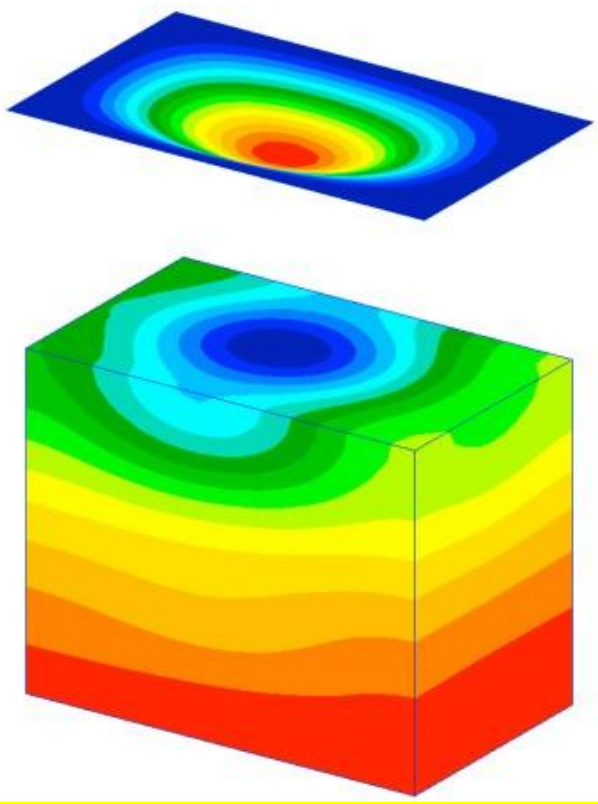

ACOUSTIC AND ELASTOACOUSTIC PROBLEMS INTERACTION FLUII-STRUCTURE

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I. INTRODUCTION

The study aims to compute acoustic natural frequencies and pressure levels in a rigid cavity and investigate the fluid-structure interaction (FSI) problem. It involves coupling an acoustic cavity with a deformable elastic plate using Patran/Nastran finite element software. Additionally, the effect of replacing air with water as the fluid is analysed, and the accuracy of finite element results compared to exact solutions is evaluated.

II. OBJECTIVES

- Calculate the natural acoustic frequencies of a rigid cavity and compare finite element (FE) results to exact solutions.
- Determine the natural frequencies and deformations of a clamped elastic plate.
- Analyse the coupled fluid-structure interaction problem (air and aluminium).
- Investigate the influence of fluid density by replacing air with water in the acoustic cavity.
- Assess frequency error for rigid cavity modes by comparing FE and exact solutions.

III. EXERCICE 1

To calculate the first ten acoustic natural frequencies of a rigid acoustic cavity, begin by creating the geometry of the cavity, ensuring its dimensions are 0.6 m × 0.5 m × 0.4 m. Next, mesh the cavity using hexahedral elements (HEX8) with a density of 10 elements per edge, resulting in a structured mesh that adequately captures the cavity's acoustic behaviour.

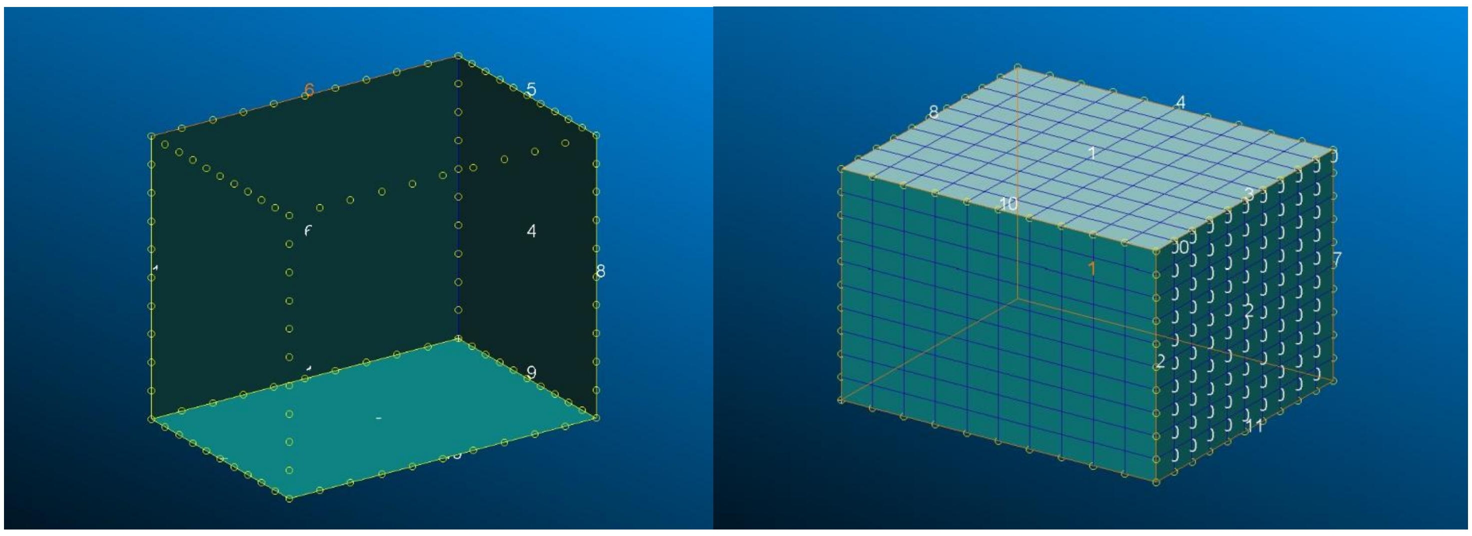
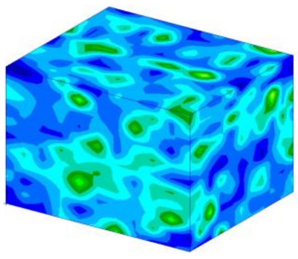
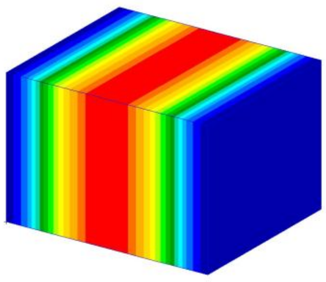
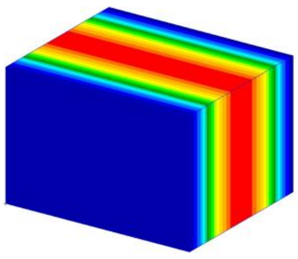
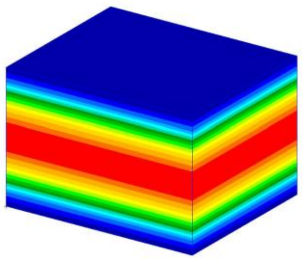
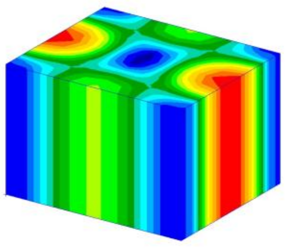
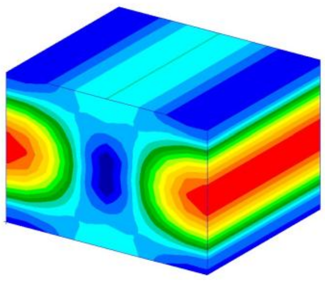
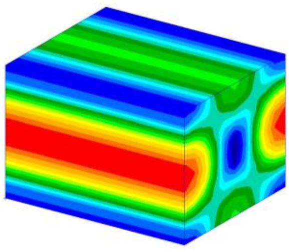
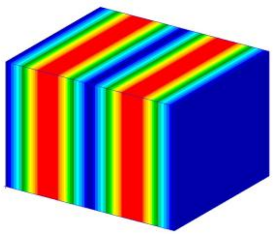
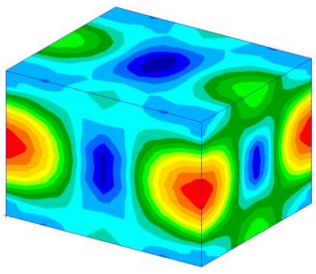


Fig1. Geometry and mesh of the Cavity

Proceed by defining the material properties for air, specifically setting its density ($\rho = 1.2kgm^{!"}$) and speed of sound (c = 340m/s). Assign these properties to the fluid domain to ensure the calculations accurately reflect the physical characteristics of the cavity.

Subsequently, adjust the modal analysis parameters by selecting "Normal Modes" under the Analysis/Solution Type menu to specify the type of calculation. Within the Analysis/Subcases menu, configure the output requests by selecting "Grid Point Force Balance" to extract the pressure distribution within the fluid domain. Once these parameters are set, initiate the calculation to compute the natural frequencies of the cavity.

Finally, display the room's pressure level corresponding to each natural frequency by using the visualization tools in the post-processing menu. This sequential process ensures the accurate determination of the acoustic natural frequencies and the associated pressure modes within the rigid cavity.

Frequencies Hertz	Pressures in the Fluid (Same View Plan)
SC1:DEFAULT, A1:Mode 1: Freq.=5.1873e-06	
SC1:DEFAULT, A1:Mode 2: Freq.=284.5	
SC1:DEFAULT, A1:Mode 3: Freq.=341.4	
SC1:DEFAULT, A1:Mode 4: Freq.=426.75	
SC1:DEFAULT, A1:Mode 5: Freq.=444.403	
SC1:DEFAULT, A1:Mode 6: Freq.=512.889	
SC1:DEFAULT, A1:Mode 7: Freq.=546.506	
SC1:DEFAULT, A1:Mode 8: Freq.=576.028	
SC1:DEFAULT, A1:Mode 9: Freq.=616.125	

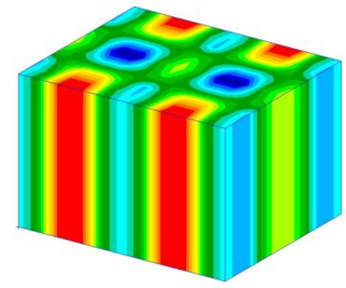
SC1:DEFAULT, A1:Mode 10: Freq.=669.599	

Fig2. Table of frequencies and illustrations of the pressure level in the cavity

On the other hand , using MATLAB we were able to calculate the 10 modes of frequencies using the formula below

$$f_{l,m,n} = 0.5c\sqrt{\frac{l}{X} + \frac{m}{Y} + \frac{n}{Z}}$$

Where c is the speed of sound in air, l,m,n are integers which varies from 1 to 10 and X,Y and Z are the dimensions of the cavity given above .

Mode Number	Analytical Frequency (Hz)	F.E. Frequency (Hz, Patran)	Error (%)
1	0.0000	5.1873×10^{-6}	Négligeable
2	283.3	284.5	0.42
3	340.0	341.4	0.41
4	425.0	426.75	0.41
5	442.6	444.403	0.41
6	510.8	512.889	0.41
7	544.3	546.506	0.40
8	566.7	576.028	1.65
9	613.6	616.125	0.41
10	660.8	669.599	1.33

Fig3. Comparison between the exact and F.E frequency

Both the analytical and finite element methods confirm that the first mode corresponds to a negligible frequency, consistent with the theoretical expectation of a uniform pressure distribution without oscillation.

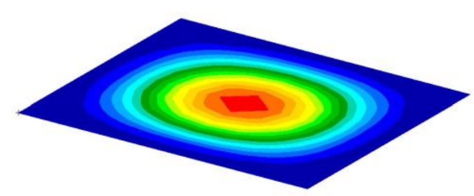
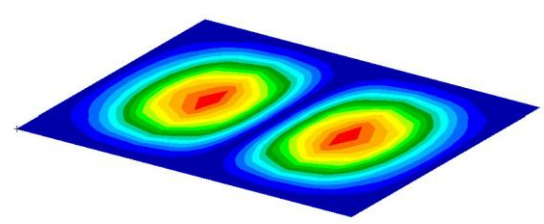
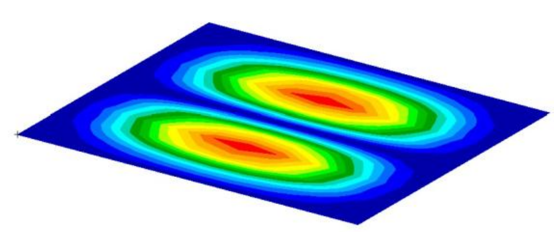
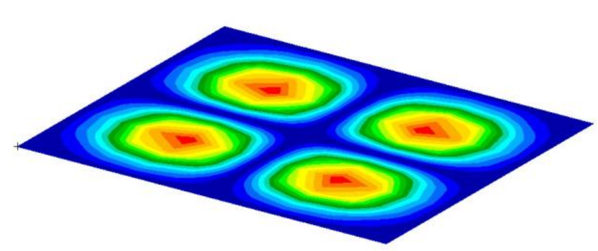
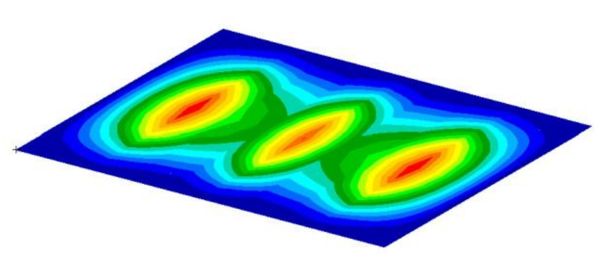
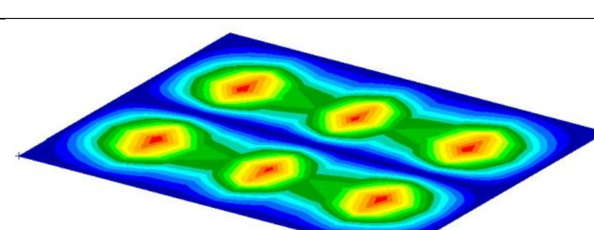
The error between the analytical and finite element frequencies is minimal, with deviations remaining below 2% for most modes, demonstrating the accuracy of the finite element approach. Slight variations observed in the higher modes, such as Mode 8 and Mode 10, may be attributed to discretization effects or numerical approximations inherent in finite element methods.

IV. EXERCICE 2

To calculate the first 10 natural frequencies of the clamped aluminium elastic plate, the procedure begins by defining the material properties: Young's Modulus (E = 70GPa), Poisson's Ratio (v = 0.33), and density (($\rho = 2700kgm^{-3}$).

The geometry of the plate is created with dimensions of 0.6m in length, 0.5m in width, and a thickness of 2m. All edges of the plate are clamped, applying boundary conditions that enforce zero displacements. The plate is then meshed using quadrilateral shell elements with a uniform mesh density to accurately capture the deformation.

A "Normal Modes" analysis is selected in the solver setup, specifying the extraction of the first 10 modes. The material properties are assigned to the model, and the simulation is executed to compute the natural frequencies and corresponding mode shapes. Postprocessing involves extracting and visualizing the first 10 natural frequencies and mode shapes to verify the results against boundary conditions.

Frequencies Hertz	Eigenmodes translational
SC1:DEFAULT, A1:Mode 1: Freq.=59.9438	
SC1:DEFAULT, A1:Mode 2: Freq.=107.829	
SC1:DEFAULT, A1:Mode 3: Freq.=135.088	
SC1:DEFAULT, A1:Mode 4: Freq.=175.349	
SC1:DEFAULT, A1:Mode 5: Freq.=187.027	
SC1:DEFAULT, A1:Mode 6: Freq.=246.426	

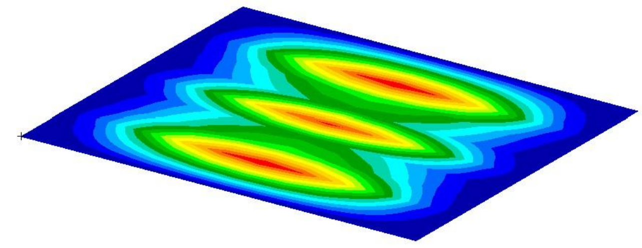
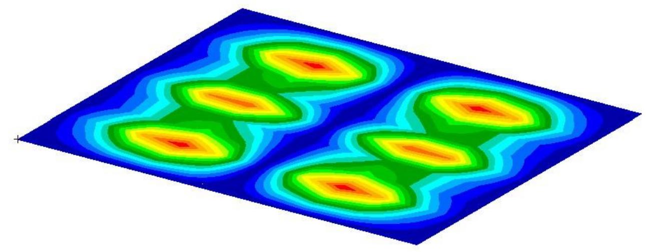
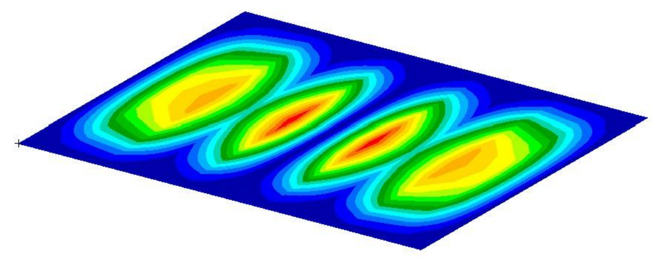
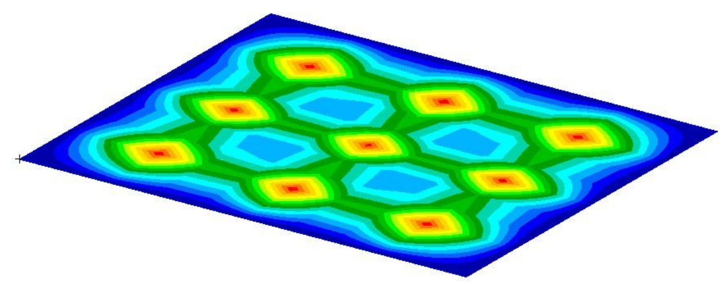
SC1:DEFAULT, A1:Mode 7: Freq.=251.523	
SC1:DEFAULT, A1:Mode 8: Freq.=286.776	
SC1:DEFAULT, A1:Mode 9: Freq.=296.832	
SC1:DEFAULT, A1:Mode 10: Freq.=346.571	

Fig4. Table of natural frequencies and illustrations of the plate

V. EXERCICE 3

To analyse the fluid-structure interaction between the acoustic cavity and the clamped aluminium plate, the procedure begins with creating the geometry of both the acoustic cavity and the plate in the same Patran file.

The acoustic cavity is meshed with hexahedral elements (Hex-Isomesh-Hex8) using a density of 10x10x10, resulting in 1000 elements and 1331 nodes. The aluminium plate is then meshed with quadrilateral elements (QUAD4) using a compatible mesh at the fluidstructure interface, ensuring 100 elements and 121 nodes.

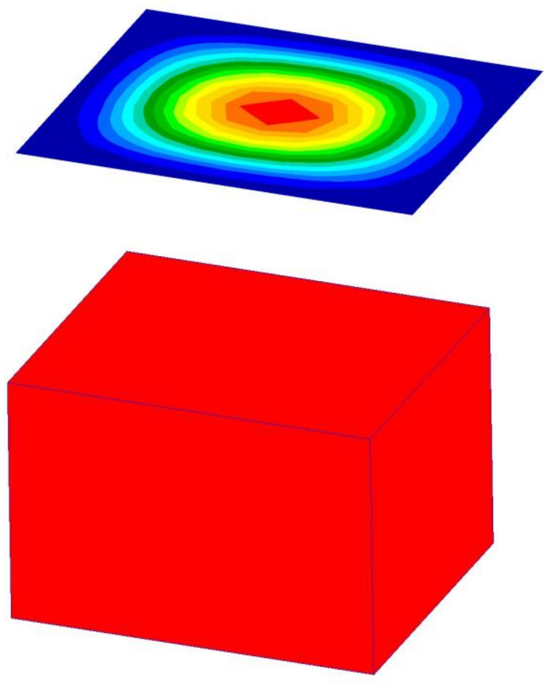
Two groups are created: the first group, named "cavity," includes the elements of the acoustic cavity, and the second group, named "plate," contains the elements of the aluminium plate. Clamped boundary conditions are applied to the plate by isolating the plate group, selecting all edge nodes, and assigning the constraints.

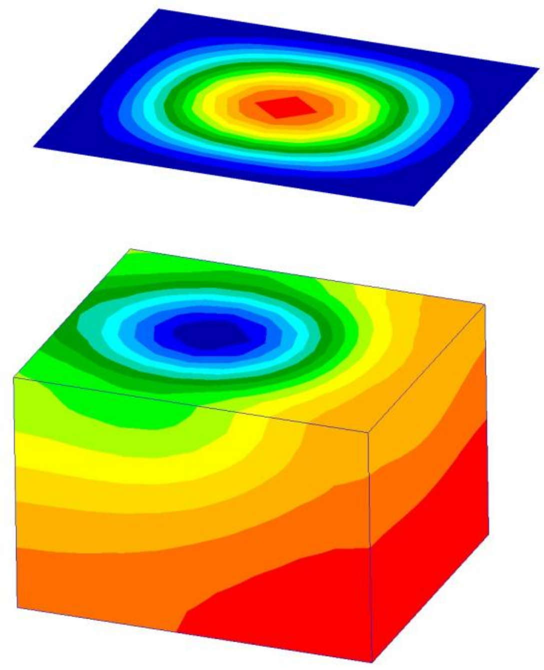
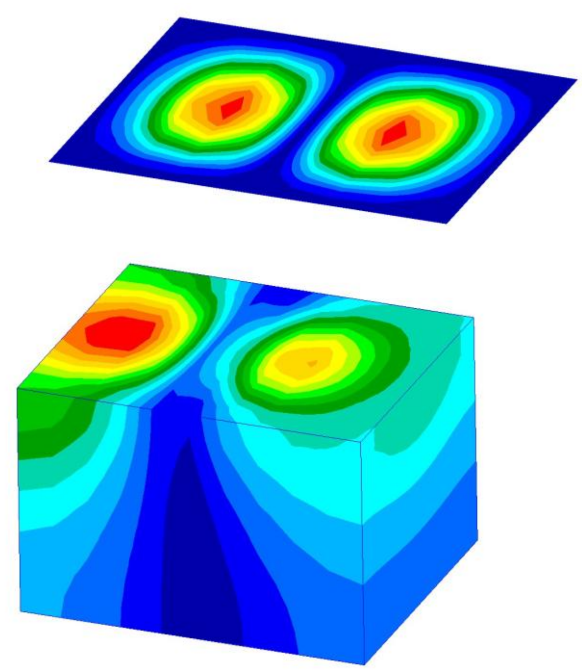
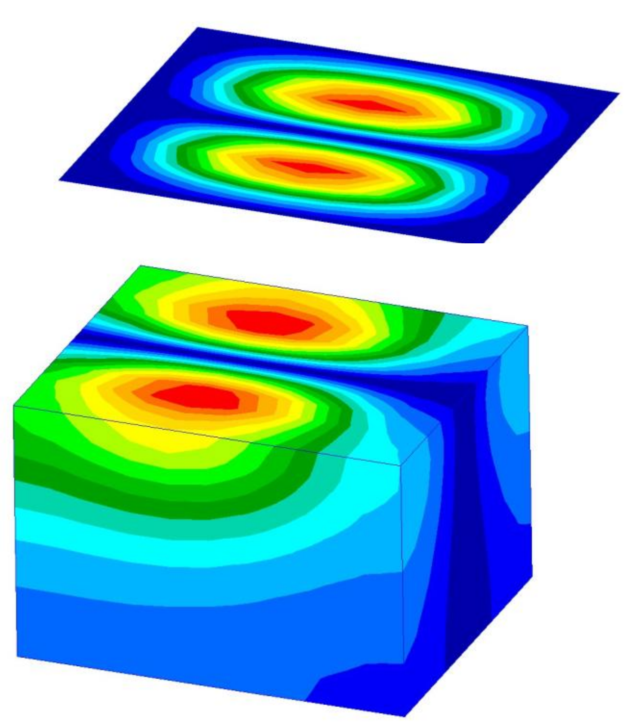
The materials for both the cavity (air) and the plate (aluminium) are defined, and their respective properties are applied. The analysis is set up by selecting "COMPLEX EIGENVALUE" with the "direct formulation" as the solution type.

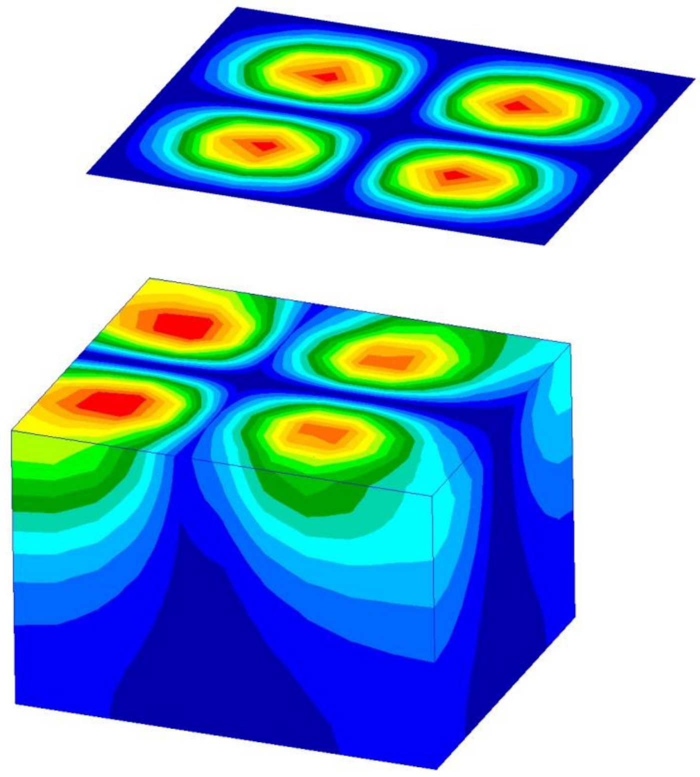
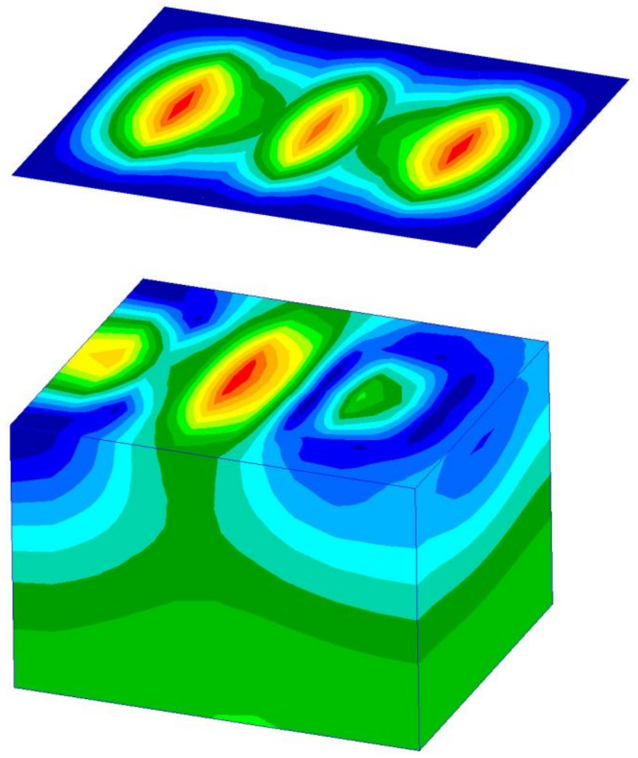
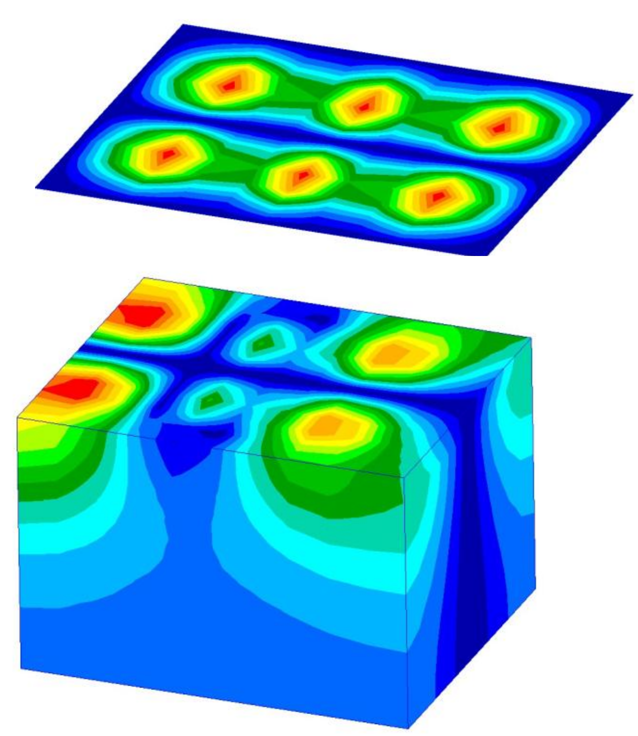
Post-processing involves visualizing the pressure levels in the acoustic cavity by selecting the cavity group and using QuickPlot to display "Eigenvectors, Translational" for the “X Component”.

Similarly, the deformation of the plate is displayed by selecting the plate group and visualizing the translational eigenvectors.

The results are then analysed to study the correspondence between the pressure levels in the cavity and the deformation of the plate, illustrating the nature of the fluid-structure interaction.

Frequencies Hertz	Eigenmodes of the coupled Problem
SC1:DEFAULT, A1:Mode 1: Freq.=0.	

SC1:DEFAULT, A1:Mode 2: Freq.=65.2271	
SC1:DEFAULT, A1:Mode 3: Freq.=106.538	
SC1:DEFAULT, A1:Mode 4: Freq.=133.64	

SC1:DEFAULT, A1:Mode 5: Freq.=174.067	
SC1:DEFAULT, A1:Mode 6: Freq.=186.135	
SC1:DEFAULT, A1:Mode 7: Freq.=245.097	

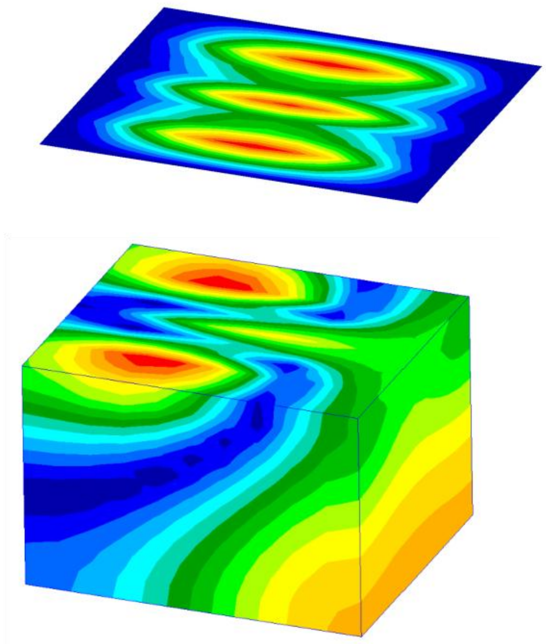
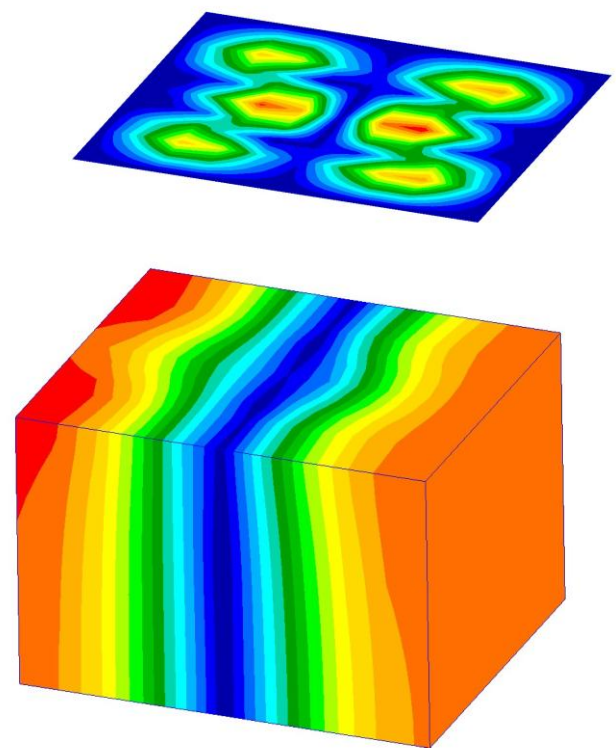
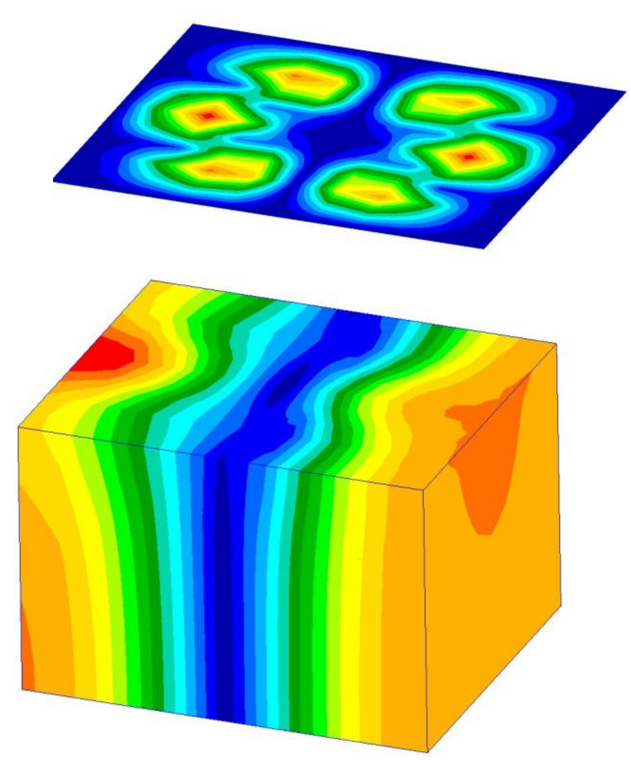
SC1:DEFAULT, A1:Mode 8: Freq.=250.132	
SC1:DEFAULT, A1:Mode 9: Freq.=280.214	
SC1:DEFAULT, A1:Mode 10: Freq.=288.227	

Fig5. Table of frequencies and illustrations of the pressure level in the cavity

Exact rigid cavity		F.E. rigid cavity		Plate only		Fluid-structure coupled problem	
Frequency	Deformed	Frequency	Deformed	Frequency	Deformed	Frequency	Deformed
0.0000		5E-6		59.9438		0	
283.3		284.5		107.829		65.2271	
340.0		341.4		135.088		106.538	
425.0		426.75		175.349		133.64	
442.6		444.403		187.027		174.067	
510.8		512.889		246.426		186.135	
544.3		546.506		251.523		245.097	
566.7		576.028		286.776		250.132	
613.6		616.125		296.832		280.214	
660.8		669.599		346.571		288.227	

Fig6. Comparison of Frequency and Deformation Results for Different Structural Models

The influence of fluid-structure coupling on the specific frequencies of the structure can be observed by comparing the frequencies of the system for three configurations: the cavity alone, the plate alone, and the coupled system of the plate and cavity.

The frequencies for the cavity alone are significantly lower than those for the plate alone, with the cavity frequencies starting from a very low value and gradually increasing. In contrast, the frequencies for the plate alone are higher and increase more steadily.

When fluid-structure coupling is introduced, the frequencies for the coupled system shift, particularly for the lower modes. For example, the first mode of the coupled system is effectively zero, indicating that the fluid-structure interaction dominates, potentially creating a degenerate mode or very low frequency.

For the second mode, the frequency of the coupled system is closer to the plate's frequency, suggesting that the fluid's influence is still present but not as strong. As the modes increase, the effect of the fluid becomes more apparent, leading to a reduction in frequency compared to the plate alone. For instance, in higher modes such as Mode 10, the frequency for the coupled system is lower than that of the plate alone, reflecting the dampening effect of the fluid.

Furthermore, to explore the effect of different fluids on the coupled system, the air in the cavity was replaced with water, which has a higher density ($\rho = 1000\ kg/m^3$) and a lower speed of sound ($c = 1500\ m/s$).

Mode Number	F.E. Frequency (Hz, Water)	F.E. Frequency (Hz, Air)
1	0	0
2	30.9908	65.2271
3	42.5875	106.538
4	76.5541	133.64
5	106.552	174.067
6	173.362	186.135
7	233.38	245.097
8	251.23	250.132
9	290.369	280.214
10	332.489	288.227

Fig7. Comparison of Frequency the cavity filled with air and water

The results demonstrate the significant influence of fluid density on the natural frequencies of the fluid-structure coupled system.

When air in the cavity is replaced with water, the natural frequencies decrease across most modes due to the higher density of water, which adds more mass to the system.

In the lower modes (Modes 2–6), the reduction is particularly pronounced; for example, the frequency of Mode 2 drops from 65.23 Hz (air) to 30.99 Hz (water), a decrease of approximately 52%.

This is because lower modes involve more global deformations, where the added mass from the fluid has a greater effect.

In contrast, the higher modes (Modes 7–10) are less affected by the fluid change, as the structural deformations are more localized and the interaction with the fluid is weaker. For instance, Mode 7 shifts slightly from 245.10 Hz (air) to 233.38 Hz (water), a reduction of about 5%. These results highlight that fluid-structure coupling is strongest in the lower modes and diminishes in higher modes.

The denser water amplifies the coupling effect compared to air, particularly in the lower frequencies, emphasizing the importance of fluid properties in dynamic analyses of coupled systems.

In addition to the above, the graph below illustrates the accuracy of the finite element (F.E.) method for a rigid acoustic cavity filled with air compared to the evolution of the frequency error for the first modes.

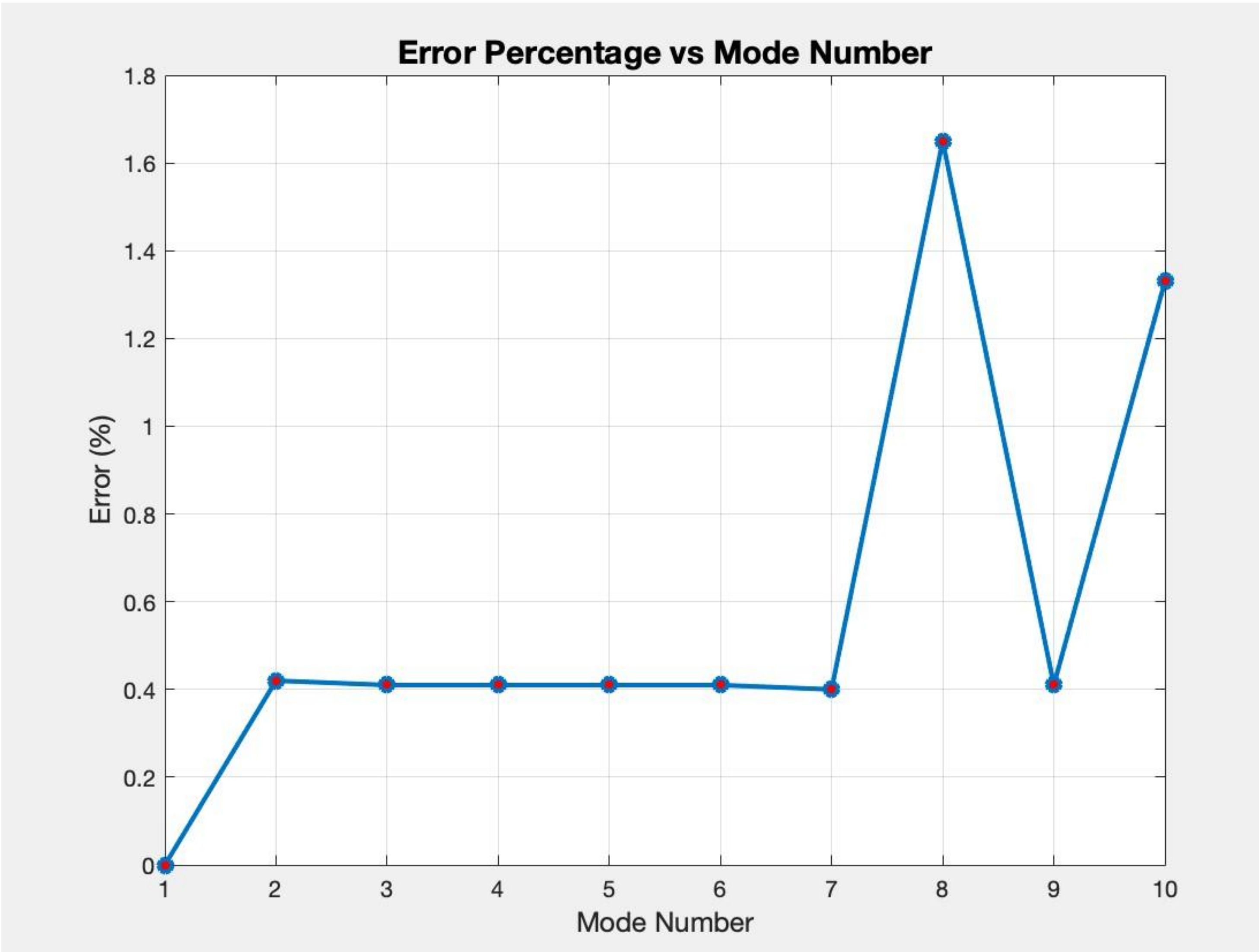


Fig8. Graph of Frequency error vs Normal modes

VI. CONCLUSION

In conclusion, the analysis of the fluid-structure coupling in the rigid acoustic cavity system has highlighted the significant impact of fluid properties on the natural frequencies of the coupled system. Both the analytical and finite element methods provided consistent results, with minimal error (below 2%) for most modes, validating the accuracy of the finite element approach. The introduction of fluid-structure coupling caused shifts in the frequencies, particularly for the lower modes, where the fluid's mass and interaction with the structure had a more pronounced effect. The first mode, corresponding to a negligible frequency, demonstrated the expected behavior of uniform pressure distribution without oscillation. As the modes increased, the influence of the fluid became less significant, with higher modes showing only slight frequency reductions.

Furthermore, replacing air with water in the cavity illustrated the role of fluid density in altering the system's dynamic behavior. The higher density of water increased the system's mass, resulting in a more substantial reduction in frequency for the lower modes. This effect was particularly evident in Modes 2–6, where global deformations were more affected by the added mass of the fluid. In contrast, higher modes (7–10) showed smaller shifts, as the structural deformations were more localized and less influenced by the fluid. These results underscore the importance of considering fluid-structure coupling, particularly in systems where fluid properties significantly affect the structural dynamics, such as in acoustic cavities and similar coupled systems.

VII. TABLE OF FIGURES

VIII. APPENDIX

Matlab code 1(analytical solution for the modes of frequencies)

```
% Constantes et dimensions de la cavité
c = 340; % Vitesse du son dans l'air en m/s (vous pouvez changer cela selon
votre cas)
X = 0.6; % Dimension de la cavité en x (en mètres)
Y = 0.5; % Dimension de la cavité en y (en mètres)
Z = 0.4; % Dimension de la cavité en z (en mètres)

% Plage de valeurs pour l, m, et n max_mode
= 10;

% Initialisation d'une liste pour stocker les fréquences
frequencies = [];

% Boucle pour calculer f_l,m,n pour chaque combinaison de l, m, n
for l = 0:max_mode     for m = 0:max_mode         for n =
0:max_mode
            % Éviter le cas où l, m, et n sont tous nuls
if l == 0 && m == 0 && n == 0                 continue;
end
            % Calcul de la fréquence
```

```
            f_lmn = (c/2) * sqrt((l/X)^2 + (m/Y)^2 + (n/Z)^2);
            % Ajouter la fréquence à la liste
frequencies = [frequencies, f_lmn];         end
end end

% Trier les fréquences dans l'ordre croissant
frequencies = sort(frequencies);

% Affichage des fréquences
disp('Fréquences de résonance (en Hz) :'); disp(frequencies);
```

Matlab code 2(Graph of modes number against error)

```
% Mode numbers (x-axis) mode_numbers
= 1:10;

% Error percentages (y-axis)
error_percentage = [0, 0.42, 0.41, 0.41, 0.41, 0.41, 0.40, 1.65, 0.41, 1.33];

% Create the plot figure;
plot(mode_numbers, error_percentage, '-o', 'LineWidth', 2, 'MarkerSize', 6,
'MarkerFaceColor', 'r');

% Labeling the axes
xlabel('Mode Number', 'FontSize', 12); ylabel('Error
(%)', 'FontSize', 12);

% Title of the plot
title('Error Percentage vs Mode Number', 'FontSize', 14);

% Display grid grid
on;
```