

Biçimsel Diller ve Soyut Makineler

Hafta7

Grammerler ve Türetme Ağaçları

$$\{a^n b^n : n \geq 0\} \qquad \{ww^R\}$$

Reguler Diller

$$a^* b^* \qquad (a + b)^*$$



A Venn diagram consisting of two concentric ellipses. The outer ellipse is labeled "Context-Free Diller" in blue text. Inside it, at the bottom, is a smaller ellipse labeled "Regular Diller" in blue text. To the left of the "Regular Diller" ellipse, within the "Context-Free Diller" ellipse, is the mathematical expression $\{a^n b^n\}$. To the right of the "Regular Diller" ellipse, also within the "Context-Free Diller" ellipse, is the mathematical expression $\{ww^R\}$.

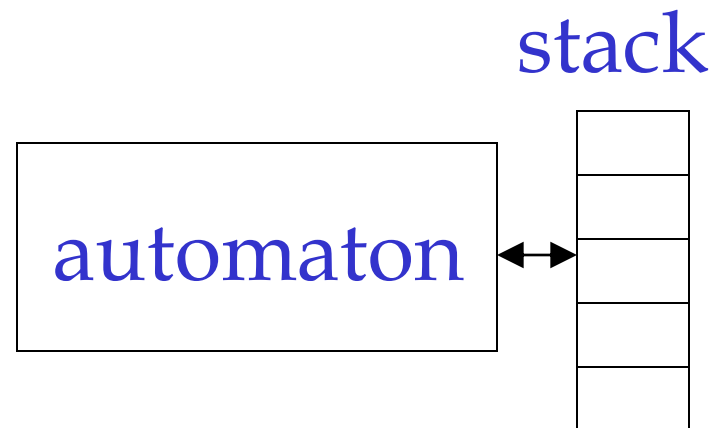
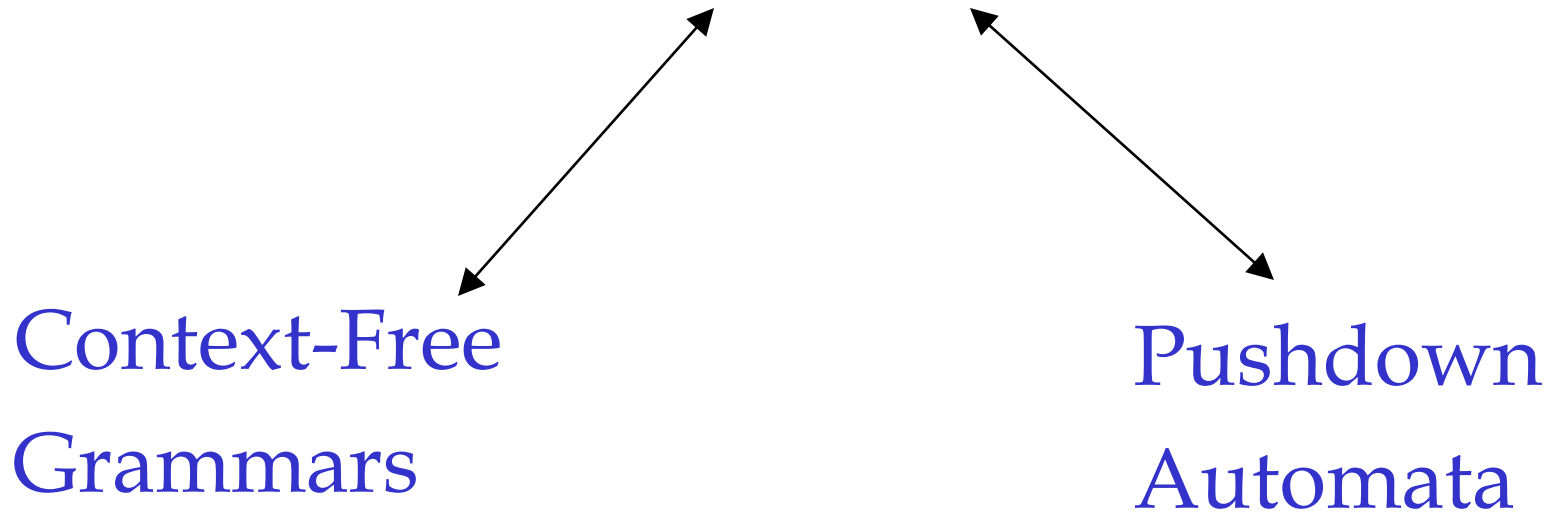
Context-Free Diller

$$\{a^n b^n\}$$

$$\{ww^R\}$$

Regular Diller

Context-Free Diller(CFL)-bağlamdan bağımsız



Bağlamdan Bağımsız Grammerler (Context-Free Grammars)

Örnek

Grammer: $S \rightarrow aSb$

$$S \rightarrow \lambda$$

Cümle türetimi : ab

$$S \Rightarrow aSb \Rightarrow ab$$
$$S \rightarrow aSb \qquad S \rightarrow \lambda$$

Grammer: $S \rightarrow aSb$

$S \rightarrow \lambda$

Cümle türetimi : $aabb$

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$



$S \rightarrow aSb$



$S \rightarrow \lambda$

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbbb$$

$$\begin{aligned} S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \\ \Rightarrow aaaaSbbbb \Rightarrow aaabbbbb \end{aligned}$$

Grammerin tanımladığı dil:

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

$$L = \{a^n b^n : n \geq 0\}$$

Biçimsel Tanım

$$G = (V, T, S, P)$$

V : Değişkenler kümesi

T : Uç simgeler (terminal)

S : Başlangıç değişkeni

P : Türetim kuralları

$$L(G) = \{w : S \overset{*}{\Rightarrow} w\}$$

Örnek

Grammer

G

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

$$G = (V, T, S, P)$$

$$V = \{S\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow aSb, S \rightarrow \lambda\}$$

Tümcesel form:

nonterminal ve termineller içerir

Örnek:

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbbb$

Tümcesel formlar

Tümce

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbbb$$

*

Yerine : $S \Rightarrow aaabbbb$ yazabiliriz

Genel olarak da

$$w_1 \Rightarrow w_2 \Rightarrow w_3 \Rightarrow \cdots \Rightarrow w_n$$

*

yerine

$$w_1 \Rightarrow w_n$$

yazabiliriz.

$$\alpha \overset{*}{\Rightarrow} \alpha.$$

If $\alpha \overset{*}{\Rightarrow} \beta$ and $\beta \Rightarrow \gamma$, then $\alpha \overset{*}{\Rightarrow} \gamma$

$$\alpha \overset{*}{\Rightarrow} \beta \quad 1. \alpha = \gamma_1, \quad 2. \beta = \gamma_n, \quad 3. \text{for } i=1,2,\dots,n-1, \gamma_i \Rightarrow \gamma_{i+1}$$

Örnek

Grammer

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Türetimler

*

$$S \Rightarrow \lambda$$

*

$$S \Rightarrow ab$$

*

$$S \Rightarrow aabb$$

*

$$S \Rightarrow aaabbb$$

Örnek

Grammer

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Türetimler

$$S \xRightarrow{*} aaSbb$$

$$aaSbb \xRightarrow{*} aaaaaaSbbbbbb$$

CFG:

$$S \rightarrow ASB \mid c$$

$$A \rightarrow \varepsilon \mid aA$$

$$B \rightarrow \varepsilon \mid bB$$

$$W = acb$$

$$S \Rightarrow ASB \Rightarrow aASB \Rightarrow aSB \Rightarrow acB \Rightarrow acbB \Rightarrow acb$$

$$S \Rightarrow ASB \Rightarrow ASbB \Rightarrow ASb \Rightarrow Acb \Rightarrow aAcb \Rightarrow acb$$

$$S \Rightarrow ASB \Rightarrow AcB \Rightarrow aAcB \Rightarrow aAcbB \Rightarrow acbB \Rightarrow acb$$

$$S \Rightarrow_{lm} ASB \Rightarrow_{lm} aASB \Rightarrow_{lm} aSB \Rightarrow_{lm} acB \Rightarrow_{lm} acbB \Rightarrow_{lm} acb \quad \text{Leftmost Derivation}$$

$$S \Rightarrow_{rm} ASB \Rightarrow_{rm} ASbB \Rightarrow_{rm} ASb \Rightarrow_{rm} Acb \Rightarrow_{rm} aAcb \Rightarrow_{rm} acb \quad \text{rightmost Derivation}$$

$$S \Rightarrow ASB \Rightarrow AcB \Rightarrow aAcB \Rightarrow aAcbB \Rightarrow acbB \Rightarrow acb \quad \text{leftmost or rightmost}$$

Örnek

$$\begin{aligned}G \quad S &\rightarrow Ab \\ A &\rightarrow aAb \\ A &\rightarrow \lambda\end{aligned}$$

Türetimler: $S \rightarrow Ab \rightarrow b$

$$S \rightarrow Ab \rightarrow aAbb \rightarrow abb$$

$$S \rightarrow Ab \rightarrow aAbb \rightarrow aaAbbbb \rightarrow aabbbb$$

$$L(G) = \{w : \overset{*}{S} \Rightarrow w\} \quad \overset{*}{S} \Rightarrow a^n b^n b$$

$$L(G) = \{a^n b^n b : n \geq 0\}$$

Örnek1

Bir CFG şu kurallarla
verilmiş olsun:

$$G \quad \begin{array}{l} S \rightarrow aSb \\ S \rightarrow \lambda \end{array}$$

Bir türetim:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$$

Bir başka türetim:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbbb$$

$$L(G) = \{a^n b^n : n \geq 0\}$$

(((()))

Örnek 2

Bir diğer CFG şu kurallarla G verilmiş olsun. Tanımladığı dili yazınız.

$$S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \rightarrow \lambda$$

$$S \rightarrow aSa \rightarrow abSba \rightarrow abba$$

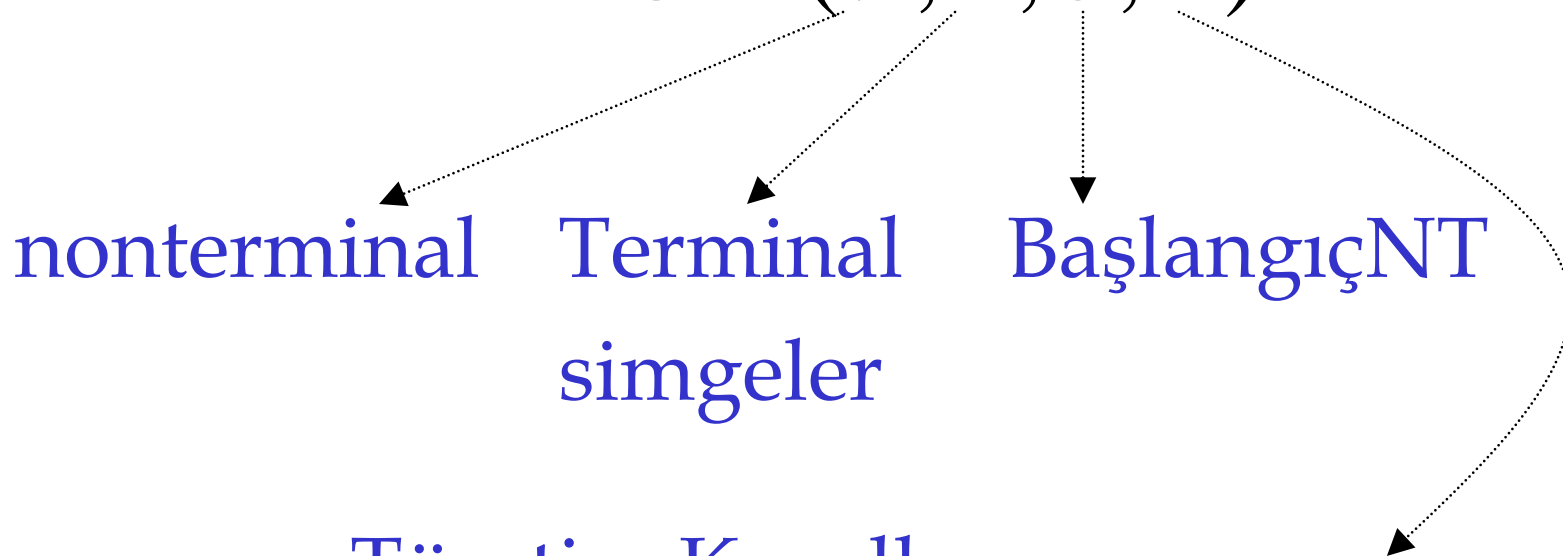
$$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abaaSaaba \Rightarrow abaadaaba$$

$$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abaSaba \Rightarrow abaaba$$

$$L(G) = \{ww^R : w \in \{a,b\}^*\}$$

Tanım: Context-Free Grammars

Grammar $G = (V, T, S, P)$



Türetim Kuralları:

Tek NT $A \rightarrow x$ Terminal yada NT
*

$$L(G) = \{w : S \Rightarrow w, \quad w \in T^*\}$$

Türetim Sırası

$$1. S \rightarrow AB$$

$$2. A \rightarrow aaA$$

$$4. B \rightarrow Bb$$

$$3. A \rightarrow \lambda$$

$$5. B \rightarrow \lambda$$

Soldan türetim (Leftmost derivation):

$$\begin{array}{ccccccccc} 1 & & 2 & & 3 & & 4 & & 5 \\ S & \Rightarrow & AB & \Rightarrow & aaAB & \Rightarrow & aaB & \Rightarrow & aaBb & \Rightarrow & aab \end{array}$$

Sağdan türetim (Rightmost derivation):

$$\begin{array}{ccccccccc} & 1 & & 4 & & 5 & & 2 & & 3 \\ S & \Rightarrow & AB & \Rightarrow & ABb & \Rightarrow & Ab & \Rightarrow & aaAb & \Rightarrow & aab \end{array}$$

$$S \rightarrow aAB$$

$$A \rightarrow bBb$$

$$B \rightarrow A \mid \lambda$$

Soldan türetim (Leftmost derivation):

$$\begin{aligned} S &\Rightarrow aAB \Rightarrow abBbB \Rightarrow abAbB \Rightarrow abbBbbB \\ &\Rightarrow abbbbB \Rightarrow abbbb \end{aligned}$$

Sağdan türetim (Rightmost derivation):

$$\begin{aligned} S &\Rightarrow aAB \Rightarrow aA \Rightarrow abBb \Rightarrow abAb \\ &\Rightarrow abbBbb \Rightarrow abbbb \end{aligned}$$

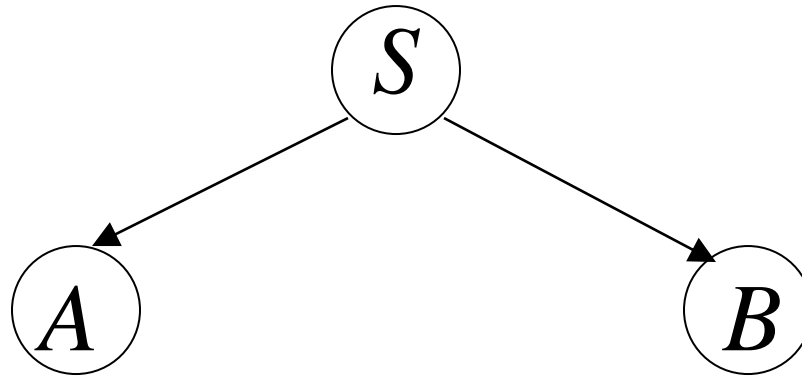
Türetim Ağaçları

$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB$$

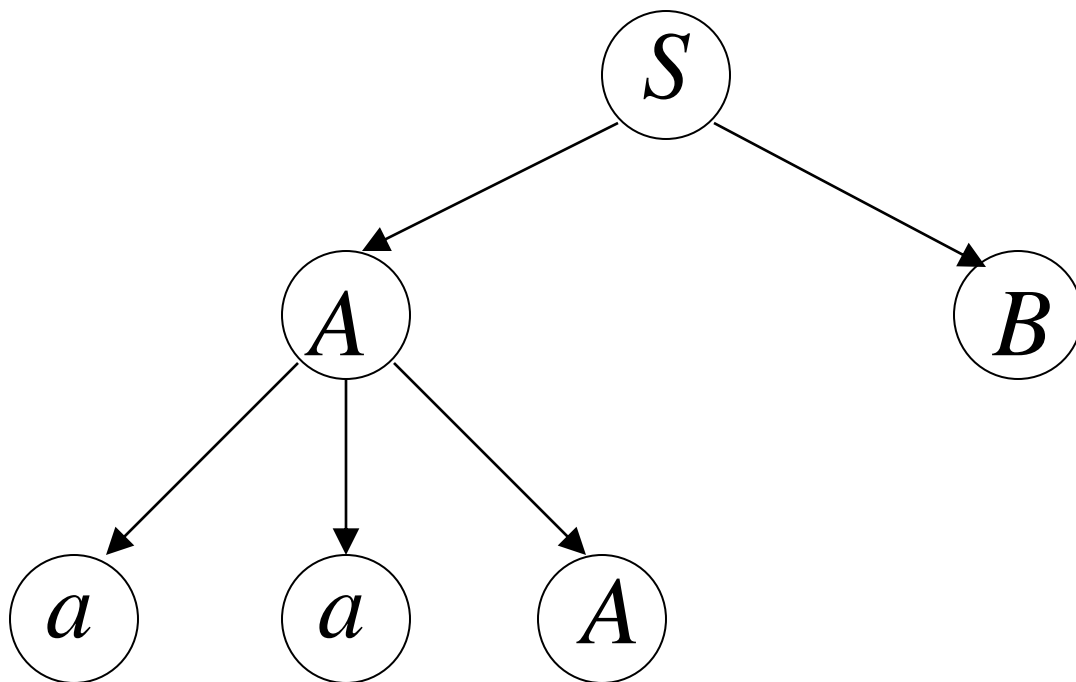


$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB$$

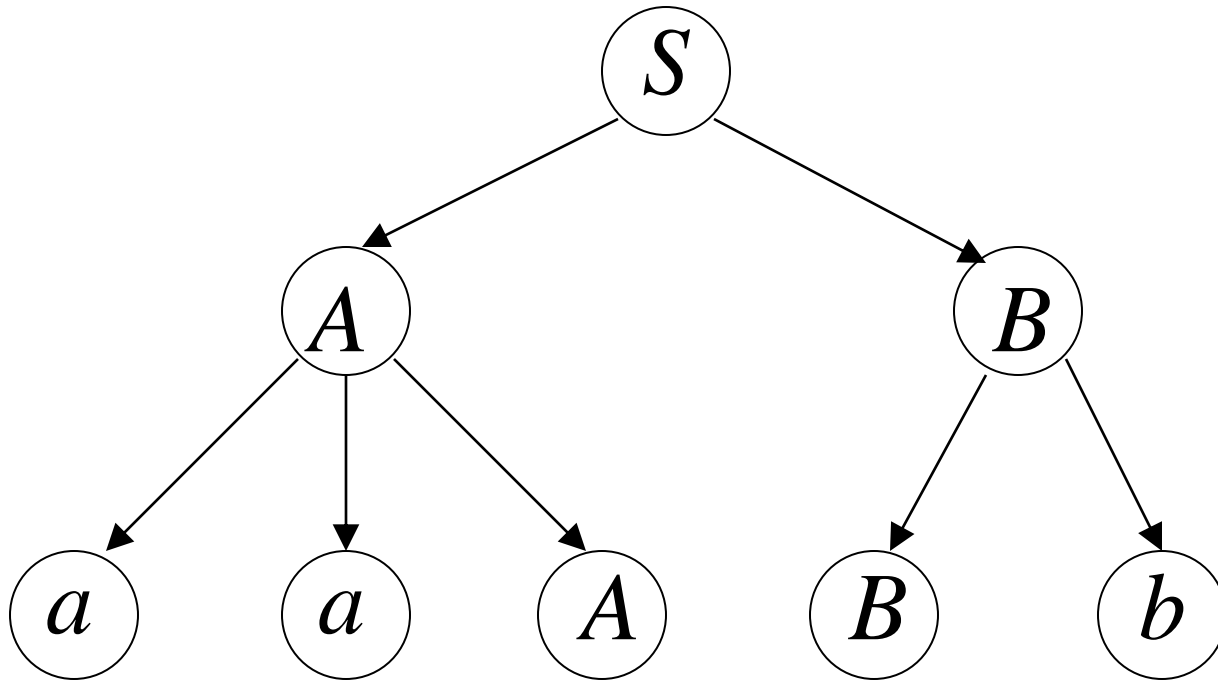


$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb$$

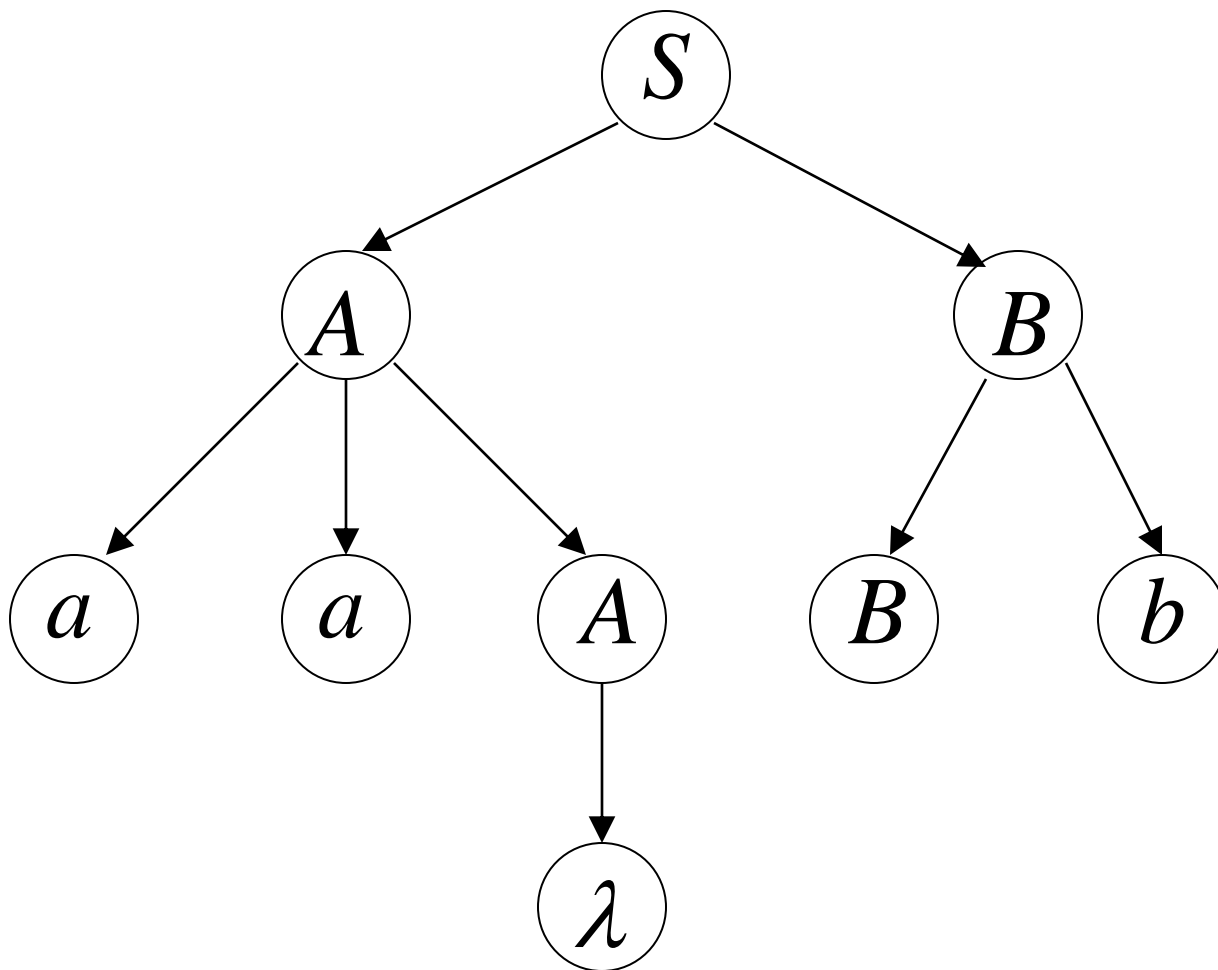


$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb \Rightarrow aaBb$$



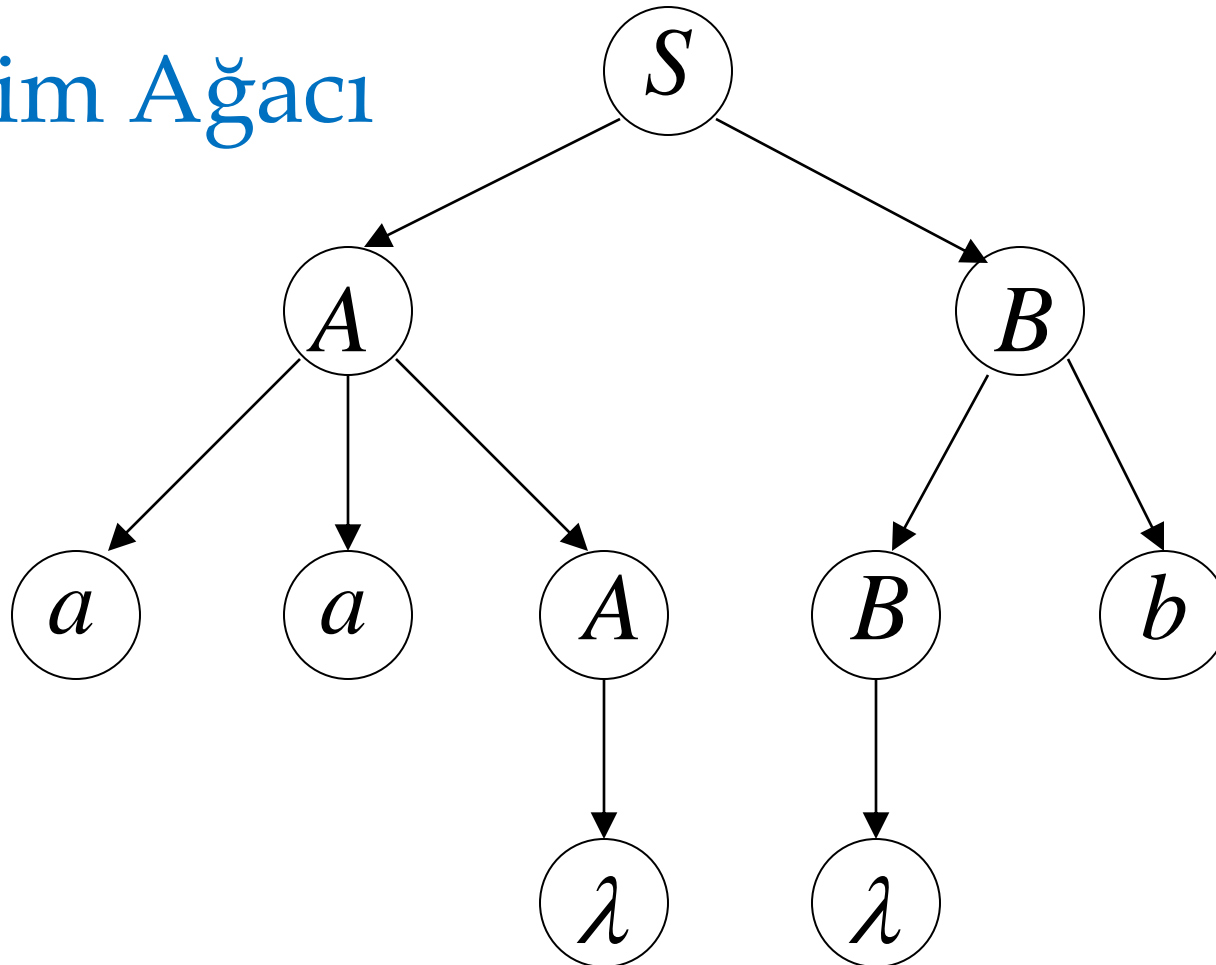
$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb \Rightarrow aaBb \Rightarrow aab$$

Türetim Ağacı



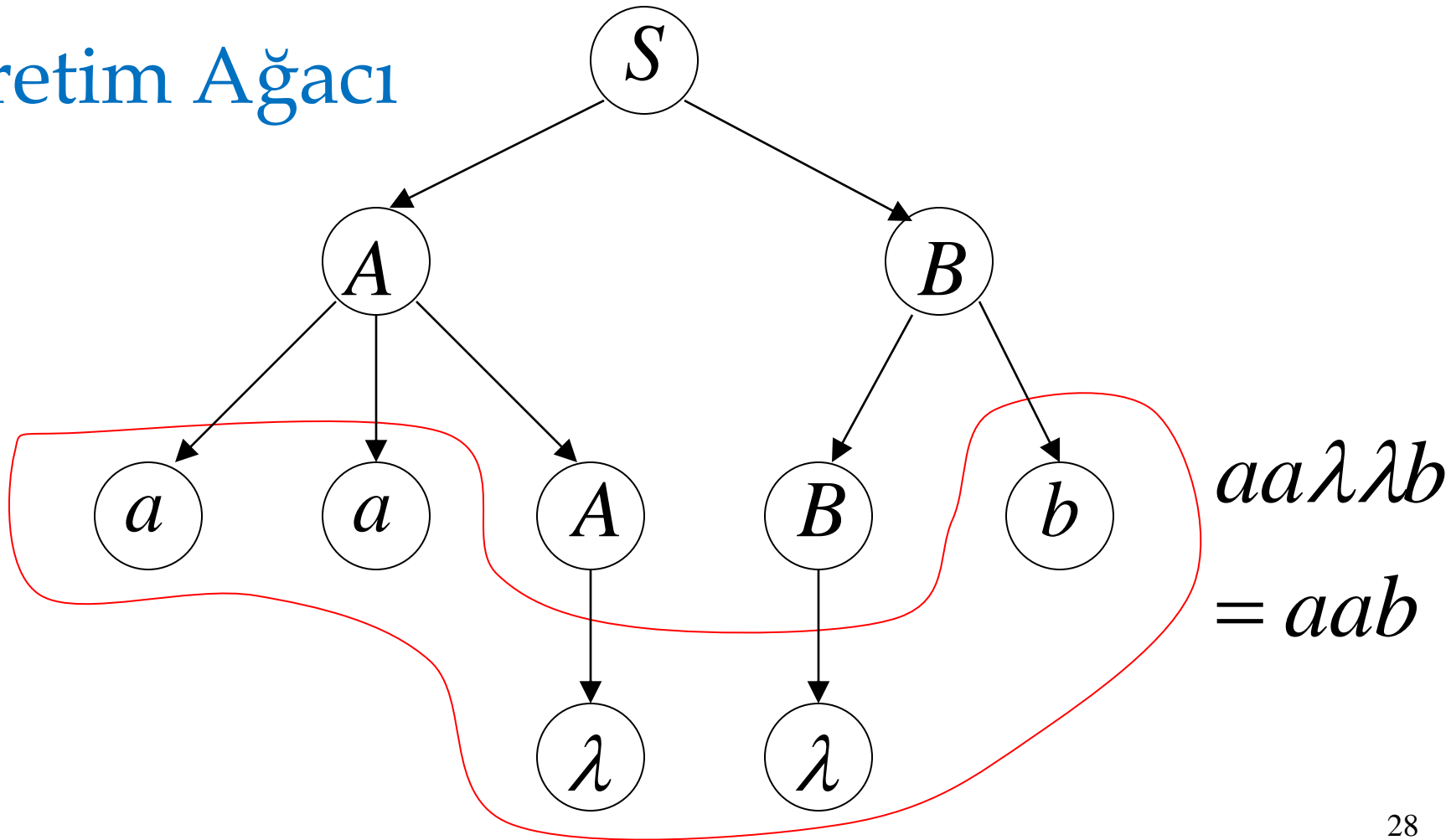
$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb \Rightarrow aaBb \Rightarrow aab$$

Türetim Ağacı



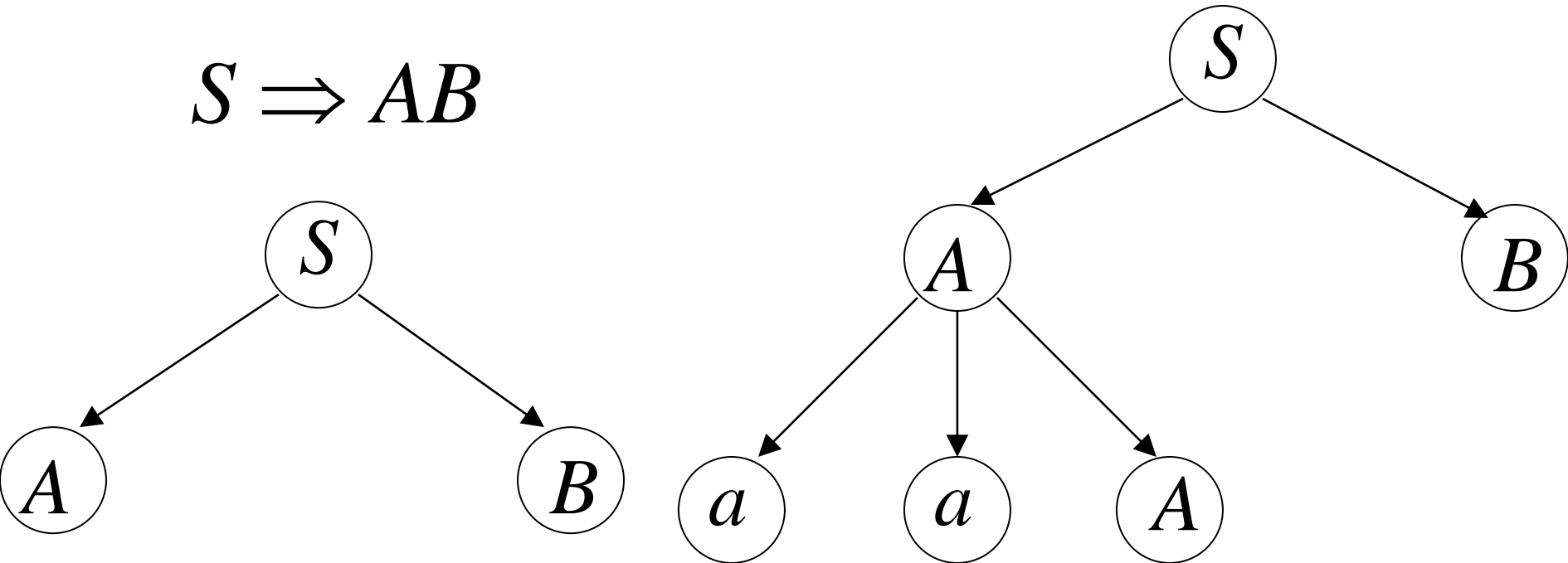
Sentential forms

$$S \rightarrow AB$$

$$A \rightarrow aaA \mid \lambda$$

$$B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB$$



Belirsiz Gramer

1. Dilde, bir karakter katarı için iki yada daha fazla en soldan türetim (leftmost derivation) dizisi varsa, CFG belirsizdir.
2. Dilde, bir karakter katarı için iki yada daha fazla en sağdan türetim (rightmost derivation) dizisi varsa, CFG belirsizdir.

$$S \rightarrow SaS \mid b$$

$$W = \text{babab}$$

leftmost

$$1. S \Rightarrow_{lm} SaS \Rightarrow_{lm} SaSaS \Rightarrow_{lm} baSaS \Rightarrow_{lm} babaS \Rightarrow_{lm} babab$$

$$2. S \Rightarrow_{lm} SaS \Rightarrow_{lm} baS \Rightarrow_{lm} baSaS \Rightarrow_{lm} babaS \Rightarrow_{lm} babab$$

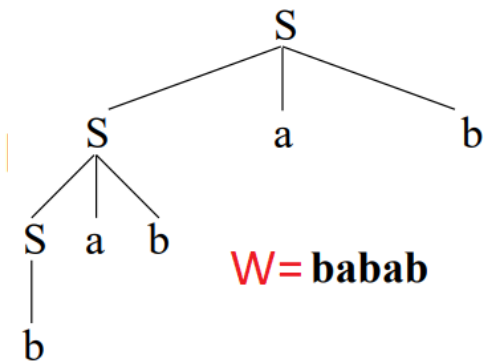
rightmost

$$1. S \Rightarrow_{rm} SaS \Rightarrow_{rm} Sab \Rightarrow_{rm} SaSab \Rightarrow_{rm} Sabab \Rightarrow_{rm} babab$$

$$2. S \Rightarrow_{rm} SaS \Rightarrow_{rm} SaSaS \Rightarrow_{rm} SaSab \Rightarrow_{rm} Sabab \Rightarrow_{rm} babab$$

$$S \rightarrow SaS \mid b \text{ yerine } S \rightarrow Sab \mid b$$

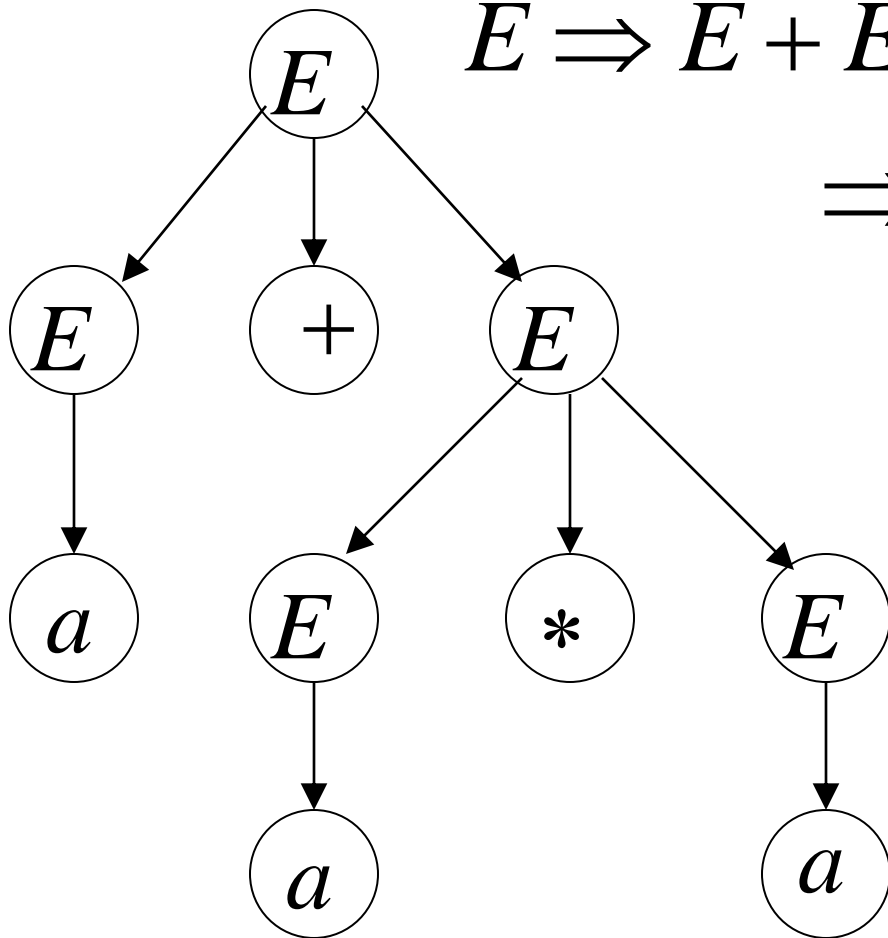
yazarak soldan gruplandırma yapılabilir



BELİRSİZLİK (Ambiguity)

$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

$$a + a * a$$



$$E \Rightarrow E + E \Rightarrow a + E \Rightarrow a + E * E$$

$$\Rightarrow a + a * E \Rightarrow a + a * a$$

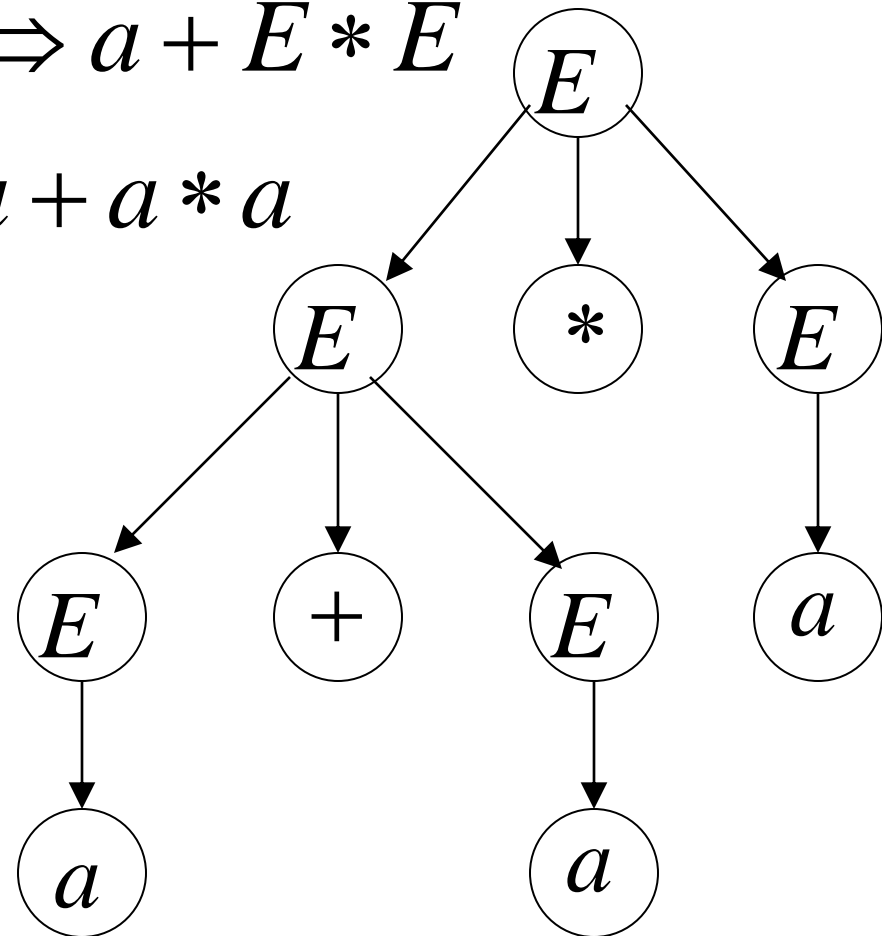
leftmost

$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

$$a + a * a$$

$$E \Rightarrow E * E \Rightarrow E + E * E \Rightarrow a + E * E \\ \Rightarrow a + a * E \Rightarrow a + a * a$$

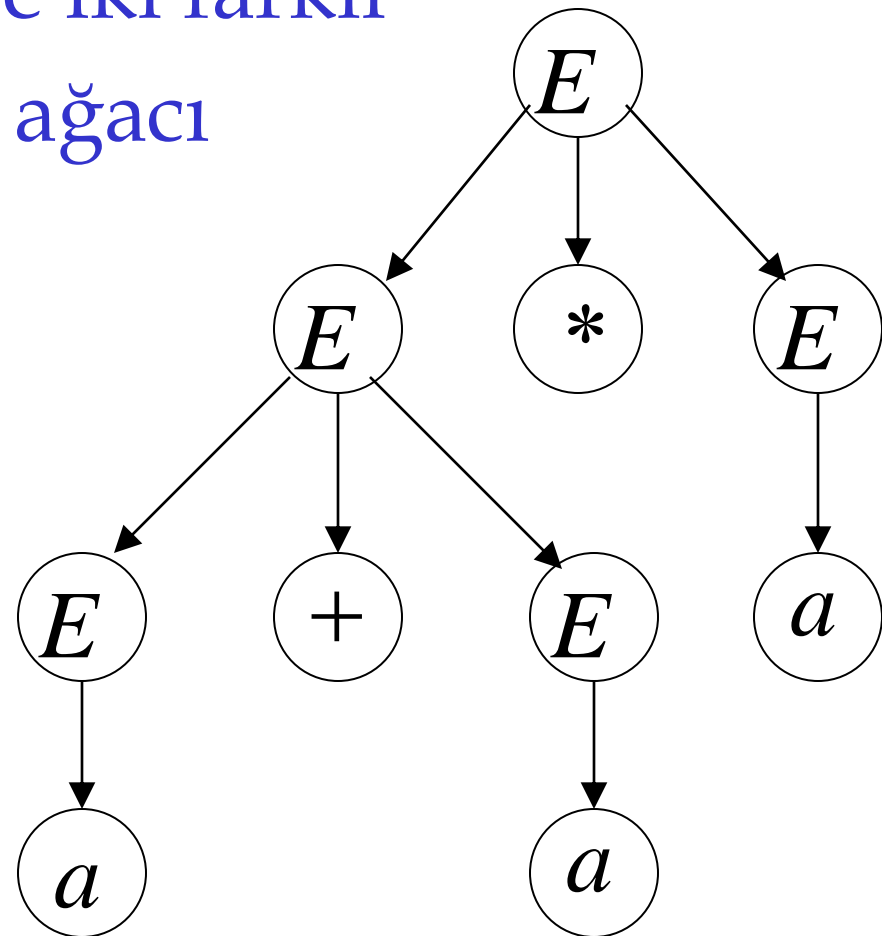
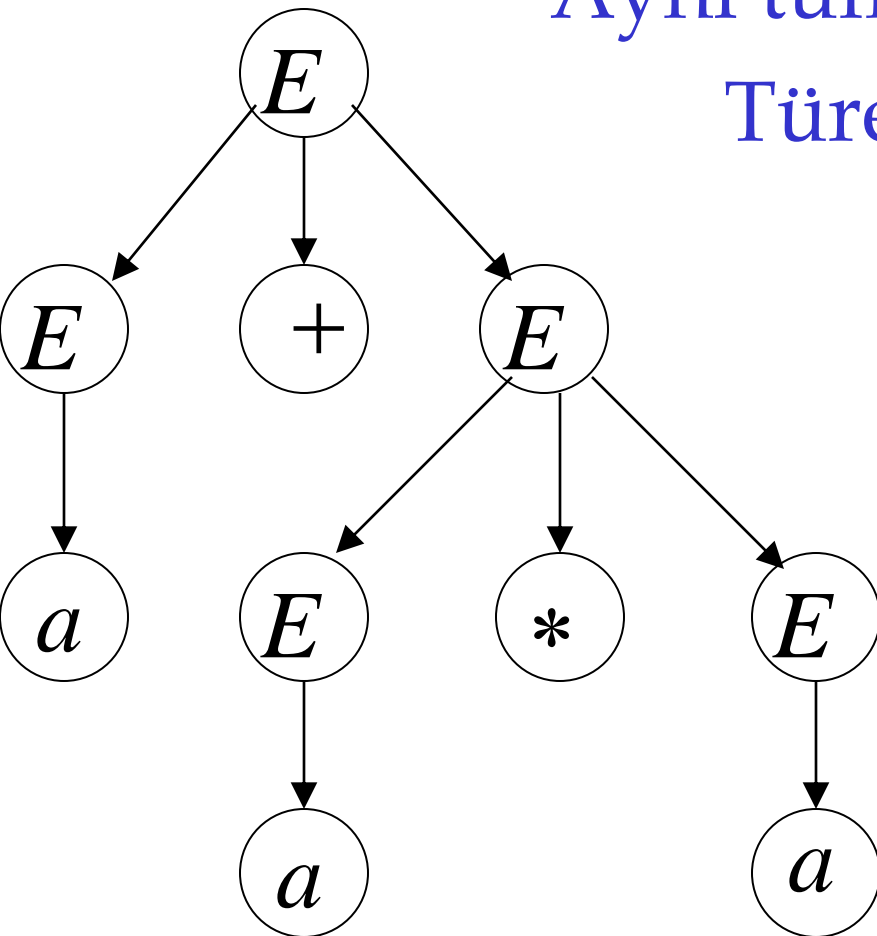
Leftmost



$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

$$a + a * a$$

Aynı tümceye iki farklı
Türetim ağacı

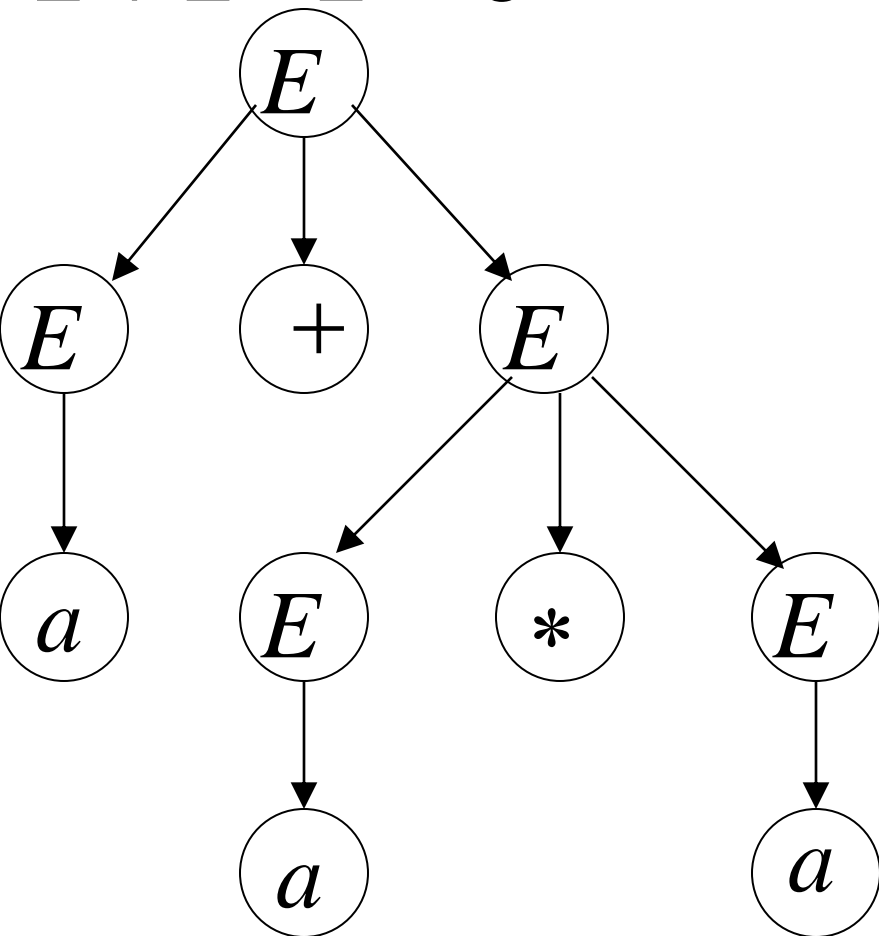


Belirsizlik neden önemlidir?

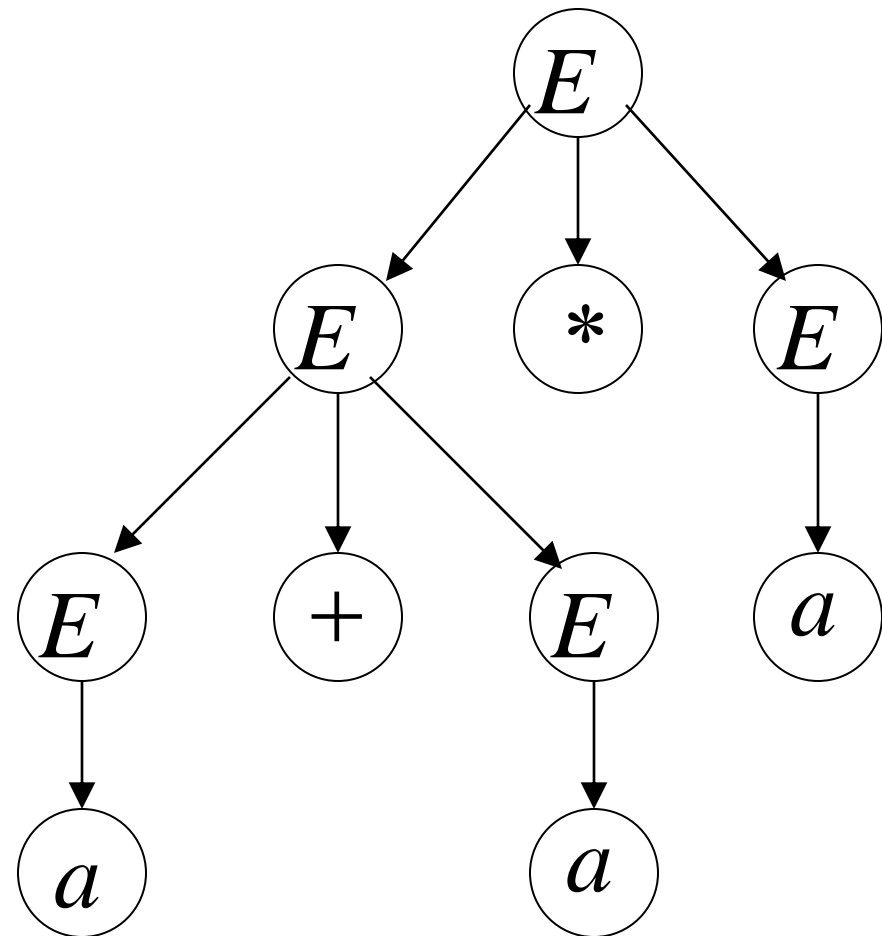
$$a + a * a$$

$a = 2$ olsun

$$2 + 2 * 2 = 6$$



$$2 + 2 * 2 = 8$$



Bir dildeki bu belirsizlikler
kaldırılmalıdır

$$E \rightarrow E + E \mid E * E \mid (E) \mid a$$

yerine

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow (E)$$

$$F \rightarrow a \quad \text{yazarız.}$$

$$\begin{aligned}
 E &\Rightarrow E + T \Rightarrow T + T \Rightarrow F + T \Rightarrow a + T \Rightarrow a + T * F \\
 &\Rightarrow a + F * F \Rightarrow a + a * F \Rightarrow a + a * a
 \end{aligned}$$

$$E \rightarrow E + T$$

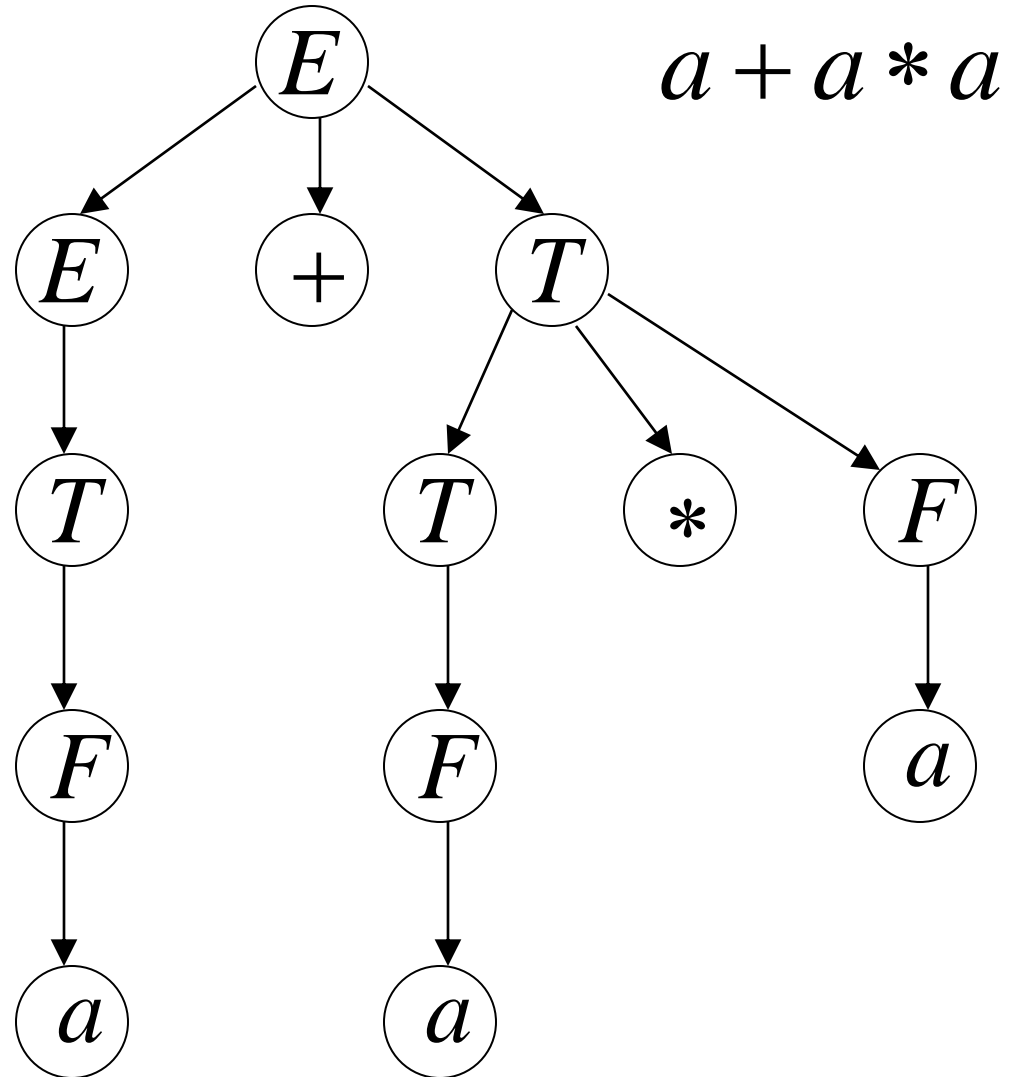
$$E \rightarrow T$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow (E)$$

$$F \rightarrow a$$



$$\mathbf{E \rightarrow E+E \mid E * E \mid E^{\wedge} E \mid id \mid (E)}$$

ÖNCELİKLER ve GRUPLANDIRMA

\wedge (right to left)

$*$ (left to right)

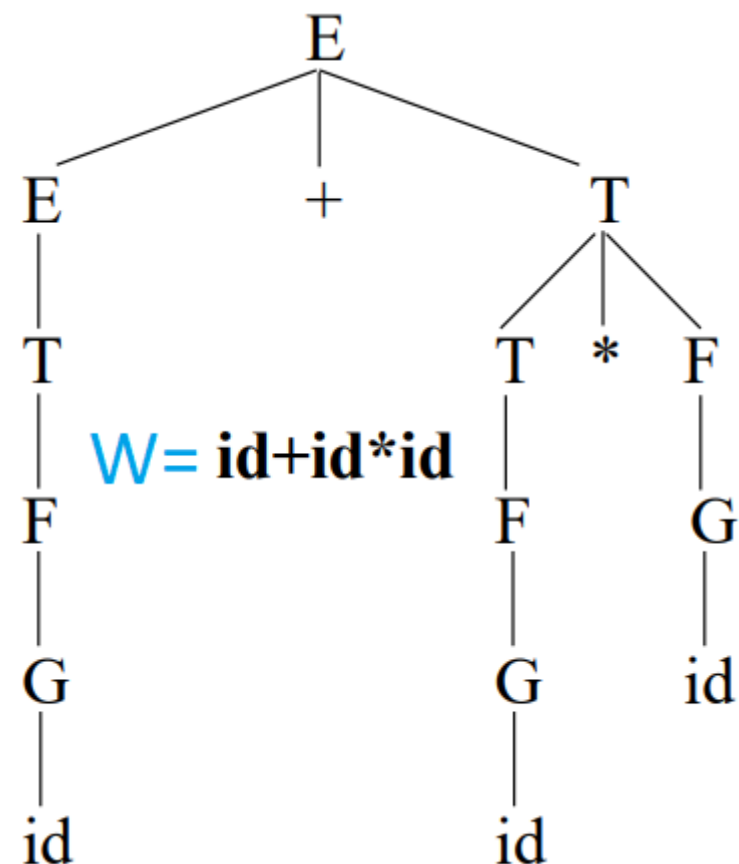
$+$ (left to right)

$$\mathbf{E \rightarrow E+T \mid T}$$

$$\mathbf{T \rightarrow T * F \mid F}$$

$$\mathbf{F \rightarrow G^{\wedge} F \mid G}$$

$$\mathbf{G \rightarrow id \mid (E)}$$



Inherently (doğası gereği) Belirsiz

$$L = \{a^n b^n c^m d^m : n \geq 1, m \geq 1\} \cup \{a^n b^m c^m d^n : n \geq 1, m \geq 1\}$$

$W = aabbccdd$.

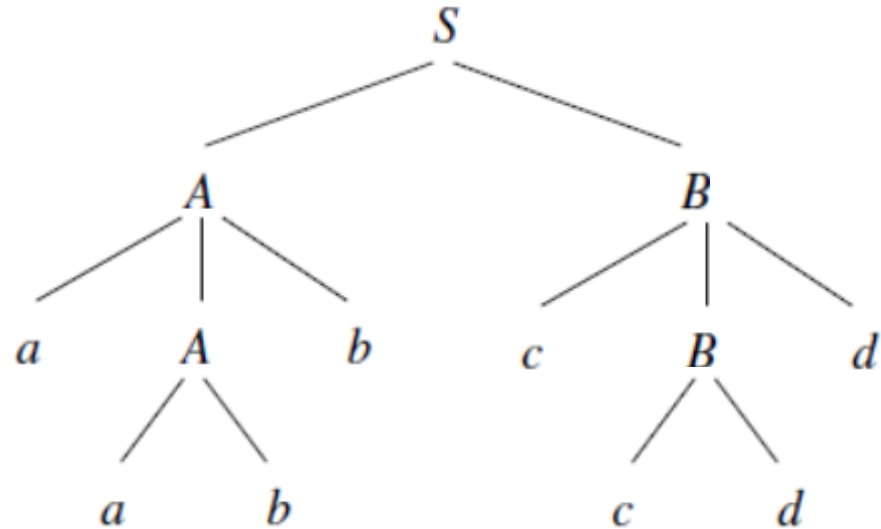
$$S \rightarrow AB \mid C$$

$$A \rightarrow aAb \mid ab$$

$$B \rightarrow cBd \mid cd$$

$$C \rightarrow aCd \mid aDd$$

$$D \rightarrow bDc \mid bc$$

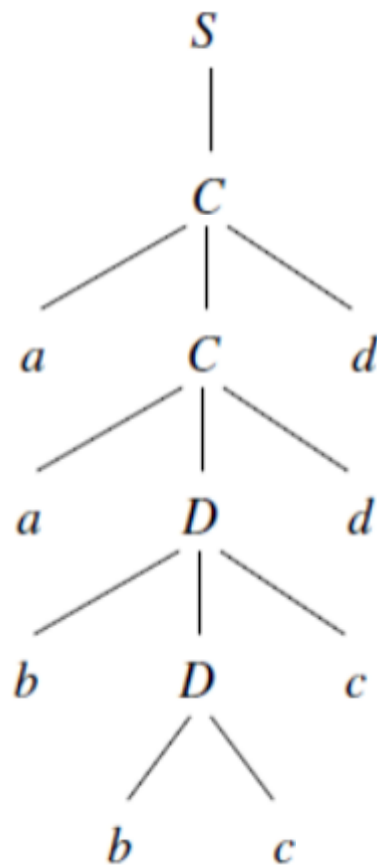


$$\begin{aligned} S &\Rightarrow_{lm} AB \Rightarrow_{lm} aAbB \Rightarrow_{lm} aabbB \\ &\Rightarrow_{lm} aabbcbBd \Rightarrow_{lm} aabbccdd \end{aligned}$$

Inherently (doğası gereği) belirsiz

$$L = \{a^n b^n c^m d^m : n \geq 1, m \geq 1\} \cup \{a^n b^m c^m d^n : n \geq 1, m \geq 1\}$$

W = aabbccdd.



$$S \rightarrow AB \mid C$$

$$A \rightarrow aAb \mid ab$$

$$B \rightarrow cBd \mid cd$$

$$C \rightarrow aCd \mid aDd$$

$$D \rightarrow bDc \mid bc$$

$$\begin{aligned} S &\Rightarrow_{lm} C \Rightarrow_{lm} aCd \Rightarrow_{lm} aaCdd \\ &\Rightarrow_{lm} aabDcdd \Rightarrow_{lm} aabbccdd \end{aligned}$$

Örnek: $\{0^n 1^m : n > m \geq 0\}$

$$S \rightarrow 0S1 \mid 0A$$

$$A \rightarrow \varepsilon \mid 0A$$

Bu kural eşit sayıda 0 ve 1 türetecek.
Fakat sıfırların sayısının 1'lerden fazla olması gerekiyor.

Şimdi 0 sayısı 1'den bir adet fazla oldu.

Hâla 0 sayısı 1'den bir adet fazla.

Şimdi 0 sayısı 1'den dilediğimiz sayıda fazla.

Örnek: Eşit sayıda 0 ve 1 içeren 0 ve 1 katarları.

$$S \rightarrow 0S1 \mid 1S0 \mid \varepsilon$$

Örnek:

$$\{0^n 1^n : n \geq 0\} \cup \{1^n 0^n : n \geq 0\}$$

$$S \rightarrow A \mid B$$

$$A \rightarrow 0A1 \mid \varepsilon$$

$$B \rightarrow 1B0 \mid \varepsilon$$

İki farklı
türetimle cevap
verildiği için
belirsizlik
vardır.

Bu gramerde belirsizlik var mıdır?

$W = \varepsilon$ bu dile ait midir? Evet bu dildedir

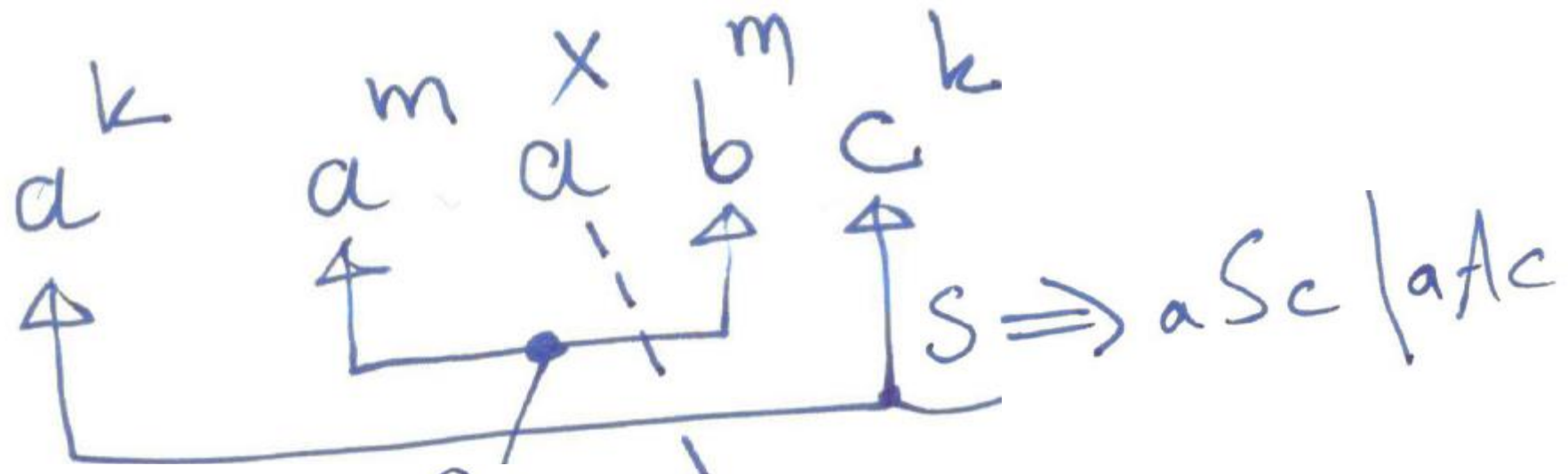
$$S \Rightarrow_{lm} A \Rightarrow_{lm} \varepsilon$$

$$S \Rightarrow_{lm} B \Rightarrow_{lm} \varepsilon$$

$$L = \{a^n b^m c^k \mid m > 0, k > 0, n > m+k\}$$

$$a^n b^m c^k \quad n = m+k \text{ o.s.g}$$

$$a^x a^k a^m b^m c^k \quad x > 0$$



$$A \Rightarrow a A b | a C b \quad C \Rightarrow a C | a$$

Her Regüler dil bir CFL'dir.

DFA'sı verilen belirli bir regüler dil için bir CFG oluşturalım

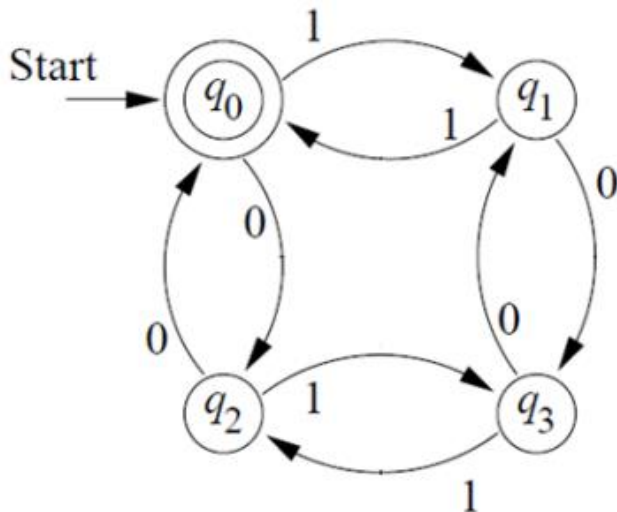
$$\text{DFA } M = (Q, \Sigma, \delta, q_0, F) \quad \mathbf{G} = (\mathbf{V}, \mathbf{T}, \mathbf{P}, \mathbf{S})$$

$$\mathbf{V} = \{ S_i \mid \text{Durumlar kümesi} \}$$

$$\mathbf{T} = \Sigma$$

$$\mathbf{P} = \{ S_i \rightarrow aS_j \mid \delta(q_i, a) = q_j \} \cup \{ S_i \rightarrow \varepsilon \mid q_i \in F \}$$

$$\mathbf{S} = S_0$$

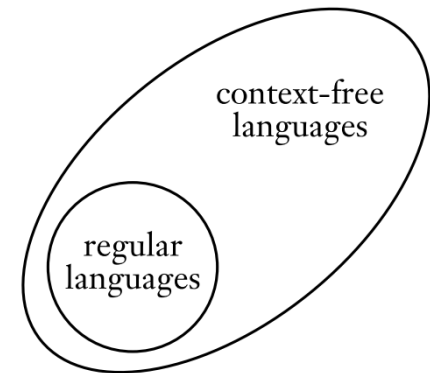


$$S_0 \rightarrow \varepsilon \mid 0S_2 \mid 1S_1$$

$$S_1 \rightarrow 0S_3 \mid 1S_0$$

$$S_2 \rightarrow 0S_0 \mid 1S_3$$

$$S_3 \rightarrow 0S_1 \mid 1S_2$$



Bir sonraki dersimiz: Gramerlerin
sadeleştirilmesi

Vize sınavında başarılar dilerim.