

Problem 1 | Linear and binary search Analyze and compare

Linear search:

```
int linearSearch(int a[], int n, int val) {  
    for (int index = 0; index < n; index++) {  
        if (val == a[index])  
            return index;  
    }  
    return -1;  
}
```

Let's consider

O_b = operation before for loop

O_i = operation inside for loop

PO_s = operations inside the if-condition

$$O_b + \sum_{index=0}^{n-1} (O_i + PO_s)$$

O , operations which equates $T(O) \rightarrow$ clock cycles.

We know $\sum_{i=x}^y 1 = (y-x) + 1$

let $O_i + PO_s = O_{is}$

$$O_b + ((n-1) - 0 + 1) O_{is}$$

$$= O_b + n O_{is}$$

$f(n)$ is a first order polynomial

$$= \boxed{c'n + c} \quad \text{Where } c' = O_{is} \quad c = O_b$$

\rightarrow it is $O(n)$

Binary search

```
int lowEnd = 0;  
int highEnd = 0;  
do { int mid = (lowEnd + highEnd) / 2;  
    if (val == a[mid]) return mid  
    else if (val > a[mid]) lowEnd = mid + 1;  
    else highEnd = mid - 1;  
} while (lowEnd <= highEnd);
```

let's consider,
Operation before do-while loop = O_b
Operation inside do-while loop = O_d
operation inside if-statement = PO_s
operation inside else-if statement = PO_e
operation inside else statement = PO_l

since in binary-search algorithm, each step search space becomes half for total n -elements of a list

$n \rightarrow \frac{n}{2} \rightarrow \frac{n}{4} \rightarrow \frac{n}{8} \dots \frac{n}{2^i}$, we can write the series as $\frac{n}{2^0} \rightarrow \frac{n}{2^1} \rightarrow \frac{n}{2^2} \rightarrow \frac{n}{2^3} \rightarrow \dots \frac{n}{2^i}$

where i is the total number of steps or iterations and after i -steps the search space is reduced to 1

$$\frac{n}{2^i} = 1 \Rightarrow n = 2^i$$
$$\Rightarrow i = \log_2 n$$

$$O_{dse} = O_d + PO_s + PO_e + PO_l$$

So, for binary search, we can write

$$O_b + O_{dse} \log_2 n \rightarrow \text{operations, } O \text{ which equates}$$
$$T(O) \rightarrow \text{clock cycles}$$
$$= \boxed{c' \log_2 n + c}; \quad c' = O_{dse}, \quad c = O_b$$

it is $O(\log n)$