# Problem 6/ Weighted Graph:



SFO — 338
1847
1465
LAX
2703
1234
DFW
803
ORD
868
743
1124
2341
189
BOS
JFK
1093
MIA — 1257
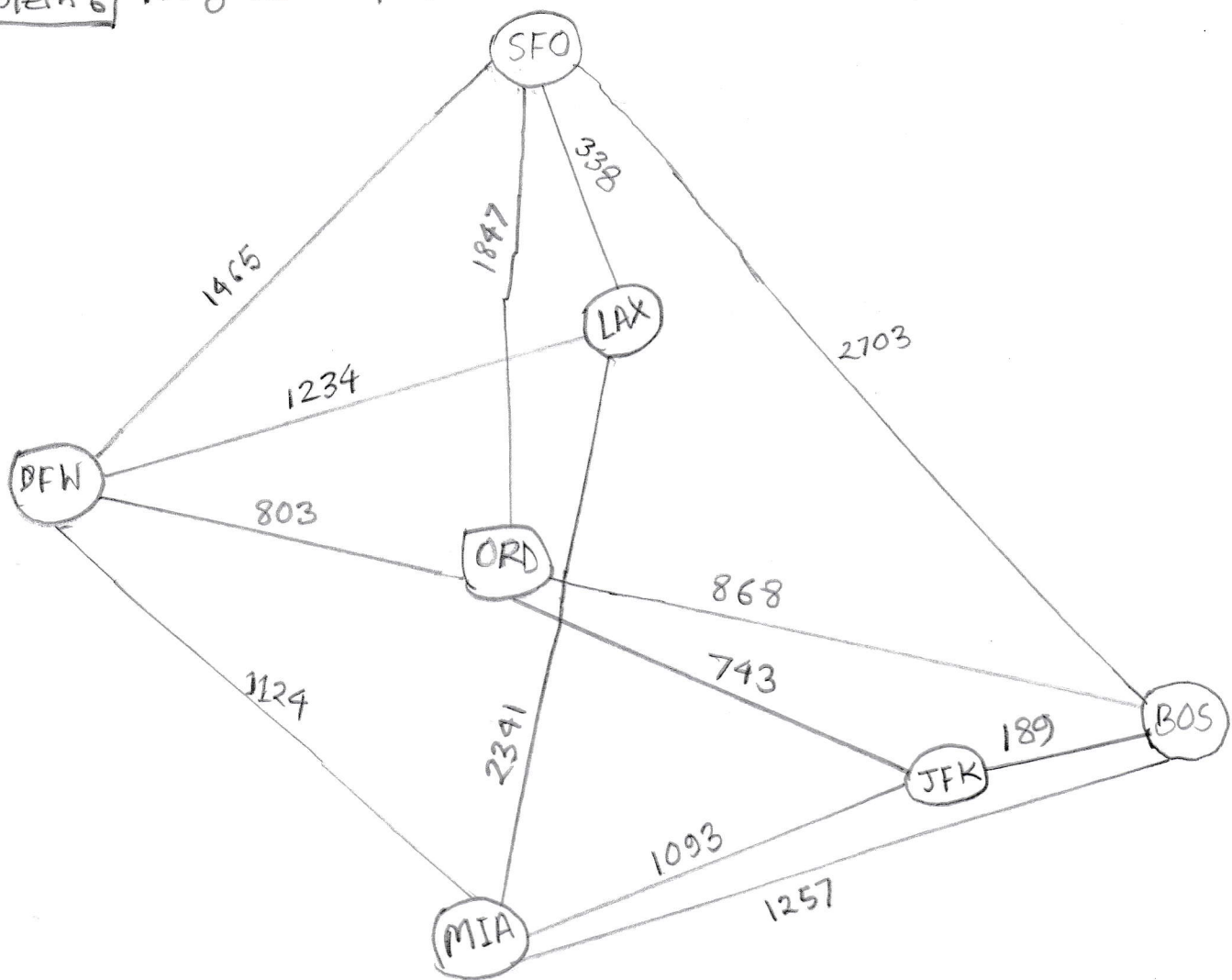
Let's analysis the graph to find the shortest distance between (ORD and LAX) and (JFK and SFO). Also, find the minimum spanning tree.

(a) Find the shortest distance between ORD and LAX

回 Start at ORD.
   * Permanent label: 0, Orders label: 1

回 Assign temporary label to neighbors of ORD:
   ORD → DFW = 802; ORD → BOS = 868; ORD → JFK = 743

回 Select small temporary label (JFK = 743) and make it permanent
   * Permanent label: 743; Order label 2

回 Assign temporary label to neighbors of JFK:
   * JFK → BOS: 743 + 189 = 932
   * JFK → MIA: 743 + 1093 = 1836
   ⇒ temporary labels: DFW: 803; BOS: 868 (< 932); MIA: 1836

回 Select the smallest temporary label (DFW = 803), make it permanent
   * Permanent label: 803; Order label 3

回 Assign temporary label to neighbors of DFW:
   * DFW → SFO: 803 + 1465 = 2268
   * DFW → LAX: 803 + 1234 = 2037
   * DFW → MIA: 803 + 1124 = 1927
   ⇒ Temporary labels:
      BOS (868); MIA (1836 → Replace by 1927); LAX (2037); SFO (2268)

回 Select the smallest temporary label (BOS = 868); make permanent
   * Permanent label: 868; Order label: 4

回 Assign temporary label to neighbors of BOS:
   * No update since all path through BOS are longer.

回 Select the smallest temporary label (LAX = 2037); make permanent
   * Permanent label: 2037; Order level: 5

回 Shortest path ORD → LAX is complete:
   Shortest Distance = | 2037 | miles

(b) Find the shortest distance between JFK and SFO

⊡ Start at JFK.
   * Permanent label: 0; Order label: 1

⊡ Assign temporary label to neighbors of JFK:
   * JFK→BOS = 189; JFK→ORD = 743; JFK→MIA = 1093
   ⇒ Temporary labels: BOS(189); ORD(743); MIA(1093)

⊡ Select the smallest temporary label (BOS=189) and make permanent
   * Permanent label: 189; order label: 2

⊡ Assign temporary label to neighbors of BOS:
   * No update as all path by BOS are longer.

⊡ Select smallest temporary label (ORD = 743)
   * Permanent label: 743; Order label: 3

⊡ Assign temporary label to neighbors of ORD:
   * ORD→DFW: 743 + 803 = 1546
   * ORD→SFO: 743 + 1847 = 2590
   ⇒ temp. label:
        MIA = 1093, DFW = 1546, SFO = 2590

⊡ Select the smallest temp. label (MIA = 1093)
   * Permanent label: 1093; order label: 4

⊡ Assign temp. label to neighbors of MIA:
   * No update since all path along MIA are longer

⊡ Select smallest temp. label (SFO = 2590)
   * Permanent label: 2590; order level: 5

⊡ JFK→ SFO shortest path is complete:
   Shortest Distance = 2590 miles

(c) Find the minimum spanning tree:

    Prim's Algorithm→   * select any vertex : LAX

                    * Select the shortest edge connected to
                      that vertex : LAX to SFO
                                Weight (338)

    * Select the shortest edge connected to any vertex
       already connected : LAX to DFW, Weight (1234)
               then,    DFW to ORD, Weight (803)
                      ORD to JFK, Weight (743)
                      JFK to BOS, Weight (189)
                      JFK to MIA, Weight (1093)

    * All vertices have been connected.
       The solution is → [338 + 1234 + 803 + 743 + 189 +
                        1093 ]
               = 4400 ←the total weight of MST

[Does the hand analysis agree with the program?]

Yes, the hand analysis result match the program
output, confirming the correctness of both methods.