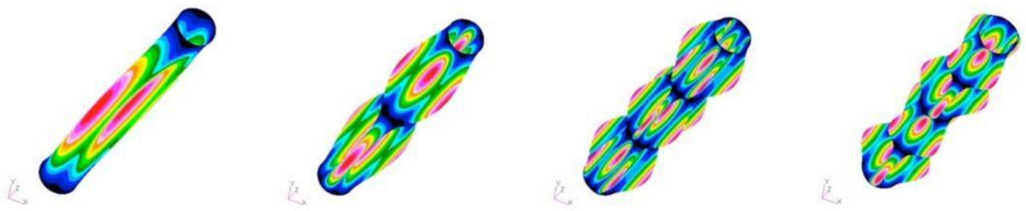


FEM MODELISATION WITH NASTRAN

EIGENMODES CALCULATION

Practical Work 2: Report

RÉSUMÉ

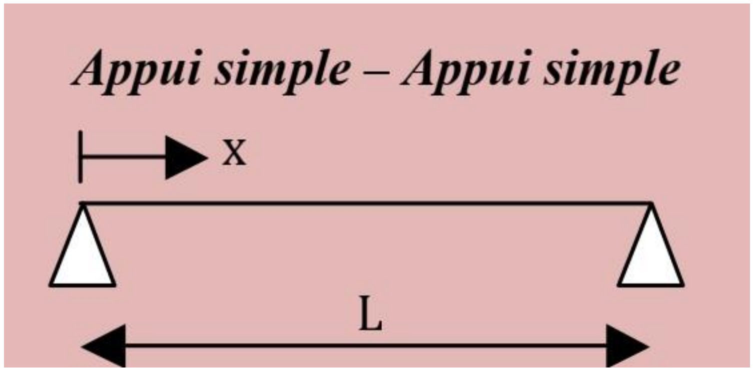
In this report, we will study the natural modes of various mechanical systems, namely a simply supported and clamped beam, a cylindrical shell, and a thick cylinder. The natural modes, also known as natural frequencies, are the characteristic vibrational configurations of a mechanical system. They are essential in the design and analysis of structures, as they help identify frequencies to avoid preventing resonance phenomena, which could lead to structural failures.

Khady sarah Sall,
Aero05

I. Eigenmodes of a Beam

1. Let’s calculate the first ten Eigenfrequencies of the beam

a) Simple support



The eigenfrequencies of the beam are given by the following general formula:

$$f_i = \frac{\lambda_i^2}{2\pi L^2}\left(\frac{EI}{m}\right)^{1/2}, \qquad i = 1,2,3,\ldots$$

$\boldsymbol{\lambda_i = i\pi}$

i	1	2	3	4	5	6	7	8	9	10
frequencies x**1.0e+04** en Hz	0.0235	0.0941	0.2118	0.3765	0.5882	0.8470	1.1529	1.5058	1.9058	2.3528

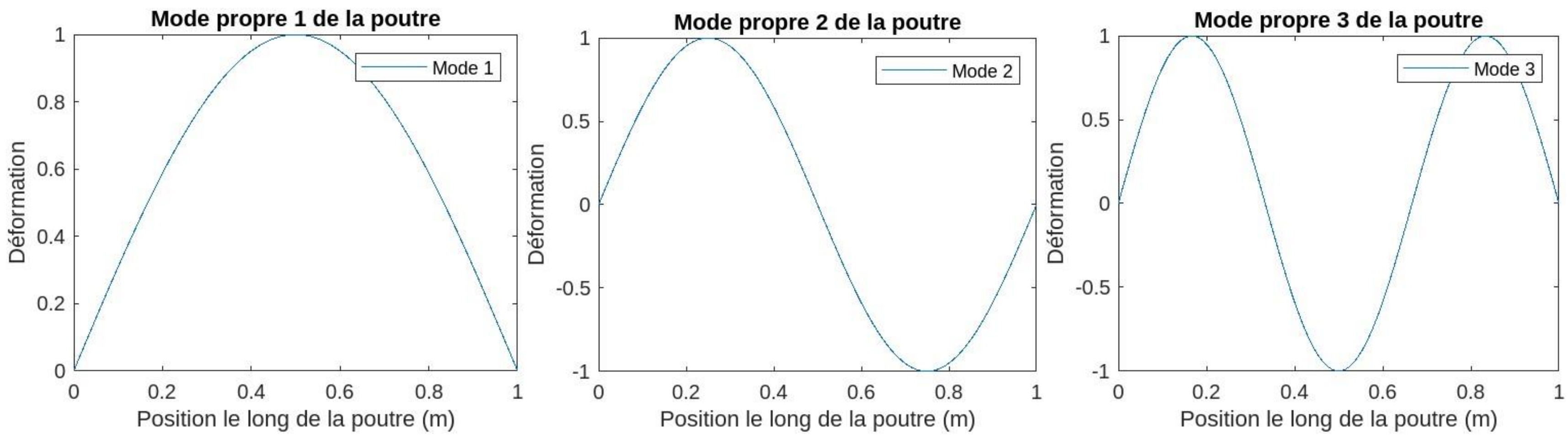
b) Fixed support

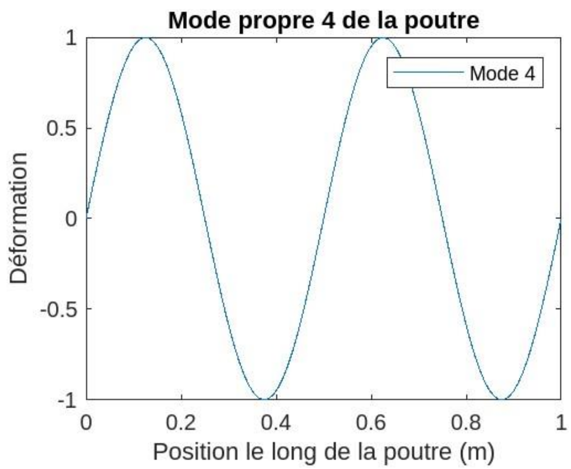
The first five λ_i are given: **4.73004074**, **7.85320462**, **10.9956079**, **14.1371655, 17.2787597** the others are determined using this formula:

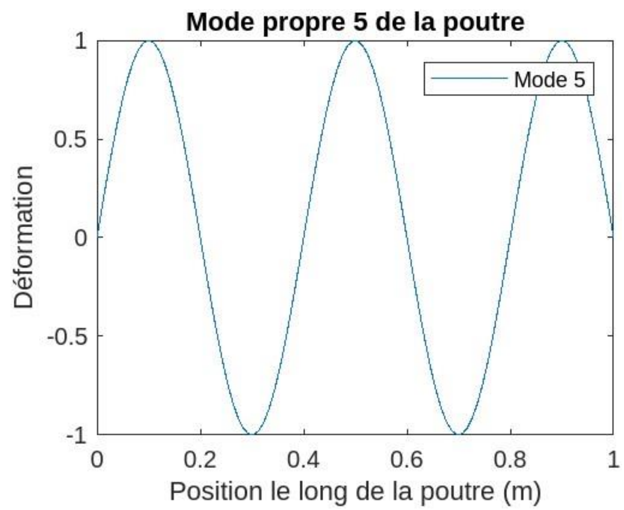
$(2i+1)\pi 2$

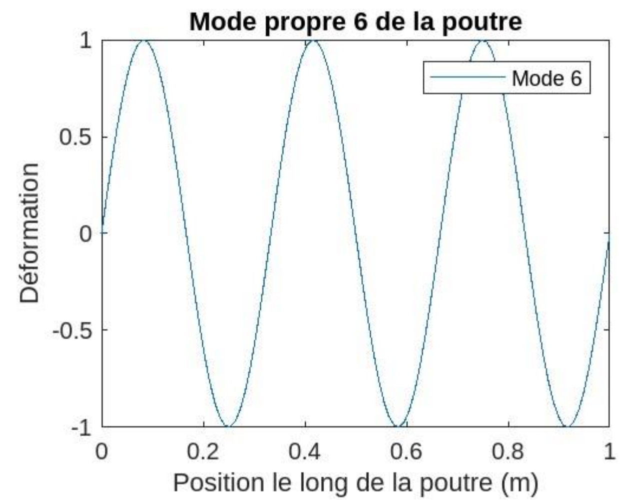
i	1	2	3	4	5	6	7	8	9	10
frequencies x**1.0e+04** en Hz	0.0533	0.1470	0.2882	0.4764	0.7117	0.9941	1.3235	1.6999	2.1234	2.5940

2. Plot of the **deformed shape** for each natural frequency, **simple support case**.




Mode propre 4 de la poutre
Mode 4
Déformation
Position le long de la poutre (m)


Mode propre 5 de la poutre
Mode 5
Déformation
Position le long de la poutre (m)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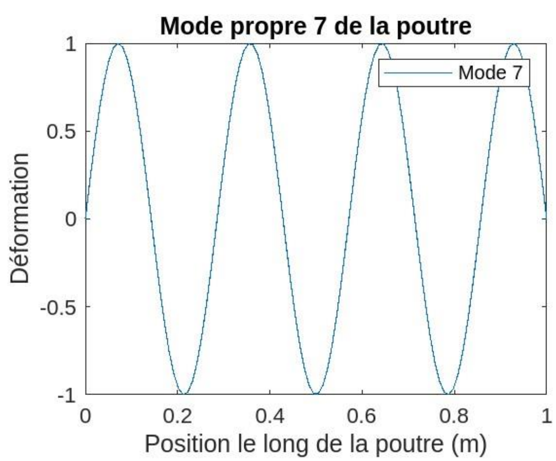
Mode 6
Déformation
Position le long de la poutre (m)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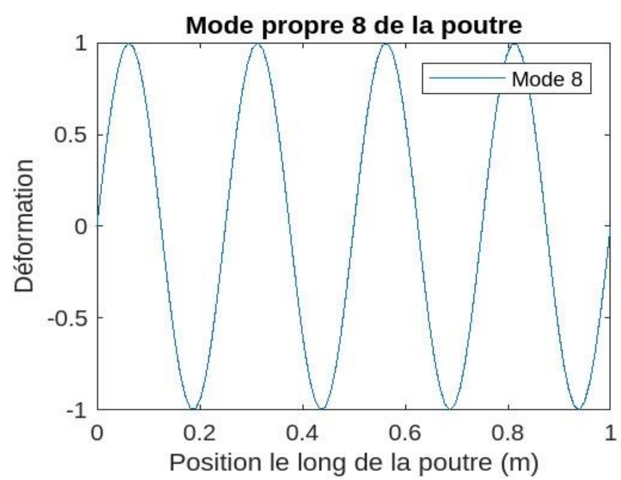
Mode 7
Déformation
Position le long de la poutre (m)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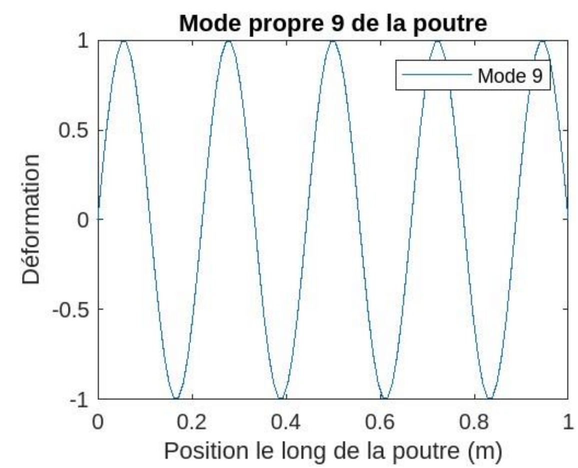
Mode 8
Déformation
Position le long de la poutre (m)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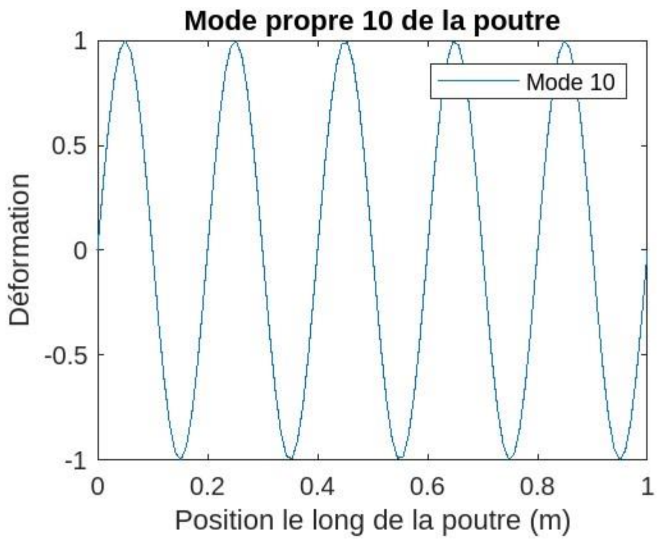
Déformation
Position le long de la poutre (m)

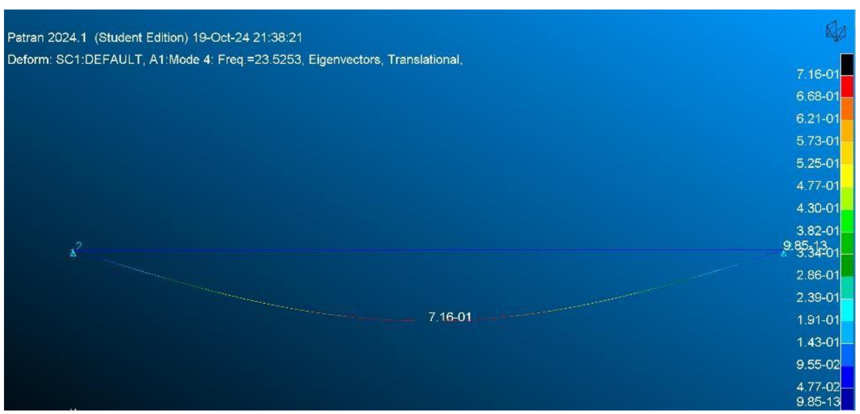


Analysis

What we **first remark** is that for each frequence the amplitude of deformation along the beam does not change we have oscillations' amplitudes between -1 and +1. This is not surprising since the deformation formula is expressed with the function **sinus** which only take values between -1 and +1.

The **second remark** is that the more the frequency increase the more vibrations (nodes or loops) of the beam we have, and it take more positions. Each natural frequency corresponds to a different mode shape of the beam as the frequency increases, the mode shapes become more complex. This means that the beam vibrates in more segments or "loops" along its length.

3. Let's calculate the **eigenmodes** for the **simple support** using Patran and Nastran



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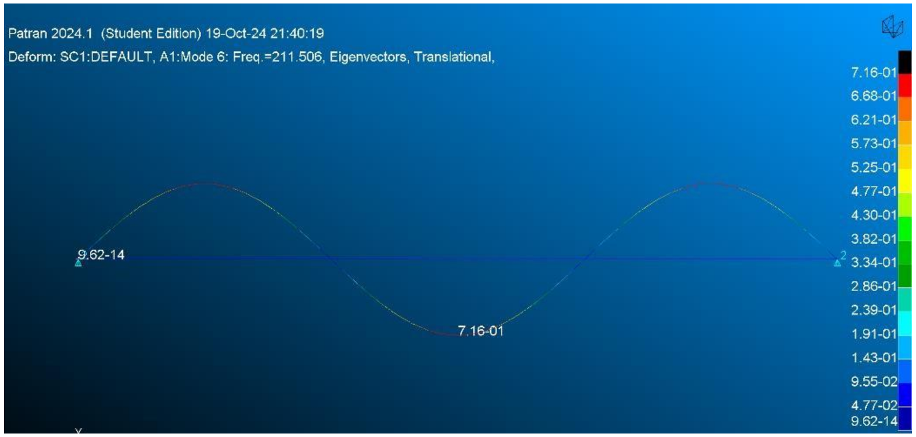
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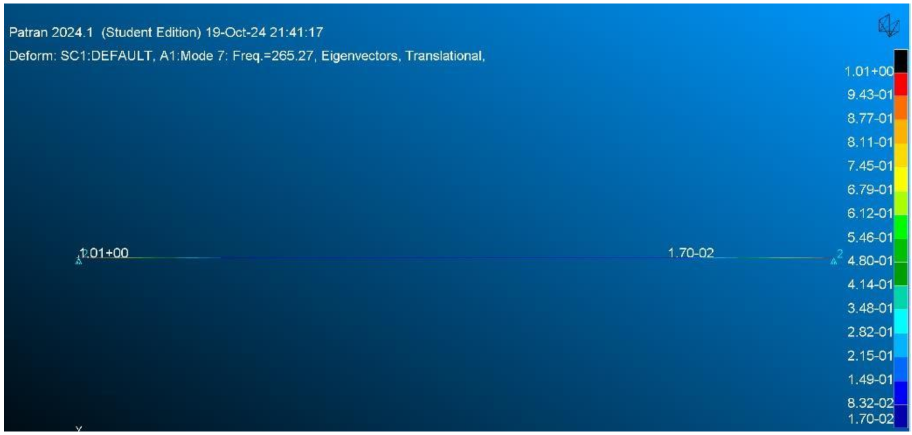
7.16-01
6.68-01
6.21-01
5.73-01
5.25-01
4.77-01
4.30-01
3.82-01
3.34-01
2.86-01
2.39-01
1.91-01
1.43-01
9.55-02
4.77-02
9.62-14
9.62-14
7.16-01

6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9.43-01
8.77-01
8.11-01
7.45-01
6.79-01
6.12-01
5.46-01
4.80-01
4.14-01
3.48-01
2.82-01
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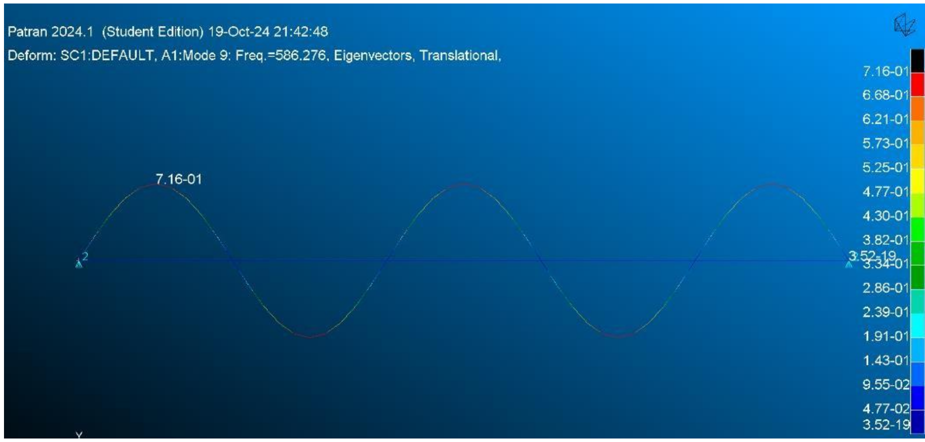
1.01+00
1.70-02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6.68-01
6.21-01
5.73-01
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4.77-01
4.30-01
3.82-01
3.34-01
2.86-01
2.39-01
1.91-01
1.43-01
9.55-02
4.77-02
3.52-19
7.16-01
3.52-19

7


Patran 2024.1 (Student Edition) 19-Oct-24 21:41:57
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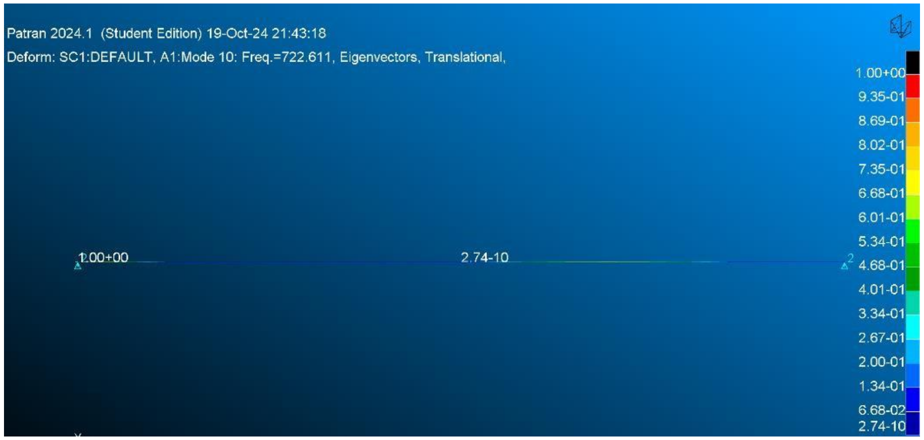
7.15-01
3.67-15
7.15-01
6.67-01
6.19-01
5.72-01
5.24-01
4.76-01
4.29-01
3.81-01
3.34-01
2.86-01
2.38-01
1.91-01
1.43-01
9.53-02
4.76-02
3.67-15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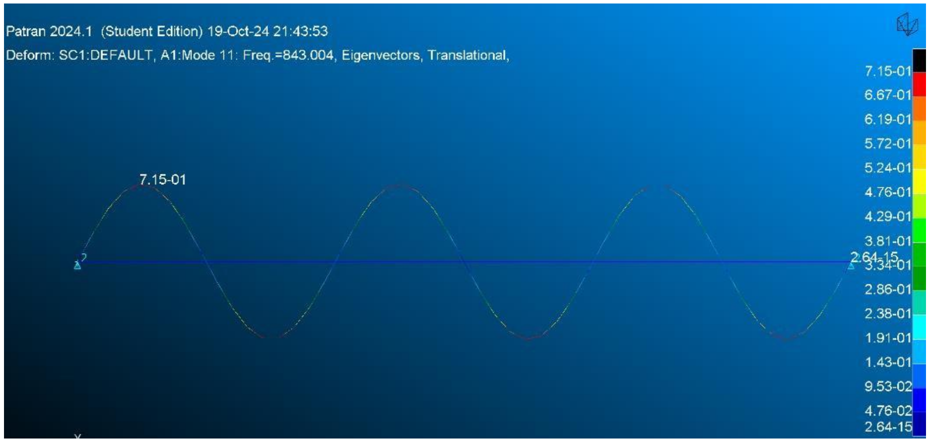
1.00+00
2.74-10
1.00+00
9.35-01
8.69-01
8.02-01
7.35-01
6.68-01
6.01-01
5.34-01
4.68-01
4.01-01
3.34-01
2.67-01
2.00-01
1.34-01
6.68-02
2.74-10

8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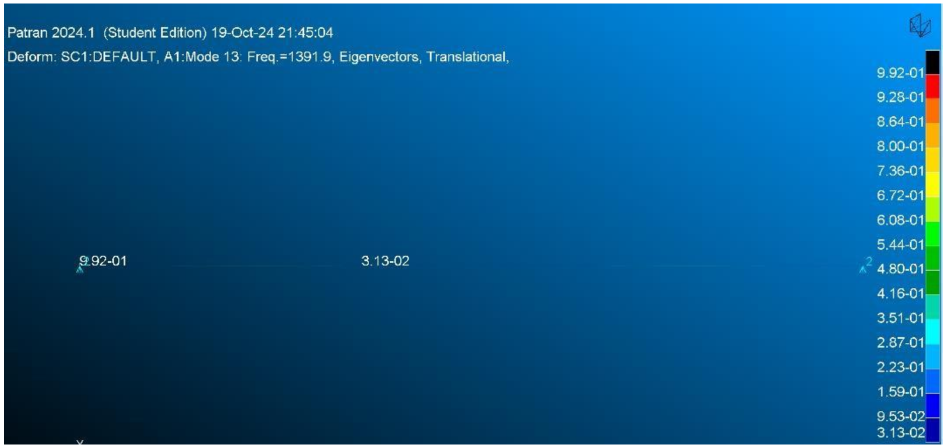
7.15-01
2.64-15
7.15-01
6.67-01
6.19-01
5.72-01
5.24-01
4.76-01
4.29-01
3.81-01
3.34-01
2.86-01
2.38-01
1.91-01
1.43-01
9.53-02
4.76-02
2.64-15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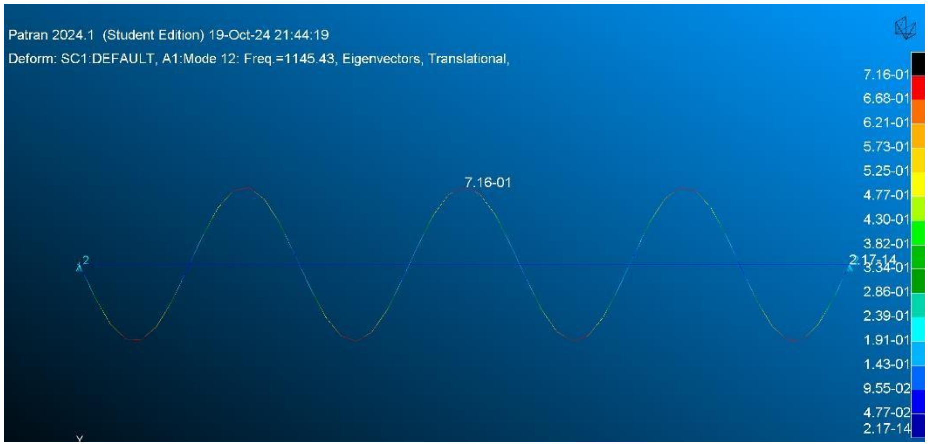
3.13-02
9.92-01
9.28-01
8.64-01
8.00-01
7.36-01
6.72-01
6.08-01
5.44-01
4.80-01
4.16-01
3.51-01
2.87-01
2.23-01
1.59-01
9.53-02
3.13-02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6.68-01
6.21-01
5.73-01
5.25-01
4.77-01
4.30-01
3.82-01
3.34-01
2.86-01
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1.91-01
1.43-01
9.55-02
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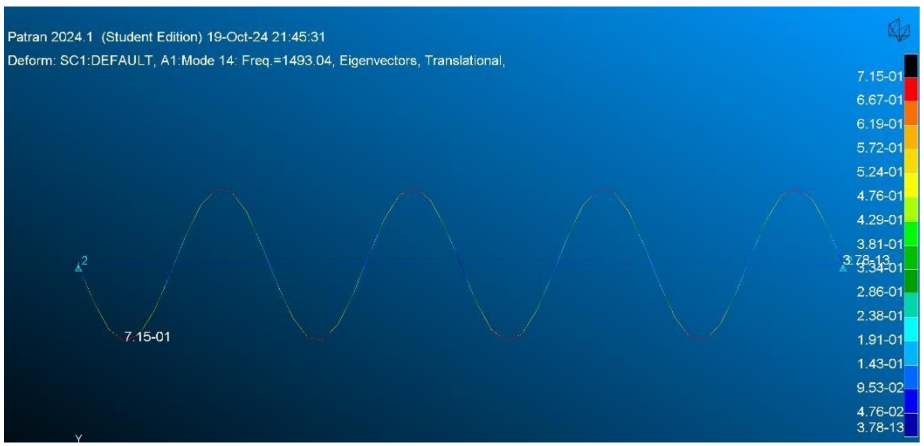
2.17-14
7.16-01
2.17-14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6.19-01
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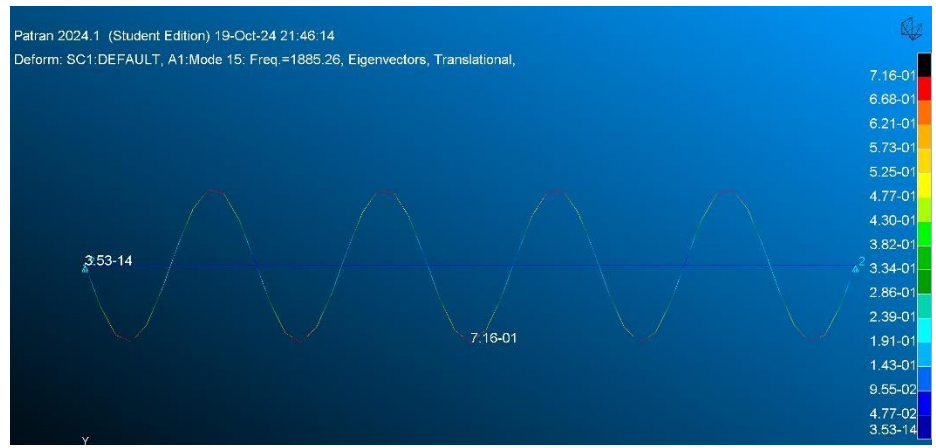
7.15-01
3.78-13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6.68-01
6.21-01
5.73-01
5.25-01
4.77-01
4.30-01
3.82-01
3.34-01
2.86-01
2.39-01
1.91-01
1.43-01
9.55-02
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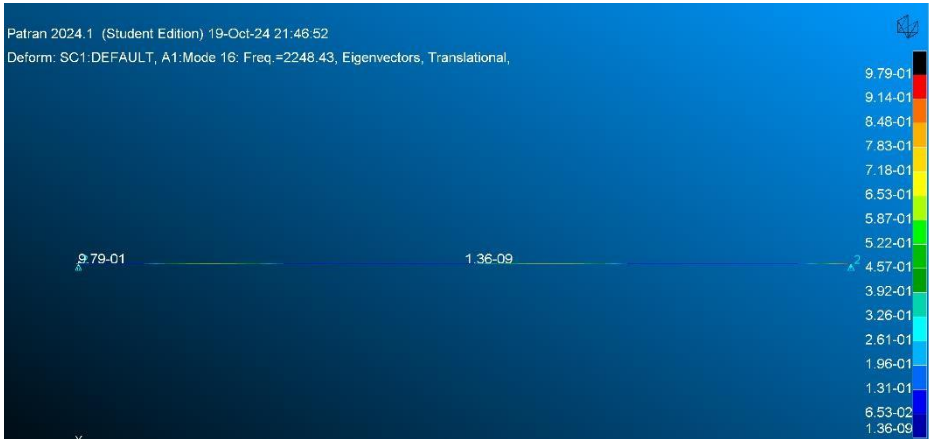
3.53-14
7.16-01



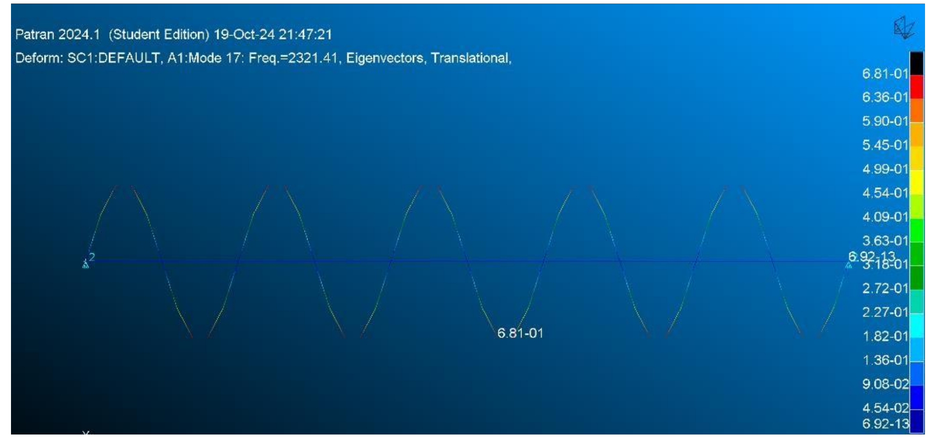
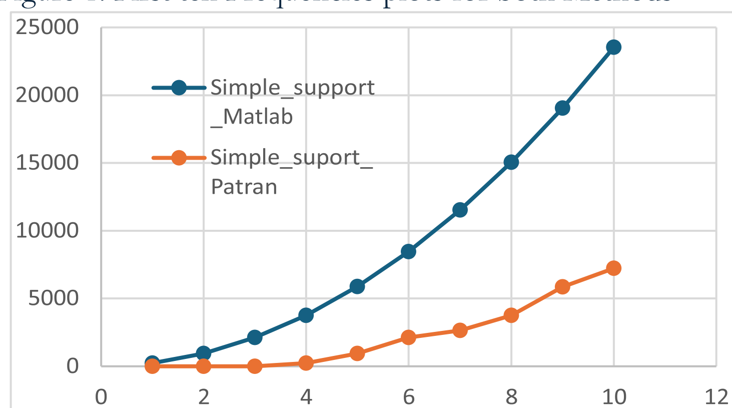


Table 1: First ten Frequencies for both methods Hz

Modes	Simple_support_Matlab	Simple_suport_Patran
1	235	0.0019525
2	941	0.00165709
3	2118	0.00189984
4	3765	235.253
5	5882	940.642
6	8470	2115.06
7	11529	2652.7
8	15058	3756.64
9	19058	5862.76
10	23528	7226.11

Figure 1: First ten Frequencies plots for both Methods



Comparison and Interpretations

When we compare the results from MATLAB and Patran, we see that MATLAB gives us analytical solutions based on theoretical equations. These are simplified and do not include things like material stiffness, non-linearities, or complex boundary conditions. Patran, on the other hand, uses a numerical method like finite elements to solve the beam equation. This type of simulation is more detailed and includes more aspects like the geometry and boundary conditions.

There is a shift between the modes in the two methods. For example, in the first three modes from Patran (out of the 50 modes calculated), we don't see any oscillations. But in MATLAB (the analytical version), from the first mode, we can already see an oscillation with one peak. Also, the frequency of the first mode in Patran is much lower than the frequency of the first mode in MATLAB. It's only the 4th mode in Patran where the frequency matches the first mode we got from MATLAB.

So, this difference is not just because of the way the calculation was done. It's possible that the first three modes from Patran are showing deformation or stiffness modes that are not picked up by the analytical method in MATLAB. Patran can also show "parasitic" modes or modes that are purely numerical, often at very low frequencies, which might be why there are no oscillations.

For example, the 7th mode in Patran shows no oscillations at all, while the 7th mode in MATLAB has several oscillations. This could be because the mode in Patran is very damped, or it doesn't play a big role in the structure's dynamics at that frequency. In Patran, the 7th mode has a frequency of 265.27 Hz, while in MATLAB, the 7th mode has a much higher frequency of 1.115×10^4 Hz.

So, we can think that MATLAB only gives us the modes where there are real vibrations, while Patran looks at more things, including modes where there aren't really any vibrations. This is why the modes don't match exactly. However, we can still find all the modes from MATLAB among the 50 modes in Patran, even if the mode numbers are different.

4. Let's calculate the **eigenmodes** for the **Fixed support** using Patran and Nastran


Patran 2024.1 (Student Edition) 19-Oct-24 22:43:21
Fringe: SC1:DEFAULT, A1:Mode 1: Freq.=53.3012, Eigenvectors, Translational, Magnitude, (NON-LAYERED)
Deform: SC1:DEFAULT, A1:Mode 1: Freq.=53.3012, Eigenvectors, Translational,
8.04-01
7.50-01
6.97-01
6.43-01
5.90-01
5.36-01
4.82-01
4.29-01
3.75-01
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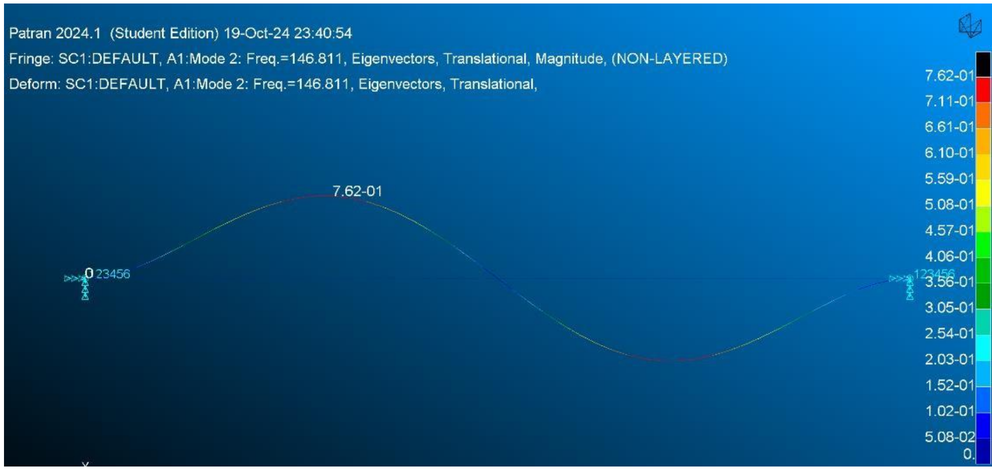
2.68-01
2.14-01
1.61-01
1.07-01
5.36-02
0.
8.04-01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Deform: SC1:DEFAULT, A1:Mode 2: Freq.=146.811, Eigenvectors, Translational,
7.62-01
7.11-01
6.61-01
6.10-01
5.59-01
5.08-01
4.57-01
4.06-01
3.56-01
3.05-01
2.54-01
2.03-01
1.52-01
1.02-01
5.08-02
0.
7.62-01

13


Patran 2024.1 (Student Edition) 19-Oct-24 23:41:56
Fringe: SC1:DEFAULT, A1:Mode 3: Freq.=262.394, Eigenvectors, Translational, Magnitude, (NON-LAYERED)
Deform: SC1:DEFAULT, A1:Mode 3: Freq.=262.394, Eigenvectors, Translational,
8.02-01
7.48-01
6.95-01
6.42-01
5.88-01
5.35-01
4.81-01
4.28-01
3.74-01
3.21-01
2.67-01
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1.60-01
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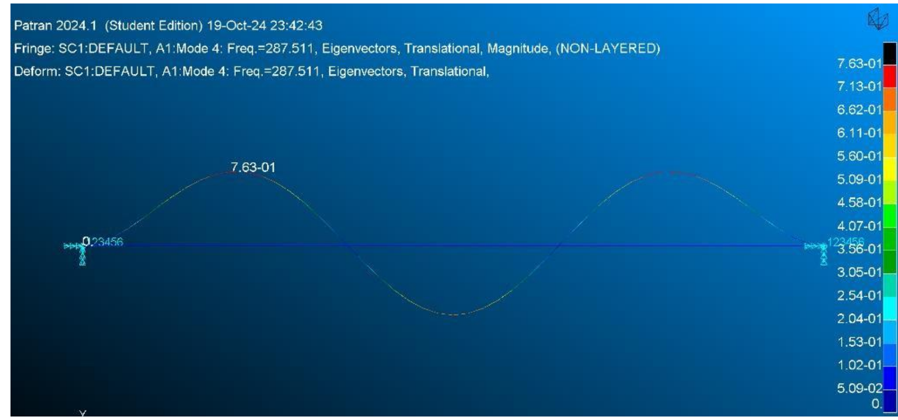
0
8.02-01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Deform: SC1:DEFAULT, A1:Mode 4: Freq.=287.511, Eigenvectors, Translational,
7.63-01
7.63-01
7.13-01
6.62-01
6.11-01
5.60-01
5.09-01
4.58-01
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1.53-01
1.02-01
5.09-02
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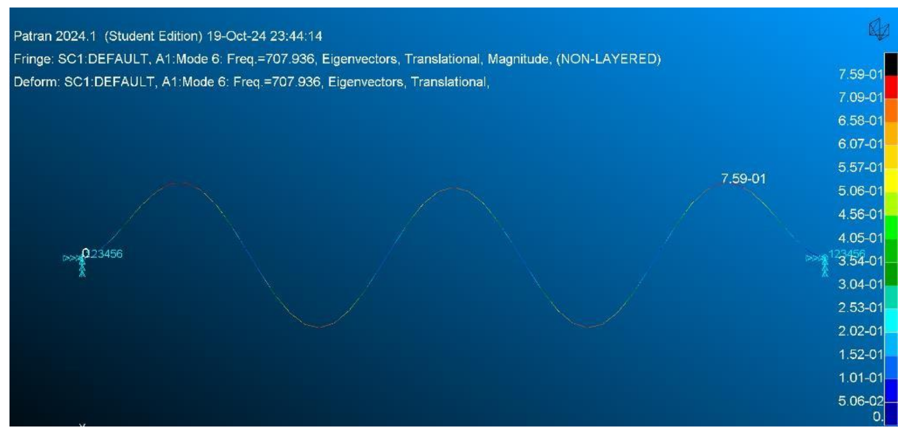
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Deform: SC1:DEFAULT, A1:Mode 6: Freq.=707.936, Eigenvectors, Translational,
7.59-01
7.59-01
7.09-01
6.58-01
6.07-01
5.57-01
5.06-01
4.56-01
4.05-01
3.54-01
3.04-01
2.53-01
2.02-01
1.52-01
1.01-01
5.06-02
0.

15


Patran 2024.1 (Student Edition) 19-Oct-24 23:43:14
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Deform: SC1:DEFAULT, A1:Mode 5: Freq.=474.652, Eigenvectors, Translational,
7.66-01
7.66-01
7.15-01
6.64-01
6.12-01
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5.10-01
4.59-01
4.08-01
3.57-01
3.06-01
2.55-01
2.04-01
1.53-01
1.02-01
5.10-02
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Patran 2024.1 (Student Edition) 19-Oct-24 23:44:40
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7.59-01
7.59-01
7.09-01
6.58-01
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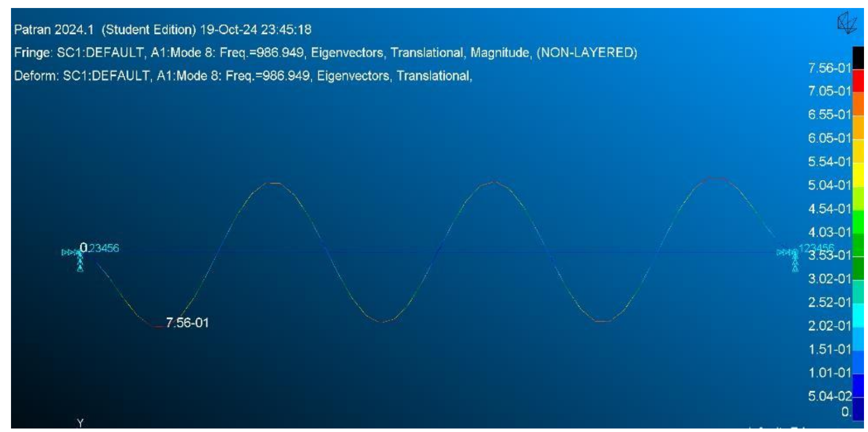
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7.56-01
7.05-01
6.55-01
6.05-01
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5.04-01
4.54-01
4.03-01
3.53-01
3.02-01
2.52-01
2.02-01
1.51-01
1.01-01
5.04-02
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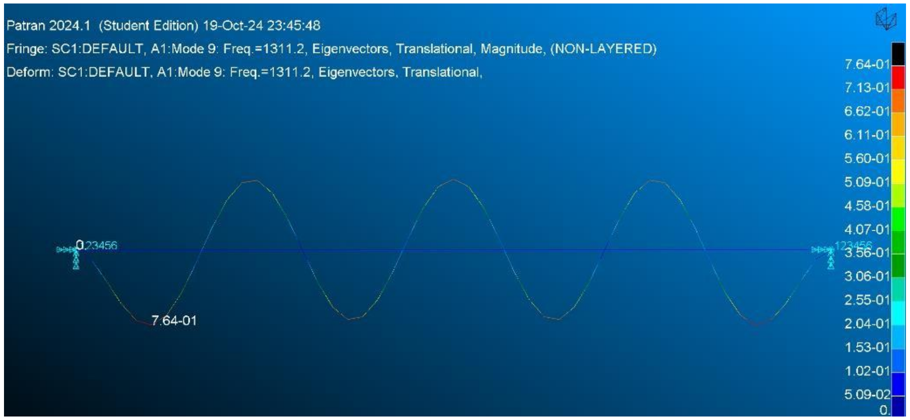
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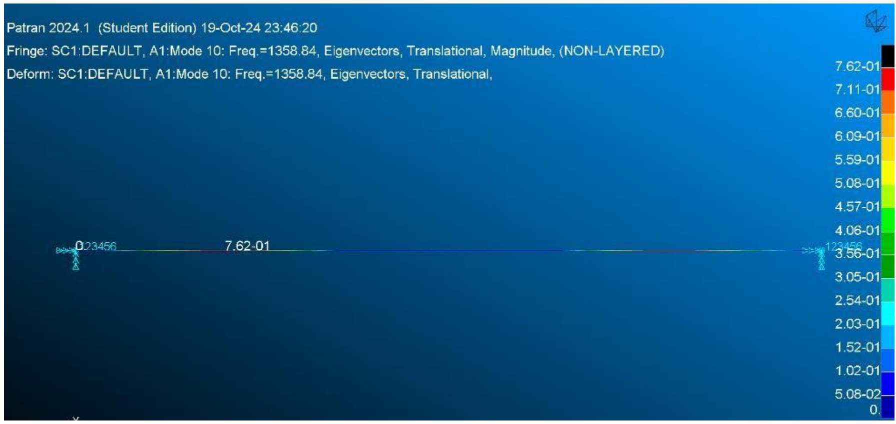
7.64-01
7.13-01
6.62-01
6.11-01
5.60-01
5.09-01
4.58-01
4.07-01
3.56-01
3.06-01
2.55-01
2.04-01
1.53-01
1.02-01
5.09-02
0.
0
7.64-01


Patran 2024.1 (Student Edition) 19-Oct-24 23:46:20
Fringe: SC1:DEFAULT, A1:Mode 10: Freq.=1358.84, Eigenvectors, Translational, Magnitude, (NON-LAYERED)
Deform: SC1:DEFAULT, A1:Mode 10: Freq.=1358.84, Eigenvectors, Translational,
7.62-01
7.11-01
6.60-01
6.09-01
5.59-01
5.08-01
4.57-01
4.06-01
3.56-01
3.05-01
2.54-01
2.03-01
1.52-01
1.02-01
5.08-02
0.
0
7.62-01

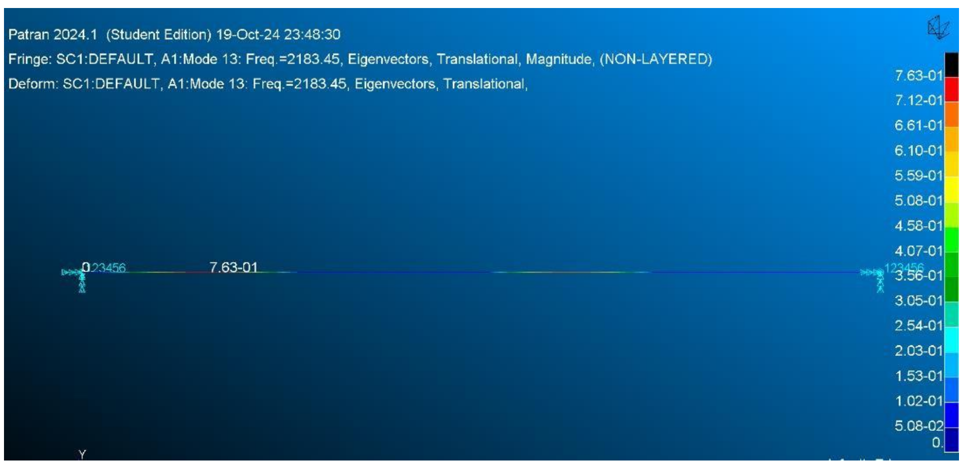
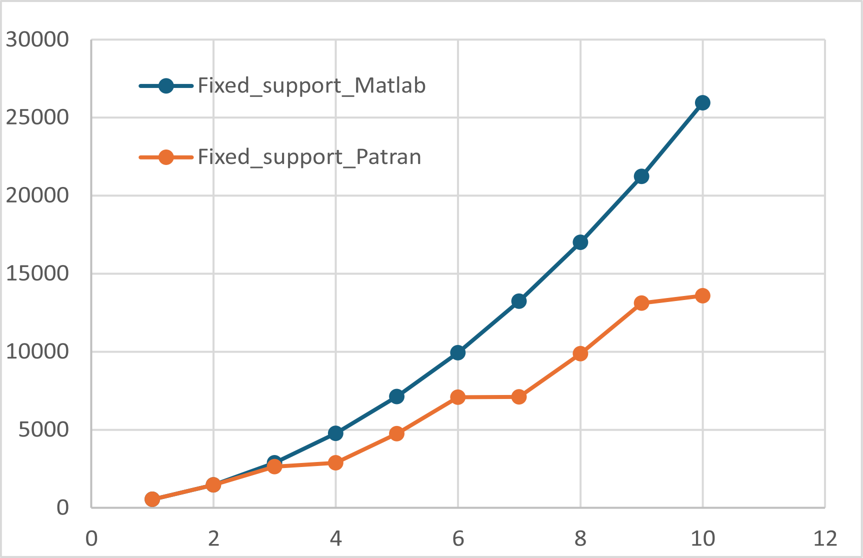


Table 2: First ten frequencies for both methods



Modes	Fixed_Support_Matlab	Fixed_Support_Patran
1	533	533.012
2	1470	1468.11
3	2882	2623.94
4	4764	2875.11
5	7117	4746.52
6	9941	7079.36
7	13235	7097.88
8	16999	9869.49
9	21234	13112

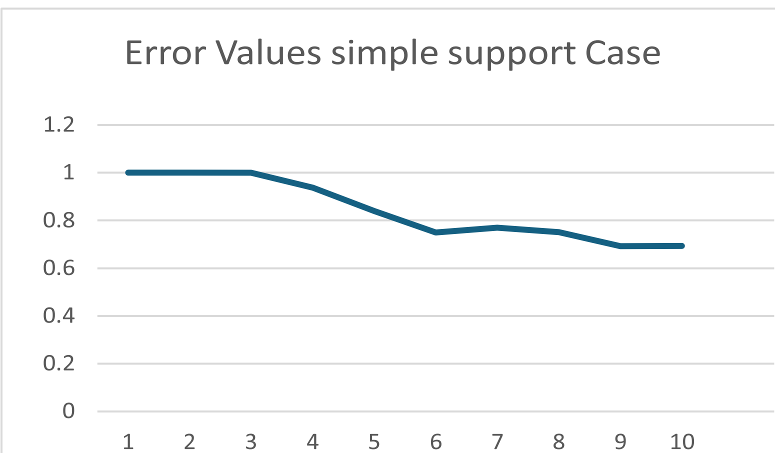
10	25940	13588.4

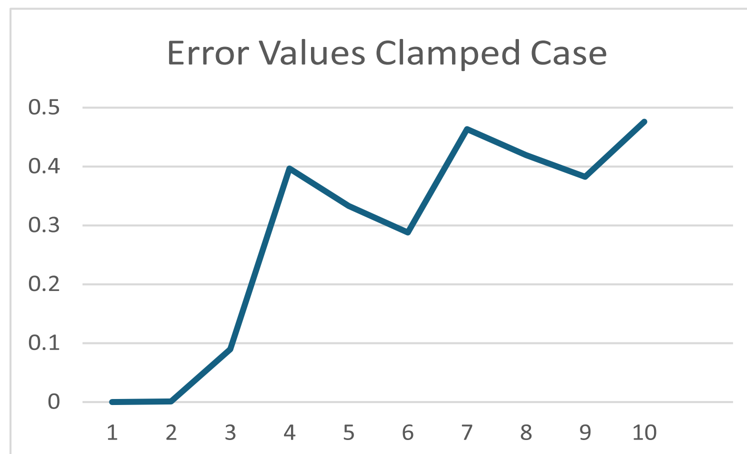
Comparison and Interpretations

Figure 2: First ten Frequencies plots for both Methods

In the clamped case, the frequencies of the first three modes are the same for both the analytical method and the numerical method done in Patran and Nastran. However, starting from the fourth mode, differences appear, especially in the seventh and tenth modes, where the gaps are more significant. This is likely due, as explained for the simple support case earlier, to the increasing complexity of the phenomena at higher frequencies, which the numerical method captures better than the simplified analytical method. We also think that since the beam is clamped, these phenomena are reduced at lower frequencies. This explains why, for modes 1, 2, and 3, the frequencies are the same for both the analytical method in MATLAB and the numerical method in Patran.

5. Let's **calculate and plot the error** for both cases





Interpretations of the errors

We remark that for the simple support case the more the natural frequencies increase the more the error decrease and seems to converge toward a single value likely 70% but it still a high value so in general case we cannot only refer to the analytical results it is not one hundred per cent reliable.
For the second case, the clamped one, the error increases when the mode increases for the mode number ten its likely equal to 50% its better than the first case but since it increases it also means that for higher modes the value of the error will be higher it does not converge at all.
We observe than a huge difference between the FEM methods and the Analytical one for both case and since the FEM methods take into account more parameters than the analytical method which is simplified it better to trust the FEM method for tests.

II. Eigenmodes of a Cylindrical shell

For this analysis, I created the geometry of the structure using a 2D shell model approach, which is well-suited for thinwalled structures. The mesh was carefully generated to ensure an accurate representation of the geometry, capturing key features and boundary conditions. The material properties, including Young's modulus, Poisson’s ratio, and density, were defined to reflect the physical characteristics of the material used. After setting up the boundary conditions and solver parameters, I ran a normal modes analysis in Nastran. Below are the results, which include the first 20 natural frequencies along with the associated deformations. The corresponding “i” and “j” indices for each mode have also been identified.

Mode 1

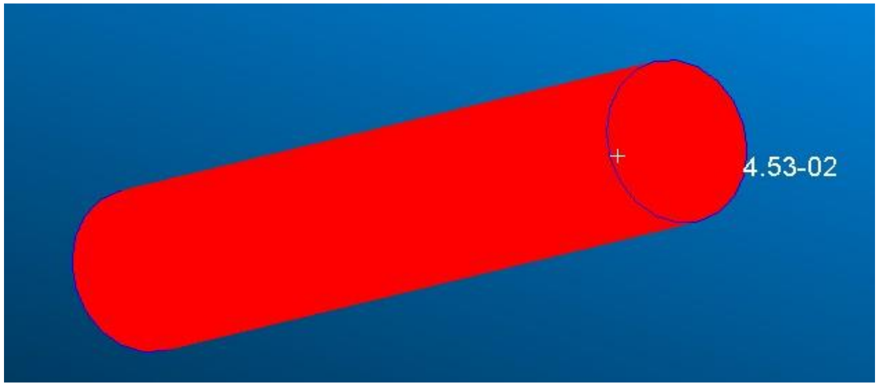


Fig1. 1^{st} natural frequency(6e-06Hz), j=0 i=0

Mode 2

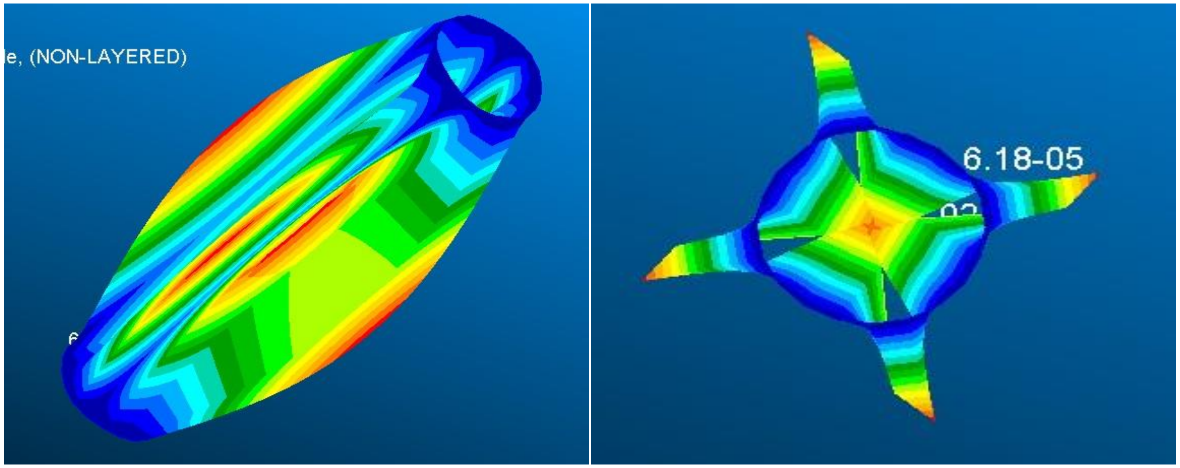


Fig2. 2^{nd} natural frequency(6.19Hz),j=1 i=4

Mode 3

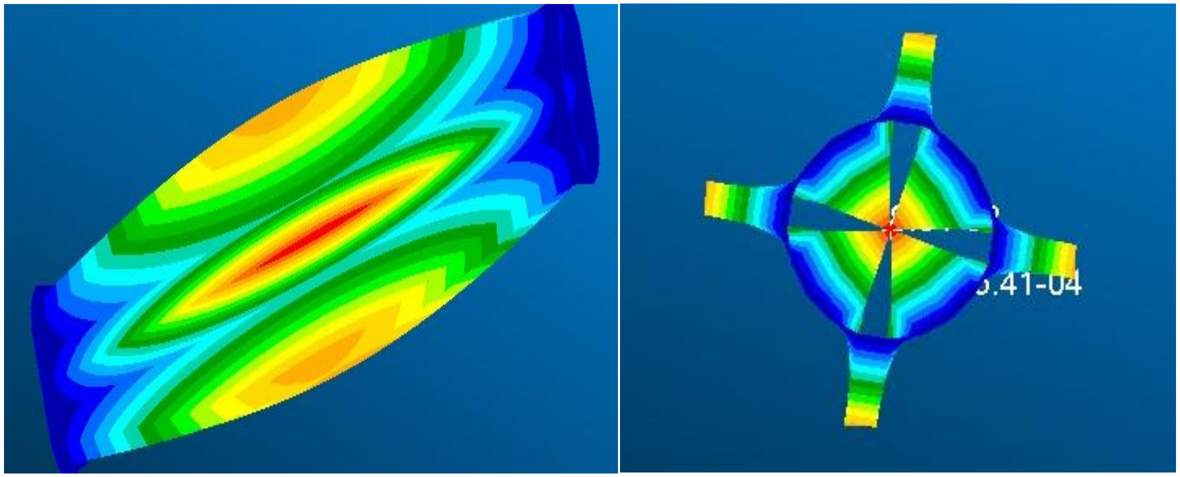


Fig3. 3rd natural frequency (6.19Hz), j=1 i=4

Mode 4

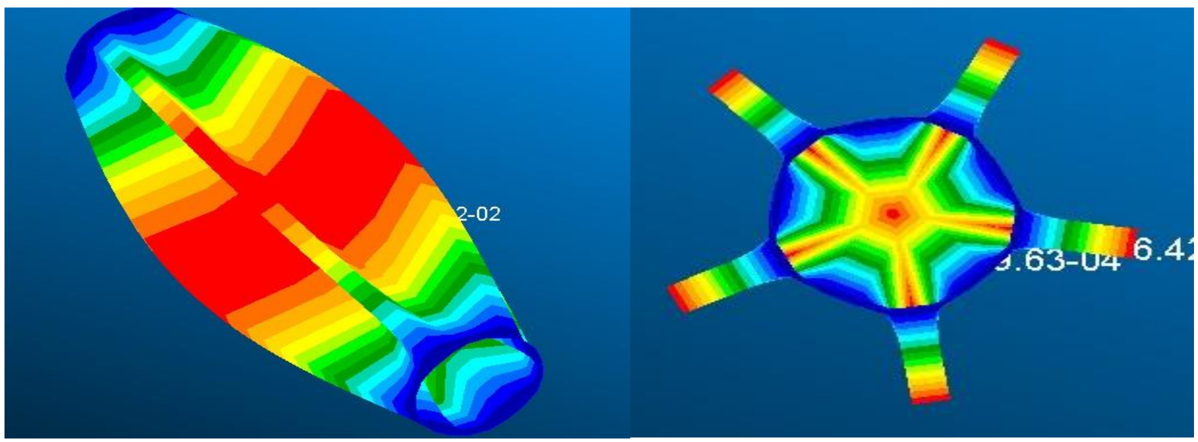


Fig4. 4th natural frequency(6.838Hz),j=1 i=5

Mode 5

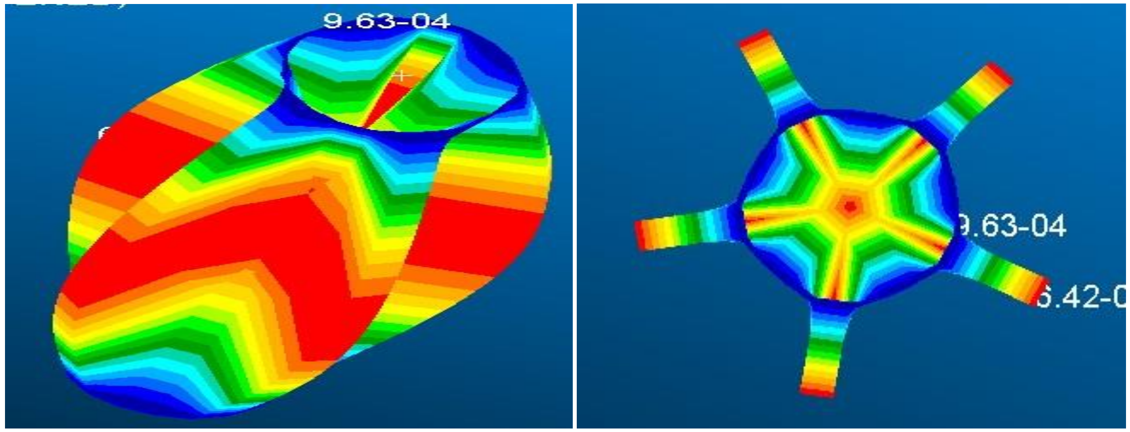


Fig5. 5th natural frequency(6.838Hz),j=1 i=5

Mode 6

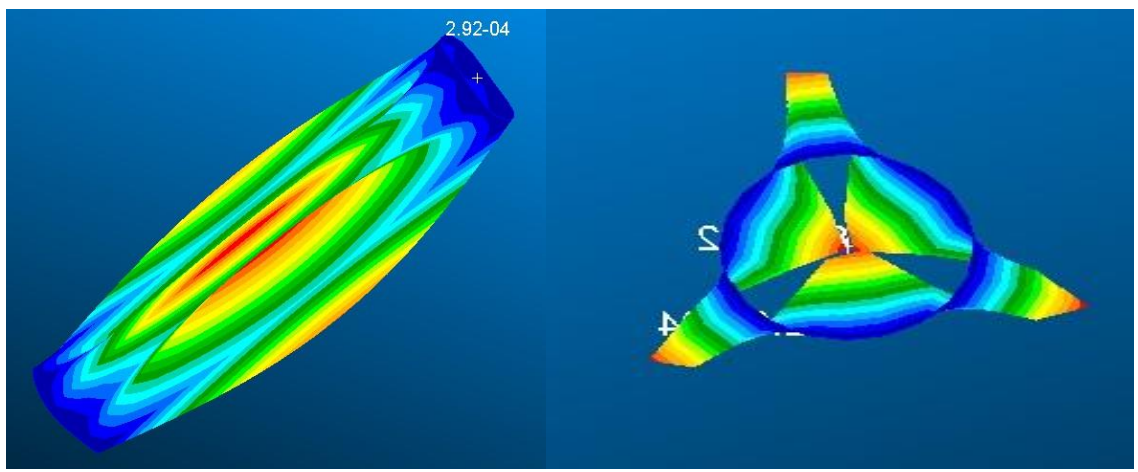


Fig6. 6th natural frequency(8.714Hz),j=1 i=3

Mode 7

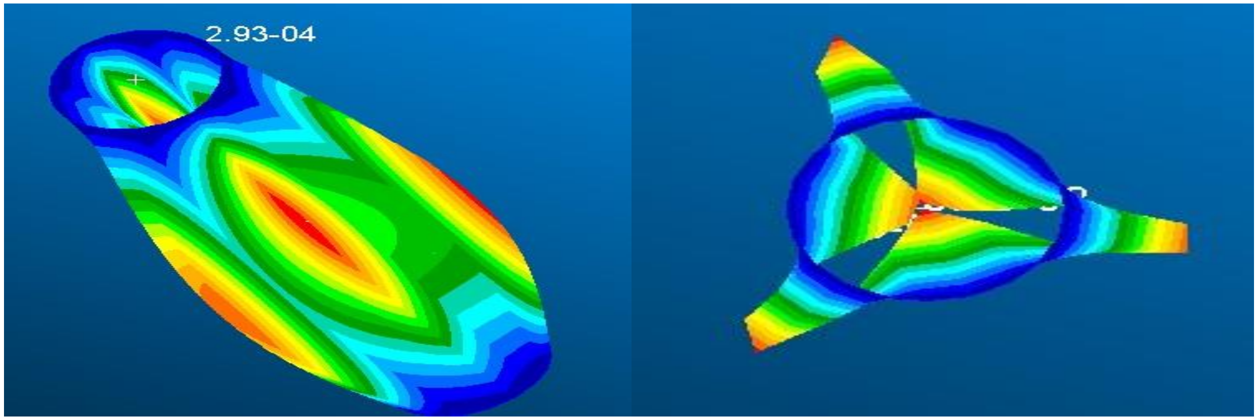


Fig7. 7th natural frequency(8.714Hz),j=1 i=3

Mode 8

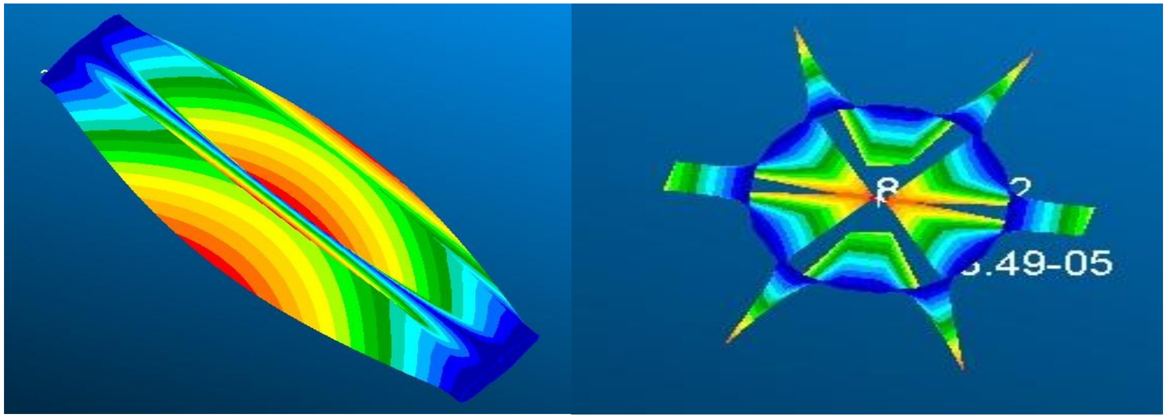


Fig8. 8th natural frequency(9.048Hz),j=1 i=6

Mode 9

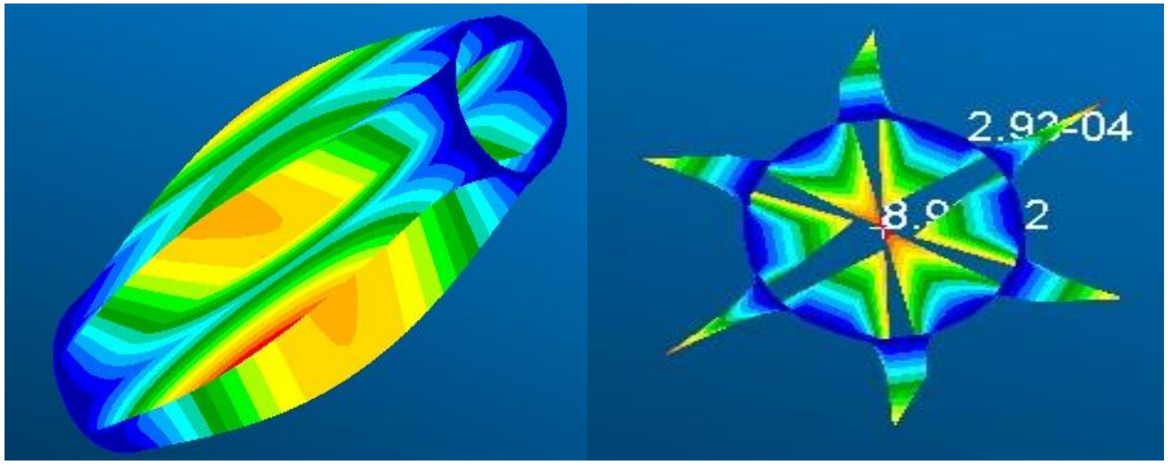


Fig9. 9^{th} natural frequency(9.049Hz),j=1 i=6

Mode 10

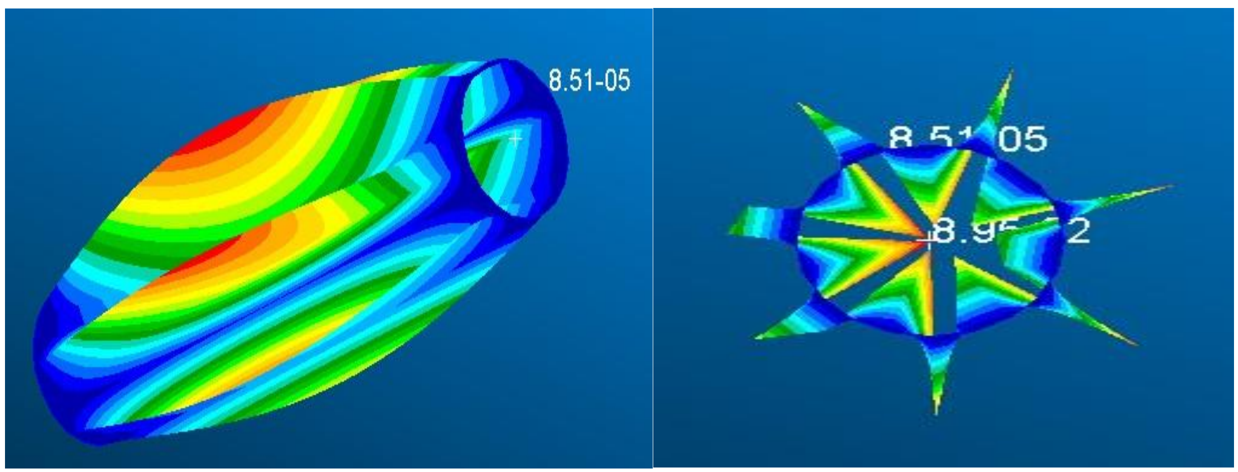


Fig10. 10^{th} natural frequency(11.891Hz),j=1 i=7

Mode 11

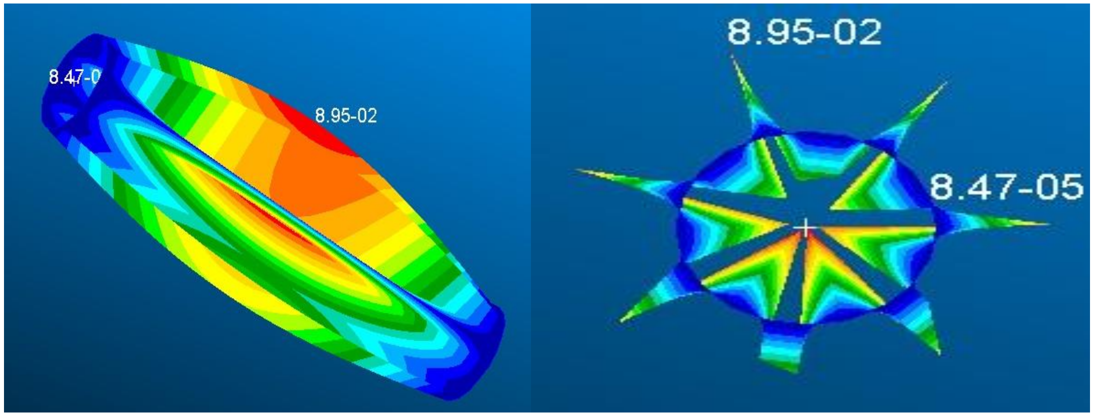


Fig11. 11th natural frequency(11.892Hz),j=1 i=7

Mode 12

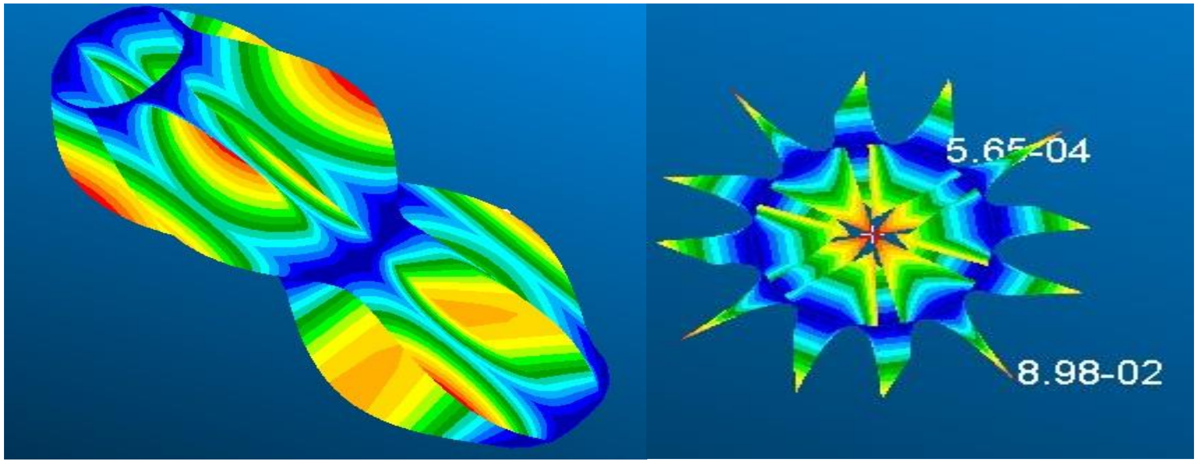


Fig12. 12th natural frequency(12.668Hz),j=2 i=6

Mode 13

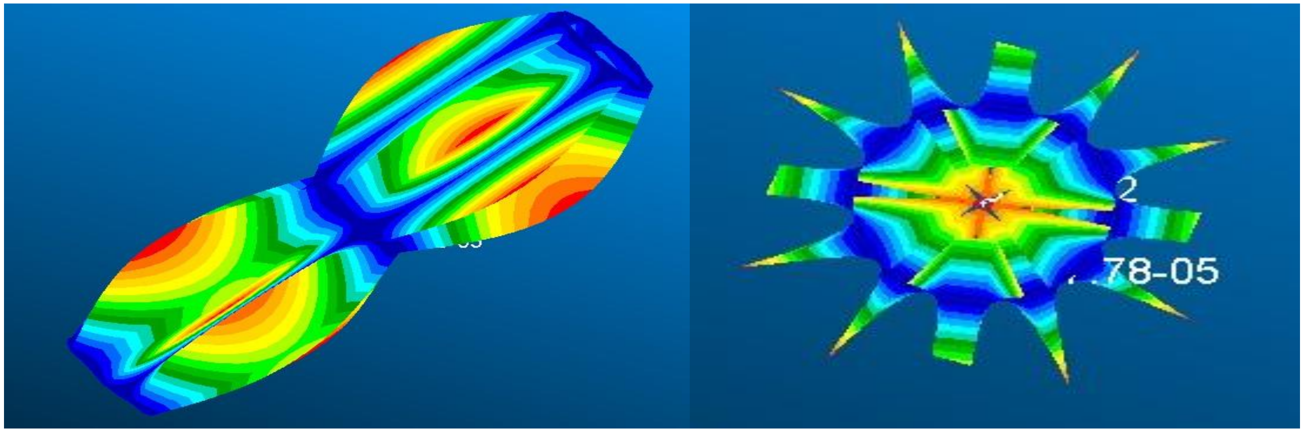


Fig13. 13th natural frequency(12.668Hz),j=2 i=6

Mode 14

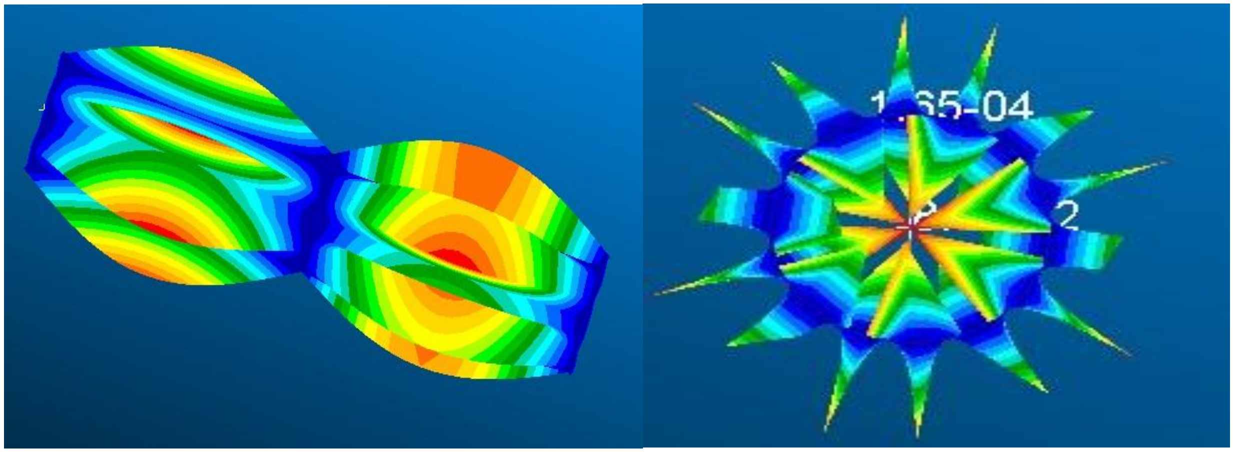


Fig14. 14th natural frequency(13.824Hz),j=2 i=7

Mode 15

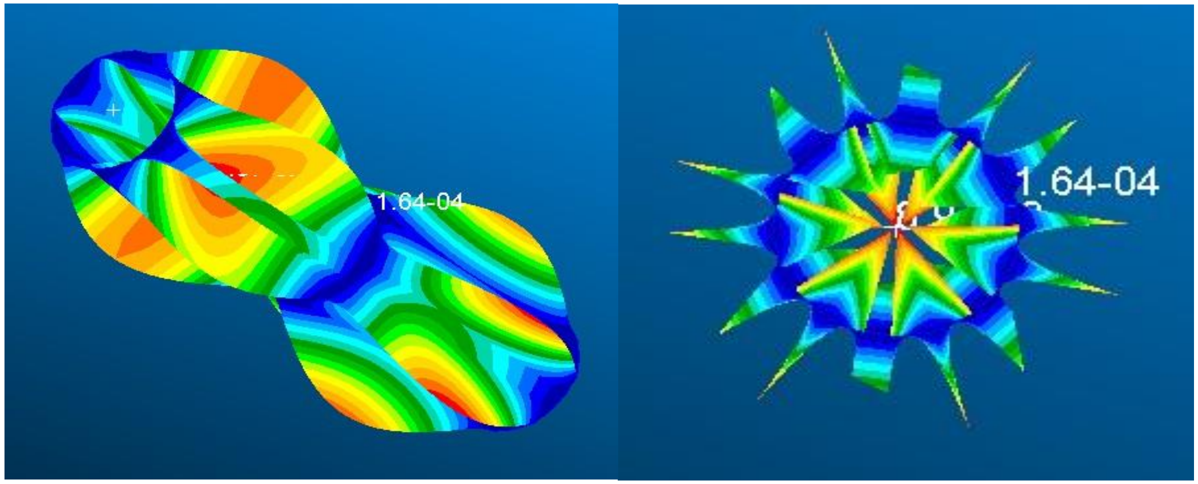


Fig15. 15th natural frequency(13.824Hz),j=2 i=7

Mode 16

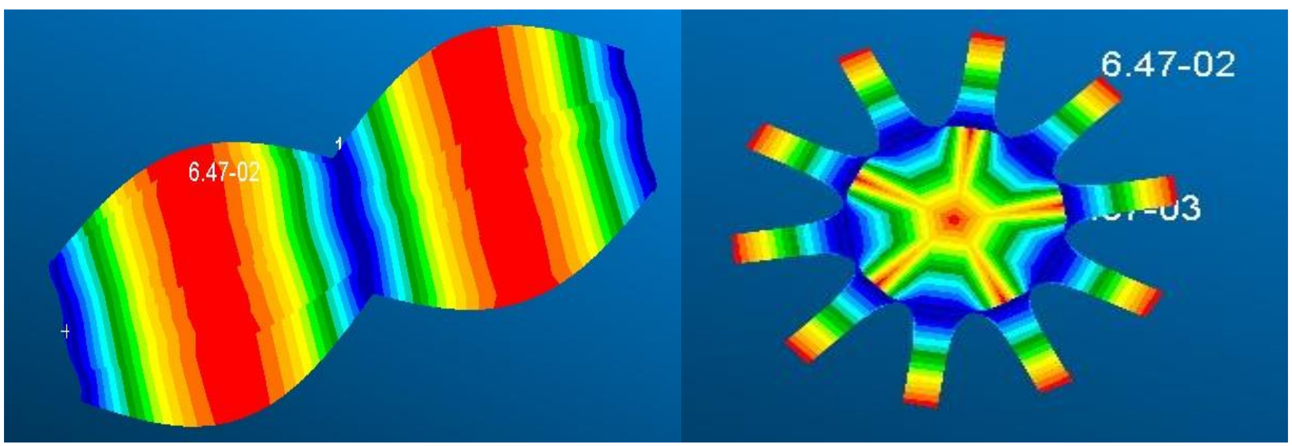


Fig16. 16th natural frequency(14.097Hz),j=2 i=5

Mode 17

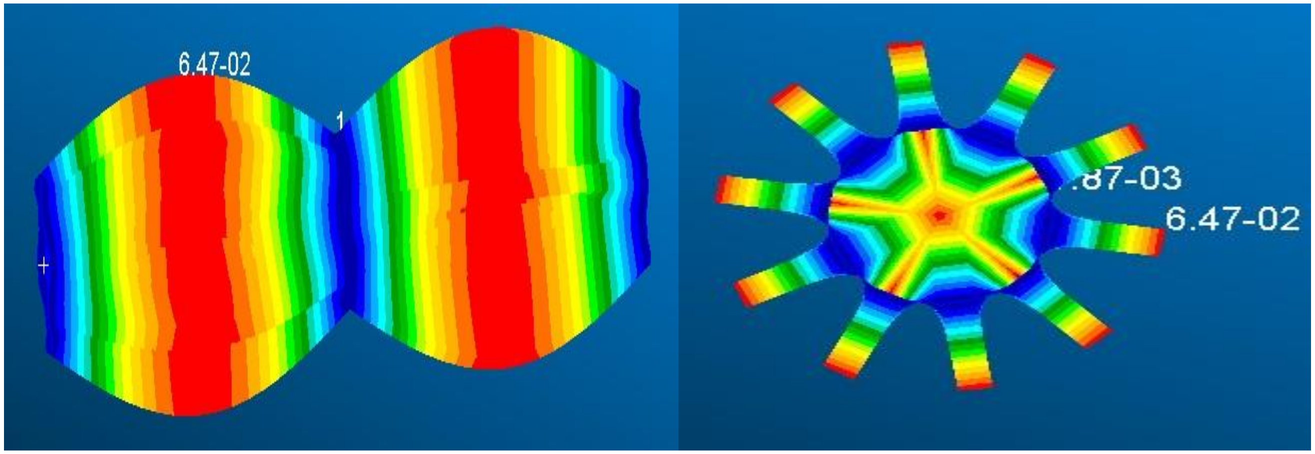


Fig17. 17th natural frequency(14.098Hz),j=2 i=5

Mode 18

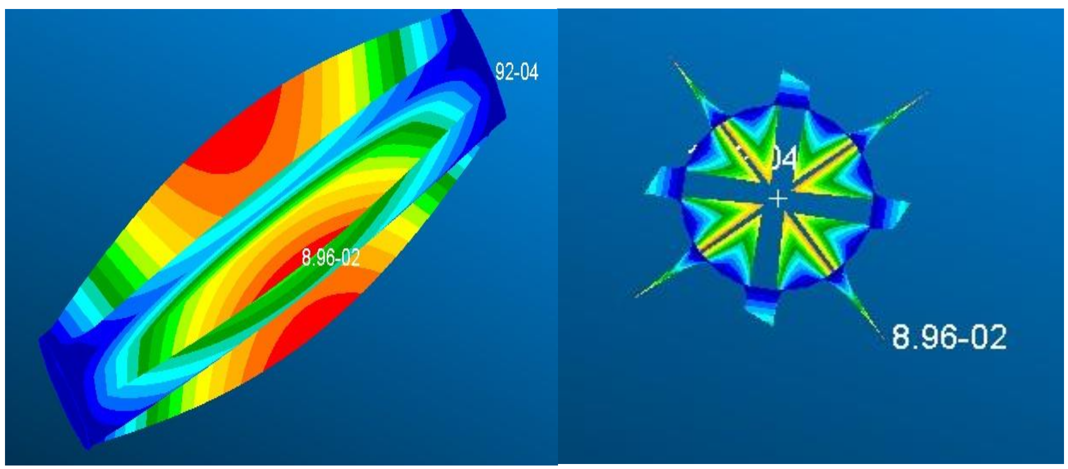


Fig18. 18th natural frequency(14.788Hz),j=1 i=8

Mode 19

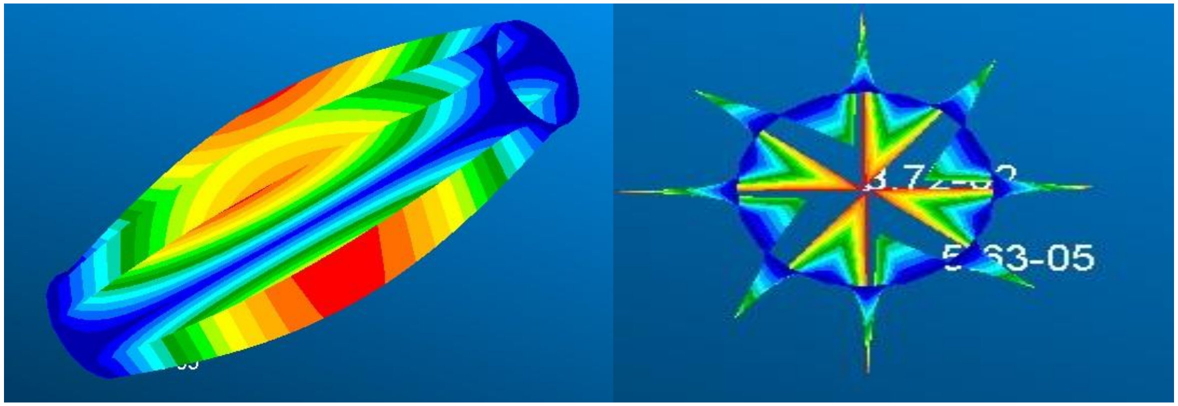


Fig19. 19th natural frequency(14.789Hz),j=1 i=8

Mode 20

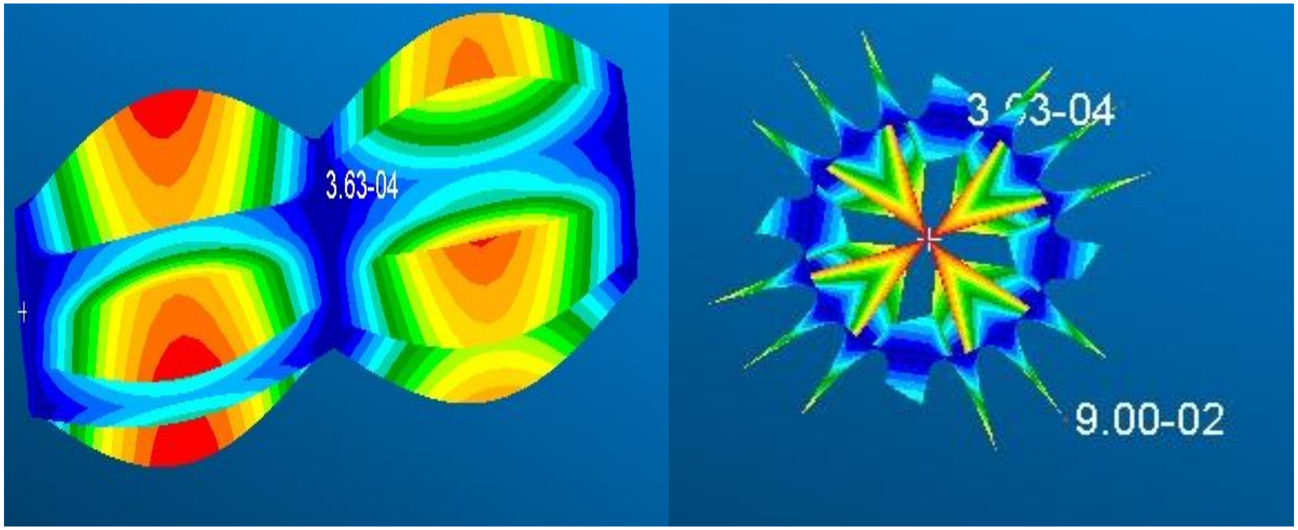


Fig20. 20th natural frequency(16.129Hz),j=2 i=8

With the circumferential mode number i and the axial mode number j established, we can now proceed to analytically calculate the natural frequencies and assess the error between these values and the Nastran frequencies using the MATLAB code below.

Explanation

This code calculates and compares the natural frequencies of a cylindrical structure using both analytical methods and results from a Nastran simulation. It first defines the material properties, such as Young's modulus, Poisson's ratio, and density, along with the cylinder's geometry, including radius, thickness, and height. The mode indices for the first 20 frequencies are specified in a matrix, where each pair of indices corresponds to a particular vibration mode. The code then calculates the natural frequencies for each defined mode using an analytical formula that involves the material properties and geometry of the cylinder.

After calculating the frequencies, the Nastran results for the same modes are loaded, and the percentage error between the analytical and Nastran frequencies is computed. The error is expressed as a percentage, showing the difference between the two sets of results. Following this, the code generates two plots: one comparing the analytical and Nastran frequencies across the mode numbers, and another showing the percentage error for each mode. Both plots are saved as PNG files for later analysis. The overall goal is to evaluate the accuracy of the analytical approach by comparing it with the more precise numerical simulation results from Nastran.

Results

Mode Number	Analytical Frequency (Hz)	Nastran Frequency (Hz)	Percentage Error (%)
1	0	0	0

2	6.4931	6.1905	4.6603
3	6.4931	6.1906	4.6598
4	7.1218	6.8388	3.9739
5	7.1218	6.8389	3.9732
6	9.2478	8.7136	5.7765
7	9.2478	8.7137	5.7759
8	9.3918	9.0502	3.6365
9	9.3918	9.0505	3.6336
10	12.51	11.8934	4.9286
11	12.51	11.8939	4.9246
12	12.8343	12.67	1.2802
13	12.8343	12.67	1.2802
14	14.1127	13.8	2.2154
15	14.1127	13.8303	2.0007
16	14.3507	14.0952	1.7806
17	14.3507	14.0954	1.7792
18	16.2373	14.7907	8.9091
19	16.2373	14.7914	8.9048
20	17.0321	16.1375	5.2522

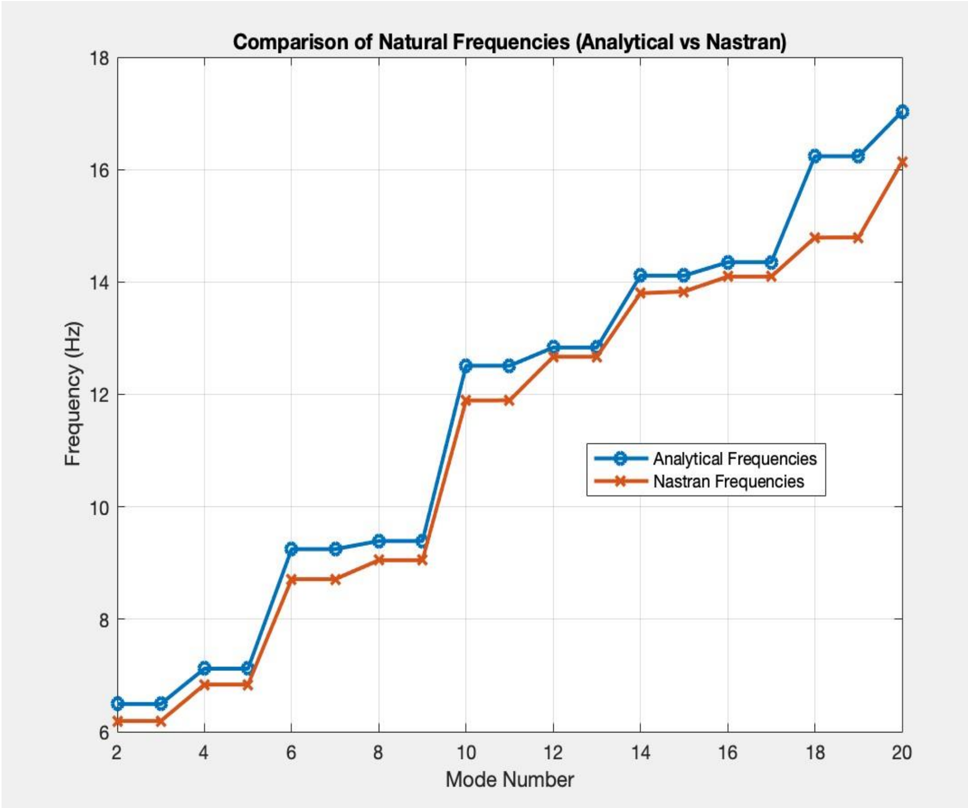


Fig21. Comparison of Natural Frequencies

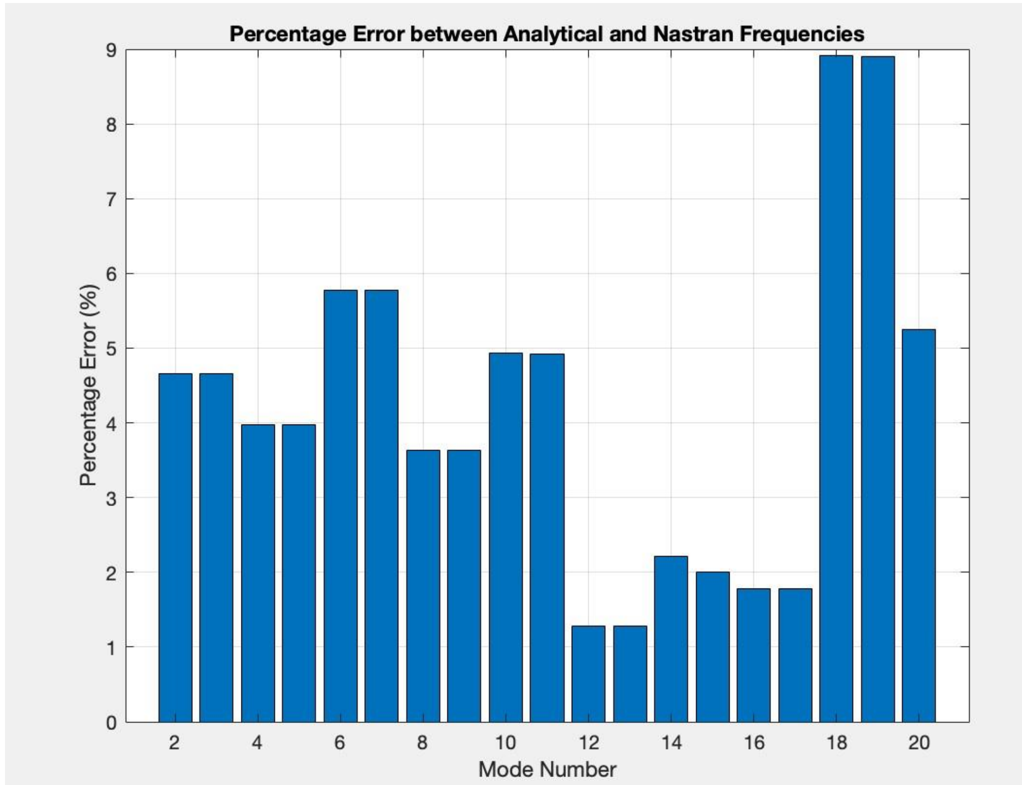


Fig22. Percentage error

The analytical frequencies calculated for the given cylindrical structure range from 6.49 Hz to 17.03 Hz, covering modes 2 through 20. These frequencies reflect the natural vibrational behavior of the structure, with lower modes (such as modes 2–7) falling between 6.49 Hz and 9.39 Hz, representing fundamental vibrations. As the mode numbers increase, the frequencies rise, reaching a maximum of 17.03 Hz for mode 20, indicating more complex and higher-energy vibrations.

The percentage error between the analytical and Nastran frequencies varies across the modes, providing insight into the accuracy of the analytical model. For several modes, particularly modes 12 through 16, the error remains low (around 1% to 3%), demonstrating that the analytical model closely approximates the Nastran results for these frequencies. For the lower and mid-range modes (2–5, 9–11), the errors are slightly higher, between 3.63% and 5.77%, reflecting a small divergence as the vibrational patterns become more complex. The most significant discrepancies occur at higher modes, such as modes 18 and 19, where the percentage errors rise to around 8.9%, suggesting that the analytical method is less accurate in predicting the behavior of higher-order vibrations.

Overall, while the analytical approach provides reasonably accurate results for lower and mid-frequency modes, its accuracy diminishes for higher frequencies, as evidenced by the increasing errors. This is typical of analytical models, which often rely on idealized assumptions, making them less effective at capturing the complexities of higher-order vibrations compared to numerical methods like Nastran.

III. Eigenmodes of Cylinder

1. We modelized the 3D Cylinder in Patran following the professor tutorial

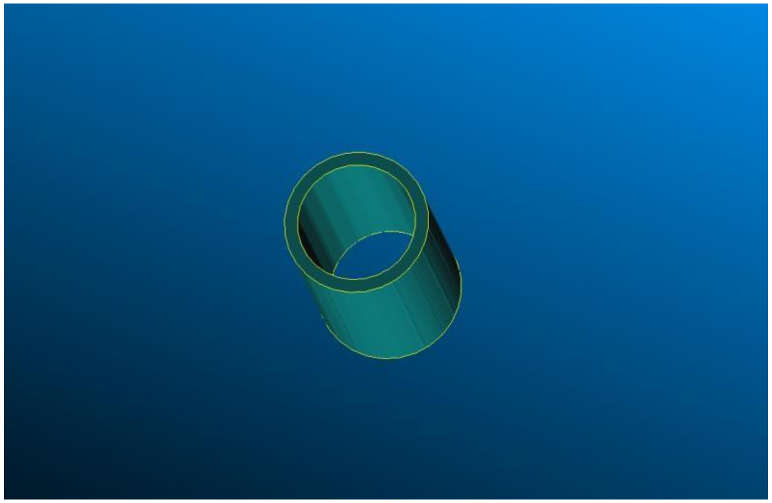


Figure 4:Solid Cylinder

2. Let’s calculate the modes and Frequencies

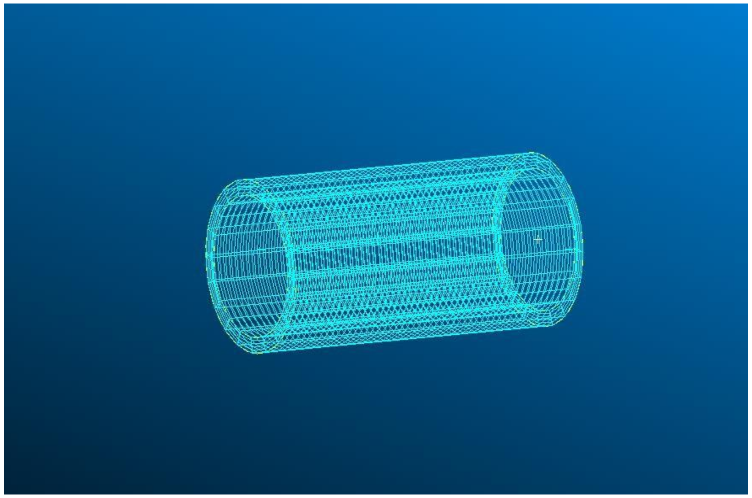


Figure 3: Meshed Cylinder

In Patran we choose to display 100 modes but only 30 of them are presents because. This choose is done to be able to view all the type of deformation the cylinder can have.

Modes	Frequencies
1	1.83067e-05
2	36.9804
3	36.9804
4	43.5176
5	43.5176
6	72.3758
7	72.3758
8	75.4497
9	82.572
10	82.5721
11	96.5248
12	96.5248

13	99.8217
14	99.8218
15	101.795
16	101.795
17	114.778
18	114.778
19	121.971
20	130.837
21	130.837
22	138.344
23	138.344
24	150.646
25	150.646
26	150.862
27	151.213
28	151.213
29	157.359
30	157.359

3. Only one mode is corresponding to null frequency it's the mode number 01 its frequency is equal to: **1.83067e-05**

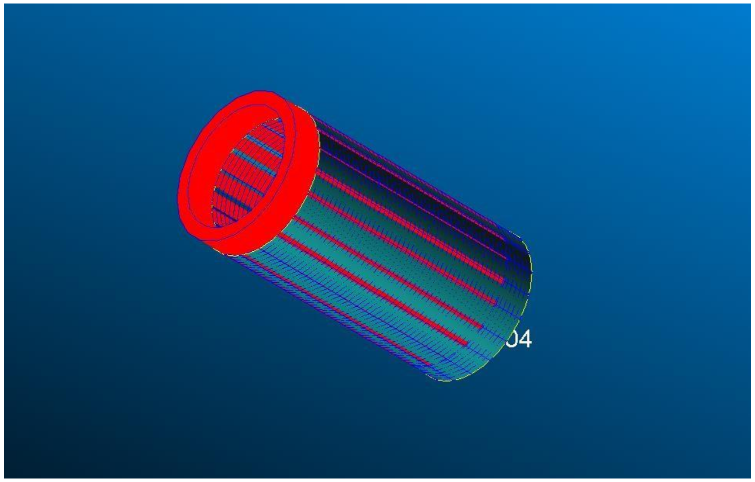


Figure 5: Mode 1, Frequency slightly Null

As we can see the mode 01 which corresponds to a null frequency is not a deformation itself but a displacement.

4. Let's Identify the various type of modes for the cylinder

a. Flexion modes Asymmetrical Modes

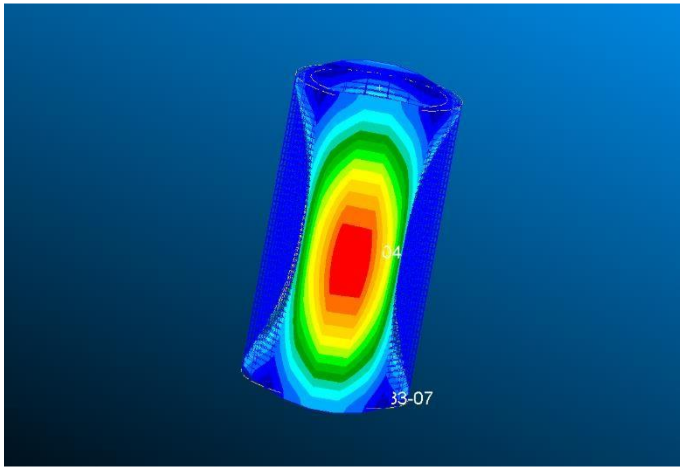


Figure 7: Flexion in mode 2

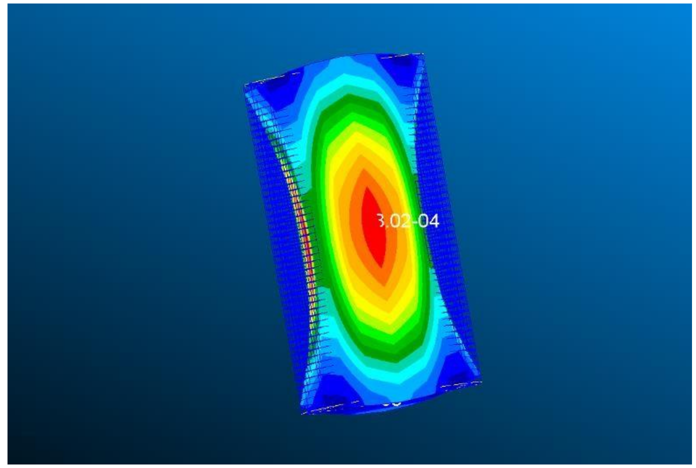


Figure 8: Flexion in mode 3

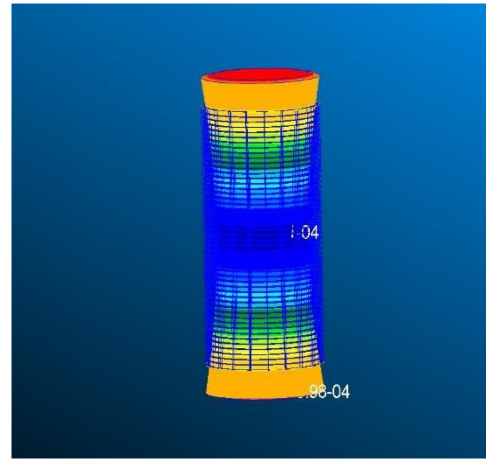


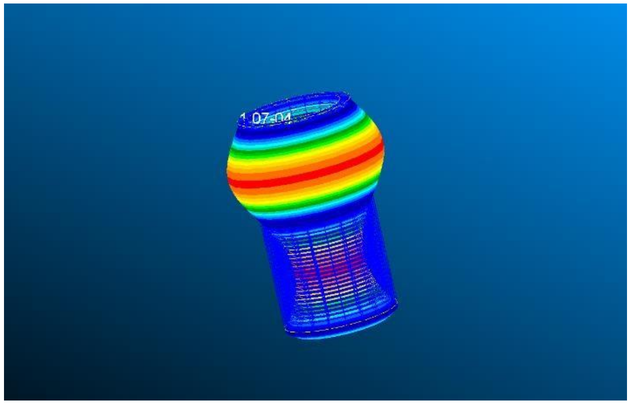
Figure 6: Flexion in mode 19

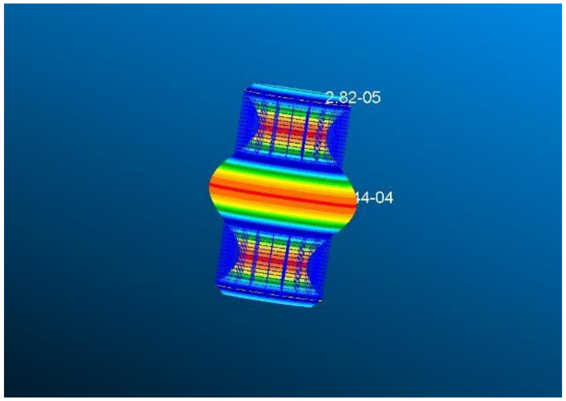
b. Asymmetrical Modes

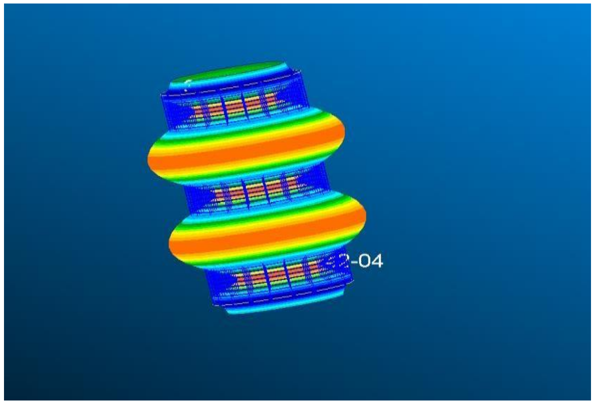
Figure 11: asymmetrical deformation: mode 31

Figure 10: asymmetrical deformation: mode 34

Figure 9:asymmetrical deformation: mode 49







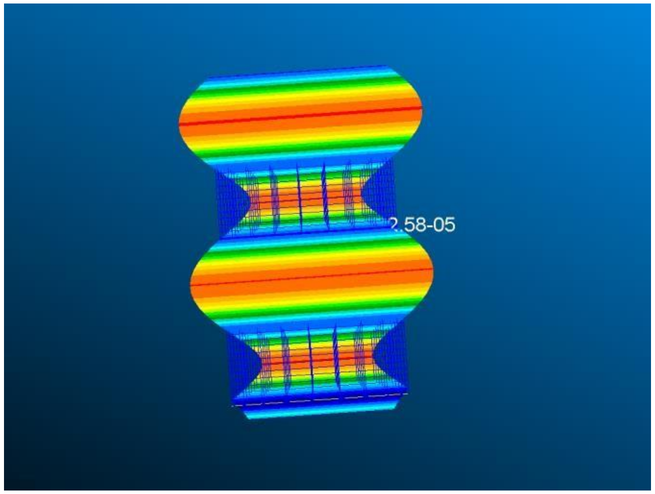


Figure 13: Asymmetrical deformation: mode 41

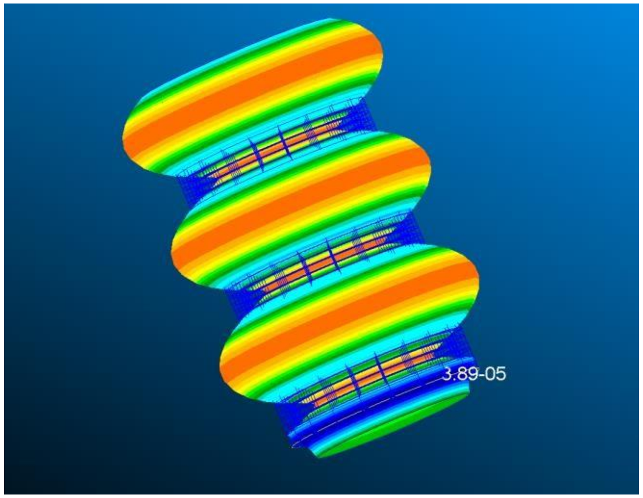


Figure 12:Asymmetrical deformation: mode 65

c. Axial Modes

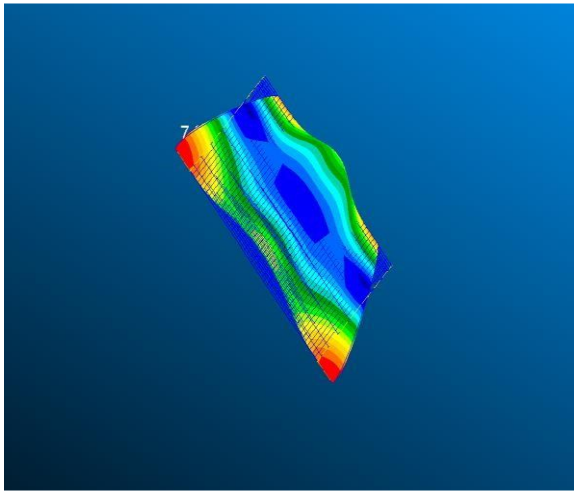


Figure 15: Axial deformation mode 24

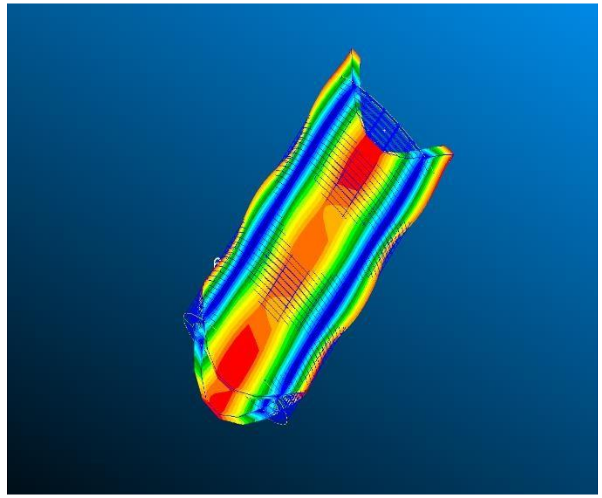


Figure 14: Axial deformation mode 43

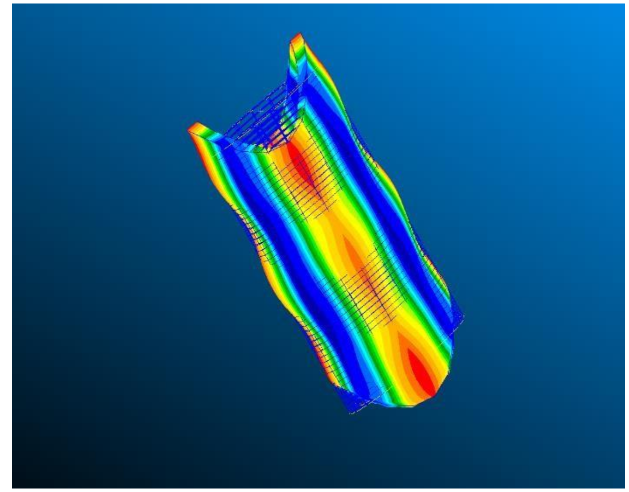


Figure 16: Axial deformation mode 44

An Axial modes or axial deformations is deformation reported along the axis of the solid which here (Z) as we can see we don't have pures axial modes because we can observe little bit of torsion this is due to our defines boundary conditions, but it is not a problem only a fact.

d. Torsion modes

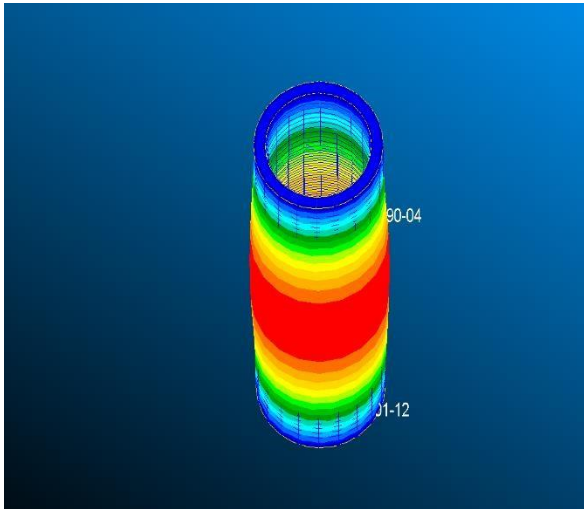


Figure 19: Torsional deformation: mode 8

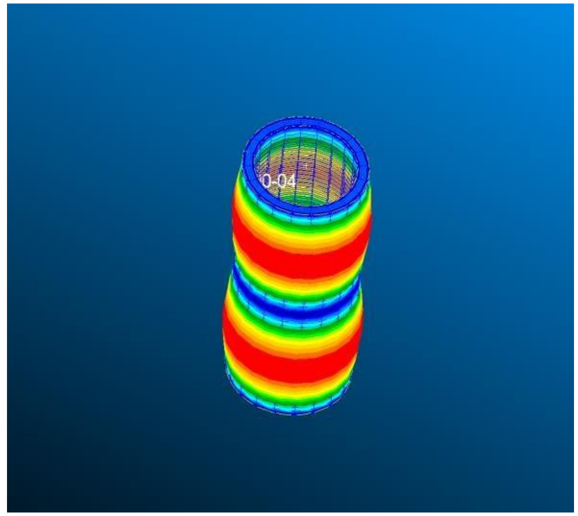


Figure 18: Torsional deformation: mode 26

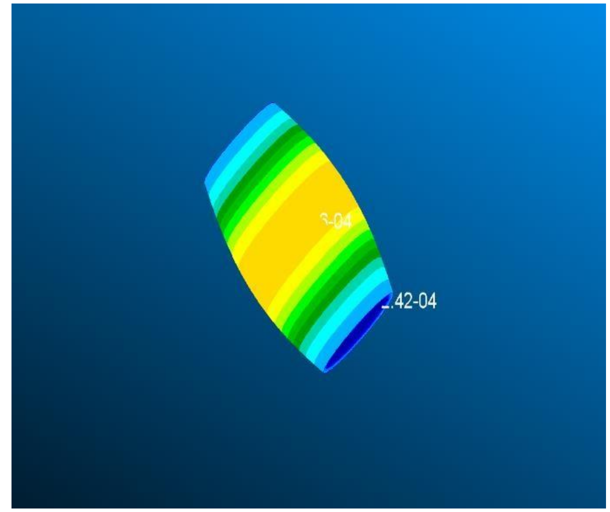


Figure 17: Torsional deformation: mode 42

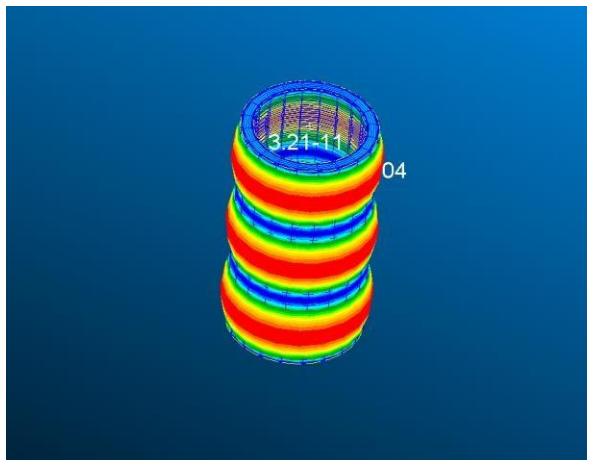


Figure 20: Torsional deformation: mode 58

5. Eigenmodes of the cylinder when it is free

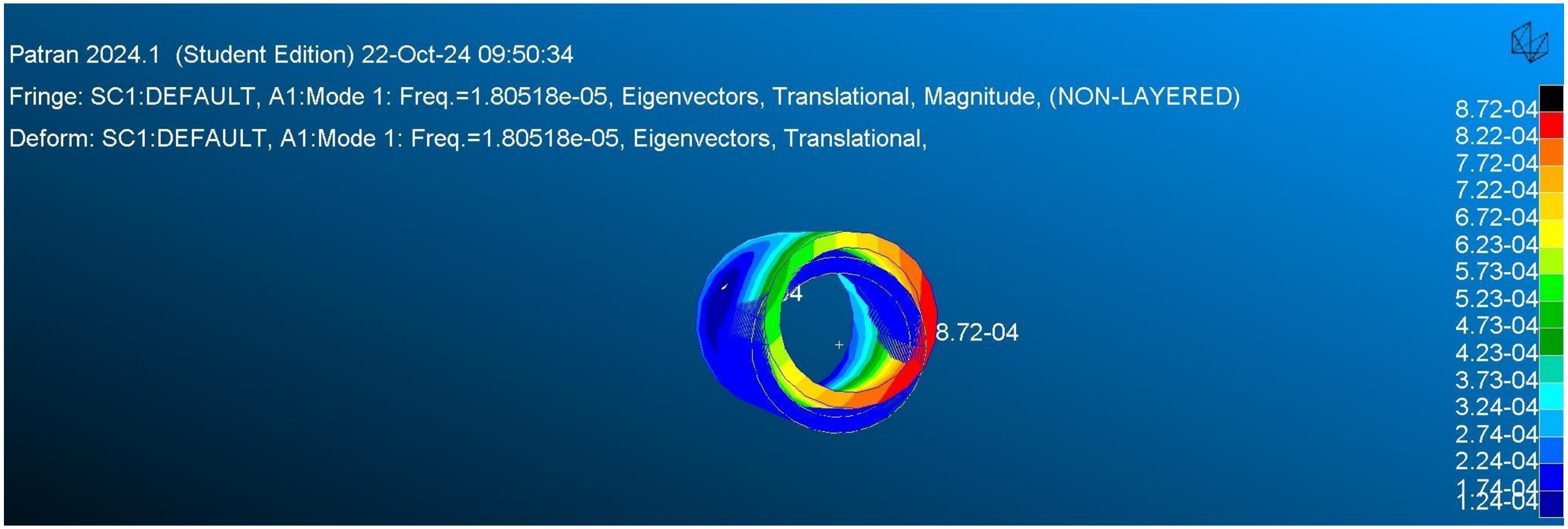
Modes	Frequencies

1	1.81E-05
2	1.55E-05
3	6.75E-06
4	1.60E-06
5	1.28E-05
6	2.00E-05
7	27.7044
8	27.7044
9	29.6883
10	29.6883
11	56.8414
12	56.8414
13	77.7746
14	77.7747
15	78.8607
16	80.0575
17	80.0576
18	83.4114
19	83.4114

The presence of these zero-frequency modes is because the structure is not constrained by boundary conditions. Physically, this means that if we apply a force or a moment to the structure, it can move or rotate freely in space without encountering any resistance.

To eliminate these modes, boundary conditions that prevent rigid body movements must be introduced, such as fixing the structure. As we saw in previous questions, even a simple support can eliminate five of these zero frequencies, so if the structure is fixed, there will be none.

Figure 21: Null frequency: mode 1



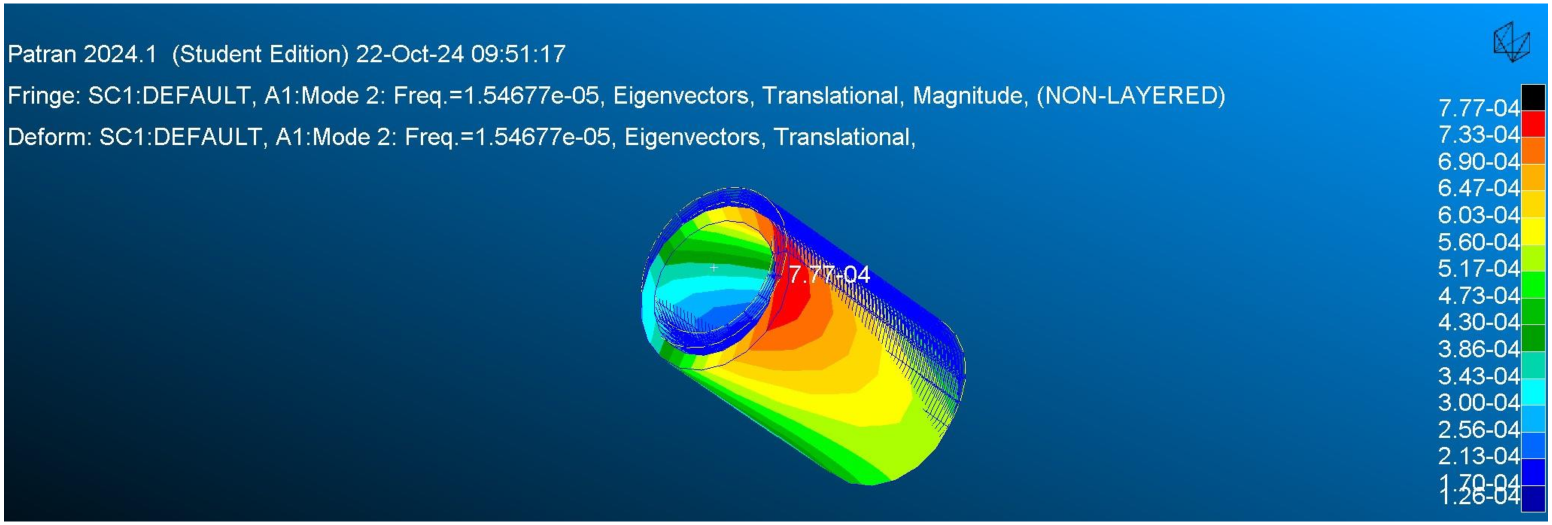


Figure 23: Null frequency: mode 2

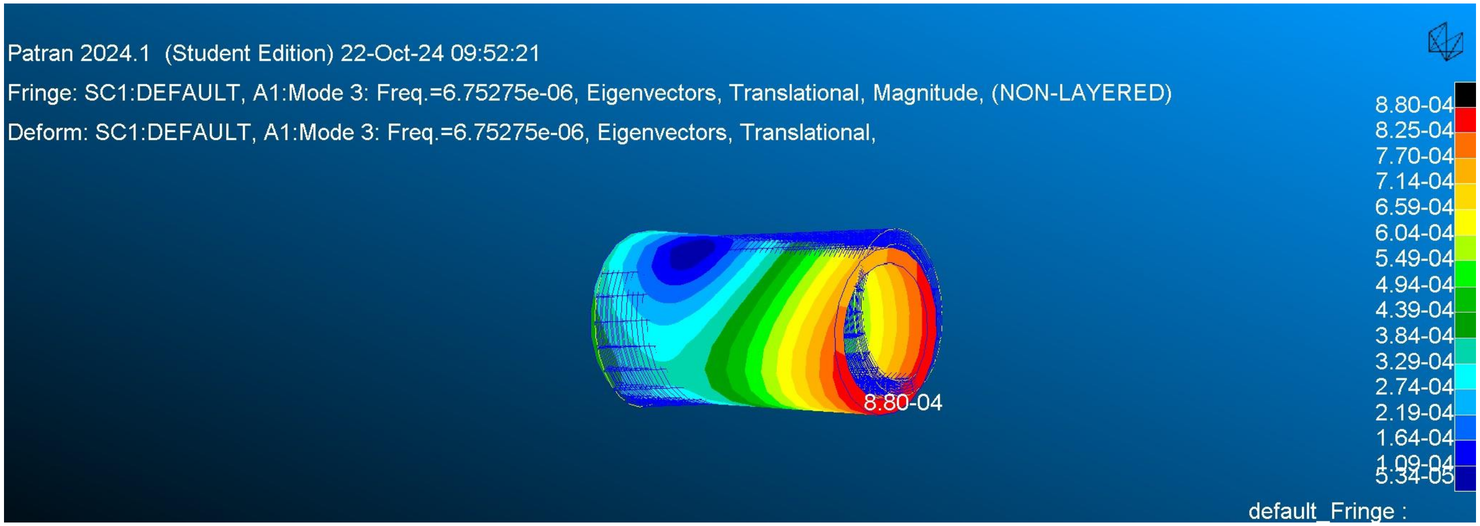
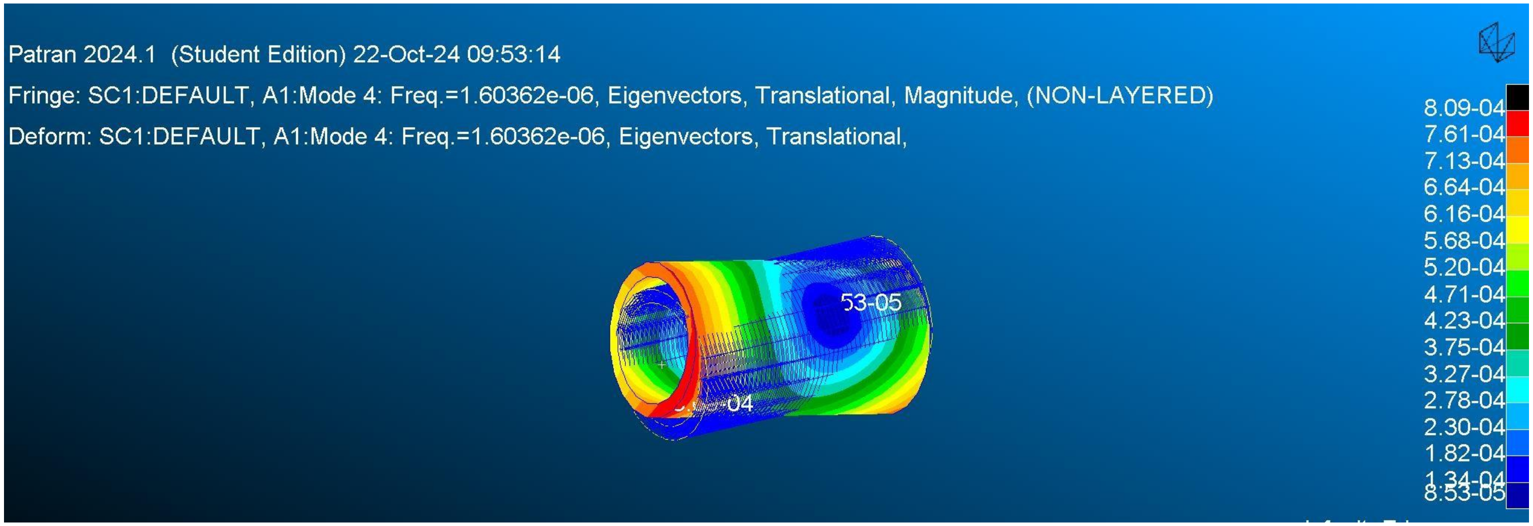


Figure 22:Null frequency: mode 3



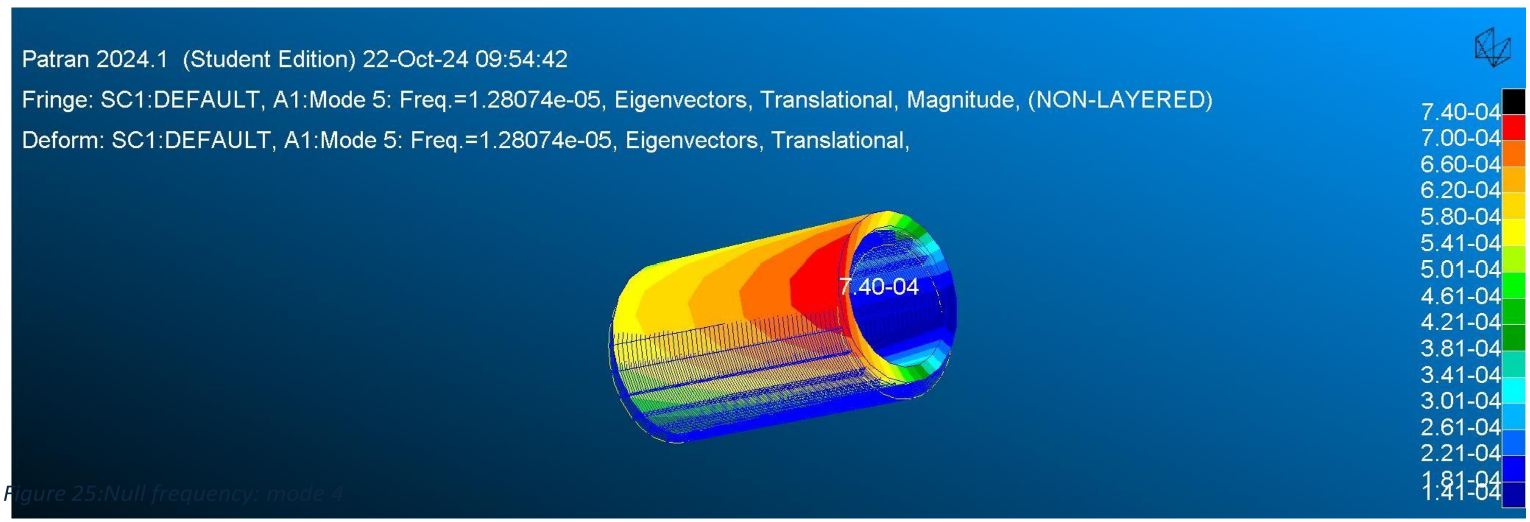


Figure 25:Null frequency: mode 4

Figure 24:Null frequency: mode 5

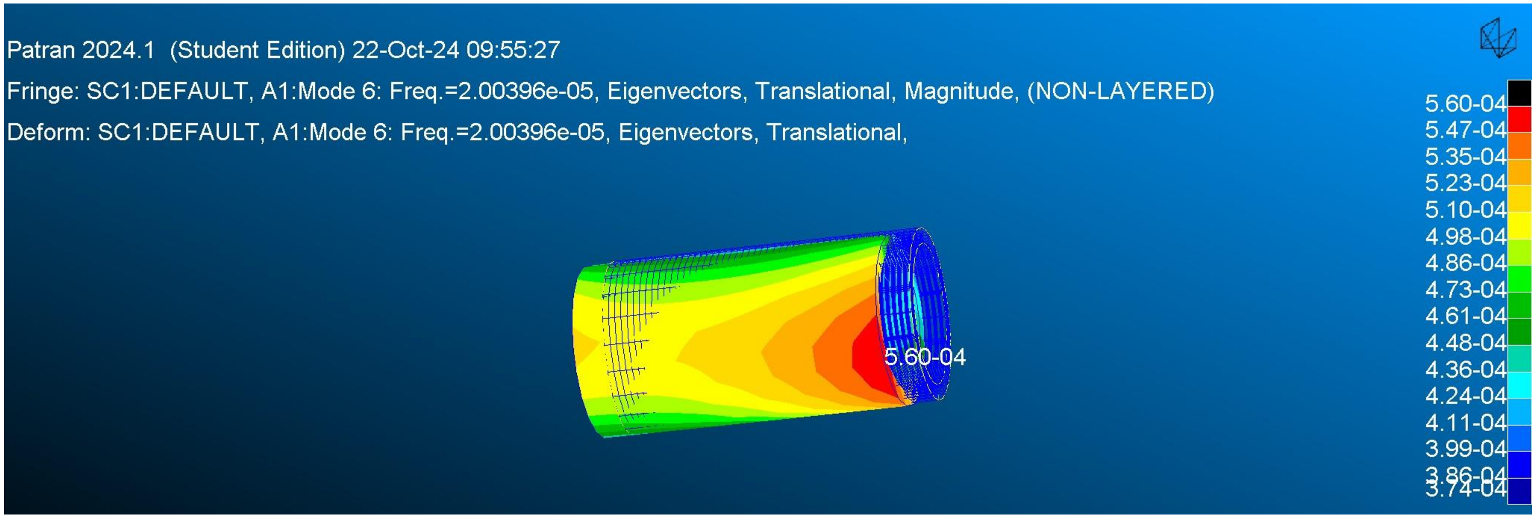


Figure 26:Null frequency: mode 6

IV. Index

Matlab code for Exercice 1:

```
%% Cas d'un appui simple
```

```
clc;close all; clear all;

% Donnees b=0.05;

% largeur h=0.01;

%hauteur

L=1; % beam lenght

D=7800; % density  E=210E11;

%young modulus p=0.33;

%poisson constant m=3.9; %

masse en kg calculee % Calcul

du moment d'inertie

I = (1/12) * b * (h^3);


% Calcul des 10 premières fréquences propres n = 10; % nombre

de fréquences propres désirées frequencies = zeros(1, n); %

initialisation du tableau des fréquences for i = 1:n     frequencies(i)

= ((i * pi)^2 / (2 * pi * L)) * sqrt((E * I) / m); end


% Affichage des fréquences disp(frequencies);


% Calcul des déformations (modes propres) et trace de la courbe x_vals

= linspace(0, L, 100); % 100 points pour un tracé plus lisse

Modes = zeros(n, length(x_vals)); % matrice pour stocker les modes
```

```
for i = 1:n     for j =
1:length(x_vals)         x =
x_vals(j);
        Modes(i, j) = sin((i * x * pi) / L);
end end

% Calcul des déformations (modes propres) x_vals = linspace(0,
L, 100); % 100 points pour un tracé plus lisse
Modes = zeros(n, length(x_vals)); % matrice pour stocker les modes

for i = 1:n     for j =
1:length(x_vals)         x =
x_vals(j);         Modes(i, j)
= sin((i * x * pi) / L);
end end
% Tracé des modes propres séparément for i = 1:n     figure;
plot(x_vals, Modes(i, :), 'DisplayName', ['Mode ' num2str(i)]);
xlabel('Position le long de la poutre (m)');
ylabel('Déformation');     title(['Mode propre ' num2str(i) ' de la
poutre']);     legend show; end %% Cas d'un encastrement
clc;close all; clear all;
```

```
% Donnees b=0.05;

% largeur h=0.01;

%hauteur

L=1; % beam lenght

D=7800; % density  E=210E11; %young

modulus p=0.33; %poisson constant

m=3.9; % masse en kg choisi arbitrairement

% Calcul du moment d'inertie

I = (1/12) * b * (h^3); %

Calcul de lambda n=10;

Lambda_i = [4.73004074, 7.85320462, 10.9956079, 14.1371655, 17.2787597];

for k = 6:n

   Lambda_i(k) = (2*k + 1) * (pi/2); % Correction de l'indice et de la formule

end

% Initialisation du tableau des fréquences frequencies_encastrement

= zeros(1, n);

% Calcul des fréquences for i = 1:n     frequencies_encastrement(i) =

((Lambda_i(i))^2 / (2 * pi * L)) * sqrt((E * I) / m); end

% Affichage des fréquences disp(frequencies_encastrement);

% Calcul des déformations (modes propres) et trace de la courbe x_vals = linspace(0, L, 100); % 100 points pour un tracé

plus lisse sigma = [0.982502215, 1.000777312, 0.999966450, 1.000001450, 0.999999937, 1, 1, 1, 1, 1]; % Coefficients pour

chaque mode Modes_encastrement = zeros(n, length(x_vals)); % Matrice pour stocker les modes
```

```
for i = 1:n
    for j = 1:length(x_vals)

x = x_vals(j);

        % Calcul du mode propre en utilisant la formule fournie

        Modes_encastrement(i, j) = cosh((Lambda_i(i) * x) / L) - cos((Lambda_i(i) * x) / L) ...

- sigma(i) * (sinh((Lambda_i(i) * x) / L) - sin((Lambda_i(i) * x) / L));     end end

% Tracé des modes propres

figure; hold on; for i = 1:n     plot(x_vals, Modes_encastrement(i, :),

'DisplayName', ['Mode ' num2str(i)]); end hold off; xlabel('Position le long de

la poutre (m)'); ylabel('Déformation en encastrement'); title('Modes propres de

la poutre encastrée'); legend show;
```

Matlab code for Exercice 2

```
%% Combined Frequency Calculation and Error Analysis

% Define the material properties and cylinder geometry
E = 210e9;  % Young's modulus (Pa) nu = 0.33;   %
Poisson's ratio rho = 7800; % Density (kg/m^3) R =
1.0;    % Radius of the cylinder (m) h = 0.001;   %
Thickness of the cylinder (m) L = 10;    % Height
(length) of the cylinder (m)
```

```
% Define the indices (i and j) for the first 20 frequencies
modes = [
    NaN, NaN; % frequency1 is not defined
    4, 1; % frequency2
    4, 1; % frequency3
    5, 1; % frequency4
    5, 1; % frequency5
    3, 1; % frequency6
    3, 1; % frequency7
    6, 1; % frequency8
    6, 1; % frequency9
    7, 1; % frequency10
    7, 1; % frequency11
    6, 2; % frequency12
    6, 2; % frequency13
    7, 2; % frequency14
    7, 2; % frequency15
    5, 2; % frequency16
    5, 2; % frequency17
    8, 1; % frequency18
    8, 1; % frequency19
    8, 2; % frequency20
];

% Preallocate frequency array
frequencies = zeros(1, 20);

% Compute the frequencies
for n = 1:20     i =
modes(n, 1);     j =
modes(n, 2);
```

```
   if isnan(i) || isnan(j)
        frequencies(n) = NaN; % Skip undefined frequencies
else
        % Compute lambda_ij
        lambda_ij = sqrt((((1 - nu^2) * ((j * pi * R) / L)^4) + ((h^2) / (12 * R^2)) * (i^2 + ((j * pi * R) / L)^2)^4) / (i^2 + ((j * pi * R) / L)^2);

        % Compute natural frequency f_ij
        frequencies(n) = (lambda_ij / (2 * pi * R)) * sqrt(E / (rho * (1 - nu^2)));
end end

% Display the calculated frequencies disp('Calculated
Analytical Frequencies (Hz):'); disp(frequencies);

%% For the Percentage Error

% Analytical frequencies (Hz) - Already computed above as 'frequencies'
% Nastran frequencies (Hz)
N = [0, 6.190533, 6.19056, 6.83883, 6.83888, 8.71364, 8.7137, 9.05024, 9.05051, ...
     11.8934, 11.8939, 12.67, 12.67, 13.8, 13.8303, 14.0952, 14.0954, 14.7907, ...
     14.7914, 16.1375];

% Initialize percentage error array
percentage_error = zeros(1, length(frequencies));

% Calculate percentage error for each frequency
for i = 2:length(frequencies) % Start from 2 since the first frequency is 0
if ~isnan(frequencies(i)) && ~isnan(N(i))
        percentage_error(i) = ((frequencies(i) - N(i)) / frequencies(i)) * 100;
else
        percentage_error(i) = NaN; % Assign NaN if frequency comparison is not possible
end end
```

```
% Display the analytical frequencies, Nastran frequencies, and percentage error
disp('Nastran Frequencies (Hz):'); disp(N);

disp('Percentage Error between Analytical and Nastran Frequencies (%):'); disp(percentage_error);

%% Plot the Natural Frequencies

% Remove NaN frequencies for plotting valid_indices
= ~isnan(frequencies); valid_frequencies =
frequencies(valid_indices); valid_nastran =
N(valid_indices);
mode_numbers = 2:20; % The mode numbers for valid frequencies

% Plot the natural frequencies (Analytical and Nastran)
figure;
plot(mode_numbers, valid_frequencies, '-o', 'LineWidth', 2, 'MarkerSize', 6, 'DisplayName', 'Analytical Frequencies'); hold
on;
plot(mode_numbers, valid_nastran, '-x', 'LineWidth', 2, 'MarkerSize', 6, 'DisplayName', 'Nastran Frequencies');
xlabel('Mode Number'); ylabel('Frequency (Hz)');
title('Comparison of Natural Frequencies (Analytical vs Nastran)');
legend('Location', 'best'); grid
on;

% Save the plot of frequencies to a PNG file saveas(gcf,
'frequency_comparison_plot.png');
disp('Frequency comparison plot saved as "frequency_comparison_plot.png".');

%% Plot the Percentage Error

% Plot the percentage error
figure;
bar(mode_numbers, percentage_error(2:end)); % Exclude the first zero mode
xlabel('Mode Number'); ylabel('Percentage Error (%)');
```

```
title('Percentage Error between Analytical and Nastran Frequencies'); grid
on;

% Save the plot of percentage error to a PNG file saveas(gcf,
'percentage_error_plot.png');
disp('Percentage error plot saved as "percentage_error_plot.png".');


%% Combined Frequency Calculation and Error Analysis
```