

Problem 7 || : Given 4 cards with 13 possible face values, calculate the probability of 1 pair, 2 pair, 3 of a kind and 4 of a kind.

4 cards are dealt from deck of 52 cards

We know, $nCr, \binom{n}{r} = \frac{n!}{(n-r)!r!}$

total number of possible 4 card hands,

$$\binom{52}{4} = \frac{52!}{(52-4)!4!} = \frac{52 \cdot 51 \cdot 50 \cdot 49}{4 \cdot 3 \cdot 2 \cdot 1} = 270725$$

Probability of One pair :

$$\binom{13}{1} = \frac{13!}{(13-1)!1!} = \frac{13!}{12!1!} = 13 \text{ [choose 1 rank for the pair]}$$

$$\binom{4}{2} = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = 6 \text{ [choose 2 suits]}$$

$$\binom{12}{2} = \frac{12!}{(12-2)!2!} = \frac{12!}{10!2!} = 66 \text{ [choose 2 other rank of remaining card]}$$

$$\binom{4}{1} \cdot \binom{4}{1} = 4 \cdot 4 = 16 \text{ [choose 1 suits for the each of remaining card]}$$

So, number of ways to get 1 pair, $13 \times 6 \times 66 \times 16 = 82368$

$$\therefore \text{Probability of 1 pair} = \frac{82368}{270725} \approx \boxed{0.304}$$

Probabilities of 2 pair :

$$\binom{13}{2} = \frac{13!}{(13-2)!2!} = \frac{13!}{11!2!} = 78 \text{ [choose 2 rank]}$$

$$\binom{4}{2} \cdot \binom{4}{2} = 6 \times 6 = 36 \text{ [choose 2 suit for each of pair]}$$

$$\binom{11}{1} = \frac{11!}{(11-1)!1!} = 11 \text{ [choose 1 other rank for remaining card]}$$

$$\binom{4}{1} = 4 \text{ [choose 1 suit for the remaining card]}$$

number of way to get 2 pair, $78 \times 36 \times 11 \times 4 = 123552$

$$\therefore P(2 \text{ pair}) = \frac{123552}{270725} \approx \boxed{0.456}$$

Mistake

For 3 of a kind:

$$\binom{13}{1} = \frac{13!}{(13-1)!1!} = 13 \text{ [choose 1 rank]}$$

$$\binom{4}{3} = \frac{4!}{(4-3)!3!} = \frac{4!}{1!3!} = 4 \text{ [choose 3 suits for the 3 card]}$$

$$\binom{12}{1} = 12 \text{ [choose 1 other rank for the remaining card]}$$

$$\binom{4}{1} = 4 \text{ [choose 1 suit for " card]}$$

number of ways get 3 of a kind, $13 \times 4 \times 12 \times 4 = 2496$

$$\therefore P(3 \text{ of a kind}) = \frac{2496}{270725} \approx \boxed{0.0092}$$

For 4 of a kind:

$${}^{13}C_1 = \binom{13}{1} = 13 \text{ [choose 1 rank for all four cards]}$$

$${}^4C_4 = \binom{4}{4} = 1 \text{ [choose all 4 suits]}$$

number of ways to get 4 of a kind, $13 \times 1 = 13$

$$\therefore \text{Probability of 4 of a kind} = \frac{13}{270725} \approx \boxed{0.000048}$$

$$P(4 \text{ of a kind}) \approx 0.000048$$

Probability of 2 pair:

$${}^{13}C_2 = \binom{13}{2} = \frac{13!}{(13-2)!2!} = \frac{13!}{11!2!} = 78 \text{ [choose 2 rank for pair]}$$

$$\binom{4}{2} \cdot \binom{4}{2} = \frac{4!}{(4-2)!2!} \cdot \frac{4!}{(4-2)!2!} = 6 \cdot 6 = 36 \text{ [choose 2 suit for each pair]}$$

Number of ways to get 2 pair, $78 \times 36 = 2808$

$$\therefore \text{Probability of 2 pair} = \frac{2808}{270725} \approx \boxed{0.0104}$$