for $(d_4)_{\text{theo}} = |\text{E} - \text{K}|/\text{K} \times 100\% =$

9. Calculation for m_{exp} in Part 5 $=$

10. % Error for $m_{\text{exp}} = |\text{E} - \text{K}|/\text{K} \times 100\% =$

QUESTIONS

1. Consider the percentage difference between the $\sum \tau_{\text{ccw}}$ and the $\sum \tau_{\text{cw}}$ for the first two parts of the laboratory when known forces are balanced. A difference of 0.5% or less is excellent, a difference of 1.0% or less is good, and a difference of 2% or less is acceptable. Based on these criteria, describe your results for the first two parts of the laboratory and defend your statement.

2. Using the same criteria as in Question 1 for the percentage differences, describe your results for the determination of mass of the meter stick in Part 3 of the laboratory and for the determination of the lever arm of the mass m_4 in Part 4 of the laboratory.

3. In all of the experimental arrangements the mass of the knife-edge clamp is ignored. Is this an approximation because its mass is small, or is there some reason it makes no contribution to the torque? State your reasoning clearly.

4. Suppose an experimental arrangement like the one in Part 2 has mass $m_1 = 0.200\,\text{kg}$ at the 0.100-m mark and a mass $m_2 = 0.100\,\text{kg}$ at the 0.750-m mark. Can the system be put in equilibrium by a 0.050-kg mass? If it can be done, state where it would be placed. If it cannot be done, state why not.

5. In Part 1 of the laboratory, what is the value of the force F_s with which the support pushes upward on the meter stick?

6. For the equilibrium conditions established in Part 4 of the laboratory, calculate the counterclockwise and clockwise torques about an axis perpendicular to the page through a point at the left end of the meter stick. Calculate the percentage difference between the net counterclockwise torque and the net clockwise torque.