Problem 1 (Hashing): Analysis document:

The distribution approximation to:

$$P(x_i = j) = e^{-1}/j!$$

calculate the expected number of cells with j elements for each j using:

Expected cells with $j = P(x=j) \times$ Table size.

Here, table size $= 512$

If $j=0$; $P(x=0) = e^{-1}/0! = 0.3679$, so $512 \times 0.3679 \approx 188$ cells with 0 elements.

If $j=1$; $P(x=1) = e^{-1}/1! = 0.3679$

So, $512 \times 0.3679 \approx 188$ cells with 1 element

If $j=2$; $P(x=2) = e^{-1}/2! = 0.1839$

So, $512 \times 0.1839 \approx 94$ cells with 2 elements

if $j=3$; $P(x=3) = e^{-1}/3! = 0.0613$

So, $512 \times 0.0613 \approx 31$ cells with 3 elements

if $j=4$; $P(x=4) = e^{-1}/4! = 0.0153$

So, $512 \times 0.0153 \approx 8$ cells with 4 elements

if $j=5$; $P(x=5) = e^{-1}/5! = 0.0031$

So, $512 \times 0.0031 \approx 2$ cells with 5 elements

if $j=6$; $P(x=6) = e^{-1}/6! = 0.0005$,

So, $512 \times 0.0005 \approx 0$ cells with 6 elements

Here is the output of the Hashing program:

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Output - Problem1_Hashing (Run)
    Hash Table Statistics:
    Buckets with 0 elements: 232
    Buckets with 1 elements: 147
    Buckets with 2 elements: 71
    Buckets with 3 elements: 35
    Buckets with 4 elements: 19
    Buckets with 5 elements: 6
    Buckets with 6 elements: 2
    Maximum collisions in a single bucket: 6

    Expected distribution (Poisson approximation):
    Buckets with 0 elements (expected): 188.354
    Buckets with 1 elements (expected): 188.354
    Buckets with 2 elements (expected): 94.1771
    Buckets with 3 elements (expected): 31.3924
    Buckets with 4 elements (expected): 7.84809
    Buckets with 5 elements (expected): 1.56962
    Buckets with 6 elements (expected): 0.261603

    RUN SUCCESSFUL (total time: 224ms)
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Max Value and Distribution (Comparison Table):

Bucket Size (Elements)	Expected (Poisson)	Observed (Simulation)
0 (Empty Buckets)	188.35	232
1 (One Element)	188.35	147
2 (Two Elements)	94.18	71
3 (Three Elements)	31.39	35
4 (Four Elements)	7.85	19
5 (Five Elements)	1.57	6
6 (Six Elements)	0.26	2

So, expected number of buckets with 0 elements was 188.35, but the simulation observed approximately 232, which is higher. Also, the number of buckets with 1 element was close to 147, while the theory predicted 188.35, these differences are due to random initialization and fall within acceptable statistical variation. Besides, buckets with 5+ elements were rare, matching theoretical expectations. The maximum expected bucket size was ~6, and only 2 buckets contained 6 elements, nearly matching the rare probability predicted by the Poisson model (0.26 buckets expected).