Problem 2: Analyze, compare and contrast bubble sort with Selection sort

Bubble sort:

```
bool swap;
do { swap = false;
     for(int i=0; i<n-1; i++) {
          if (a[i] > a[i+1]) {
               int temp = a[i];
               a[i] = a[i+1];
               a(i+1] = temp;
               swap = true;
   } while (swap);
```

let consider, O_b = operations before do-While loop

O_d = operations inside do-While Loop

O_i = operations inside for-loop

PO_s = operation inside if-statement

in this case, the outer do-While can require n-1 iteration (Worst-case); until no swap are made

So, we can write,

$$O_b + \sum_{k=0}^{n-2}\left(O_d + \sum_{i=0}^{n-2}(O_i + PO_s)\right); \qquad \sum_{j=x}^{Y} 1 = (Y-x)+1$$

let, $O_i + PO_s = O_{is}$

$$\sum_{i=0}^{n-2} O_{is} = ((n-2)-0+1)\,O_{is}$$

$$= (n-1)\,O_{is}$$

so, $O_b + \sum_{K=0}^{n-2} \left(O_d + (n-1) O_{is}\right)$

$= O_b + \left(((n-2) - 0 + 1) * \left(O_d + (n-1) O_{is}\right)\right)$

$= O_b + (n-1) * \left(O_d + (n-1) O_{is}\right)$

$= O_b + (n-1) O_d + (n-1)^2 O_{is}$

$= O_b + n O_d - O_d + n^2 O_{is} - 2n O_{is} + O_{is}$

$= n^2 O_{is} + (O_d - 2O_{is}) n + (O_b - O_d + O_{is})$

second order polynomial in n $\rightarrow$

$\boxed{c'' n^2 + c' n + c;}$ $c'' = O_{is}$, $c' = (O_d - 2O_{is})$

$c = (O_b - O_d + O_{is})$

it is $O(n^2)$

Algorithm Analysis (selection sort):

```
int indx, min;
for (int pos=0; pos<n-1; pos++) {
        min = a[pos];  indx = pos;
    for (int i = pos+1; i<n; i++) {
        if (a[i] < min) {
                min = a[i];
                indx = i;
        }
    }
      a[indx] = a[pos];
      a[pos] = min;
}
```

Let's consider,

O_b = operation before for loop

O_{pos} = Operation inside pos for loop

O_i = Operations inside i for loop

PO_s = operations inside if - condition

We can write from the above code:

$$O_b + \sum_{pos=0}^{n-2} \left(O_{pos} + \sum_{i=pos+1}^{n-1} (O_i + PO_s) \right) \quad \text{----} (1)$$

O, operation equates $T(O) \rightarrow$ clock cycle

We know, $\sum_{J=x}^{y} 1 = (y-x)+1$

Let,

$$O_i + PO_s = O_{is}$$

so, $$\sum_{i=pos+1}^{n-1} (O_i + PO_s)$$

$$= \left((n-1)-(pos+1)+1\right) O_{is}$$

$$= (n-pos-1)\, O_{is}$$

from equation (1)

$$O_b + \sum_{pos=0}^{n-2} \left(O_{pos} + (n-pos-1)\right) O_{is}$$

$$= O_b + \sum_{pos=0}^{n-2} \left(O_{pos} + (n-1)\, O_{is} - pos\, O_{is}\right)$$

$$= O_b + (n-1)\left(O_{pos} + (n-1)\, O_{is} - O_{is} \sum_{pos=0}^{n-2} pos\right.$$

$$= (n-1)^2 O_{is} + (n-1)\, O_{pos} + O_b - \frac{(n-2)(n+1)}{2} O_{is}$$

it is a second order polynomial in n

$$\left(O_{is} - \frac{O_{is}}{2}\right) n^2 + \left(-2O_{is} + O_{pos} + 1/2\, O_{is}\right) n$$

$$+ \left(2O_{is} - O_{pos} + O_b\right)$$

$$\boxed{c'' n^2 + c' n + c}$$

$c'' = O_{is}/2$, $c' = \left(O_{pos} - 3/2\, O_{is}\right)$

$c = \left(2O_{is} - O_{pos} + O_b\right)$

it is $O(n^2)$