Problem 5: Derive the order of the error with the respect to the sin and cosine approximations.

A) The Tylor series expansion for Sine,

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Find 1st neglected term for $\sin\left(\frac{1}{N}\right)$,

$$\sin\left(\frac{1}{N}\right) = \frac{1}{N} - \frac{\left(\frac{1}{N}\right)^3}{3!} + \frac{\left(\frac{1}{N}\right)^5}{5!} - \frac{\left(\frac{1}{N}\right)^7}{7!} + \cdots$$

1st neglected term is, $-\frac{\left(\frac{1}{N}\right)^3}{3!}$

the power of $\left(\frac{1}{N}\right)$ in first neglected term is 3

The order of the error for $\sin\left(\frac{1}{N}\right) = \frac{1}{N} + O\left(\frac{1}{N^3}\right)$

so, error $\sin\left(\frac{1}{N}\right) = O\left(\frac{1}{N^3}\right)$

B) The Tylor Series expansion for cosine,

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

find 1st neglected term for $\cos\left(\frac{1}{N}\right)$

$$\cos\left(\frac{1}{N}\right) = 1 - \frac{\left(\frac{1}{N}\right)^2}{2!} + \frac{\left(\frac{1}{N}\right)^4}{4!} - \frac{\left(\frac{1}{N}\right)^6}{6!} + \cdots$$

1st neglected term is, $\frac{\left(\frac{1}{N}\right)^4}{4!}$

the power of $\left(\frac{1}{N}\right)$ is 4

the order of the error for $\cos\left(\frac{1}{N}\right) = 1 - \frac{1}{2N^2} + O\left(\frac{1}{N^4}\right)$

Error $\cos(1/N) = O\left(\frac{1}{N^4}\right)$