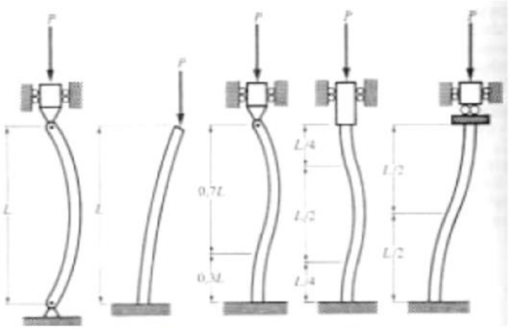

STABILITY STUDY AND LINEAR BUCKLING CALCULATION

FEM Practical Work number 3

ABSTRACT

This lab report explores the phenomenon of buckling and its defining characteristics, focusing on the effect of different boundary conditions. We analyzed buckling in beams under various constraints (fixed-free, pin-pin, and fixed-pin), as well as in a simply supported truss beam and a square plate with symmetrical, non-uniform loading. Both analytical and finite element methods were used, with results showing close alignment, especially for the square plate where the error was minimal (0.2%). This study highlights the importance of understanding buckling behavior to predict and prevent structural deformations.

– Khady Sarah SALL

IPSA, AERO.5

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I. INTRODUCTION

In engineering and structural mechanics, stability plays a fundamental role in the design and evaluation of compression systems, particularly under conditions where load-bearing capacity and structural integrity are crucial. A system is considered in stable equilibrium when any displacement from its equilibrium position invokes restorative forces that return it to this position. However, systems subjected to compressive forces are susceptible to instability, which can lead to abrupt changes in equilibrium as load levels exceed a certain critical threshold. This phenomenon, known as buckling instability, can occur even at stress levels below the material's ultimate strength, defining the maximum permissible loads for such systems.

Buckling analysis is essential to ensure that structures subjected to compression remain within safe load limits. For structural engineers, it is crucial to determine these limits accurately, as buckling can lead to sudden and potentially catastrophic failure. This report examines the fundamental principles of buckling, utilizing analytical and computational methods to assess structural stability under compressive loads. The study includes a detailed exploration of Euler's formula, which provides a basis for calculating the critical load for columns with various boundary conditions, and extends to finite element methods (FEM) to address complex structures.

The purpose of this report is to analyze the maximum allowable compressive stresses in structures, specifically beams, trusses, and plates. Through a combination of analytical calculations and finite element analysis (FEA) using NASTRAN software, we will assess the critical load thresholds under different boundary conditions, comparing theoretical results with FEM data to verify accuracy and understand the behavior of each structural element under load.

II. AIMS AND OBJECTIVES

The primary aim of this report is to analyze the buckling behavior of structural elements under compressive loads using both analytical methods and finite element analysis (FEA). By comparing theoretical calculations with computational simulations, this study aims to verify the reliability of critical load predictions for different boundary conditions and structural configurations.

Objectives:

- Understand Buckling Principles
- Apply Analytical Buckling Formulas
- Explore Finite Element Analysis (FEA) for Buckling
- Compare Analytical and FEA Results
- Analyzing Influence of Boundary Conditions on Stability
- Evaluate Structural Safety Against Buckling

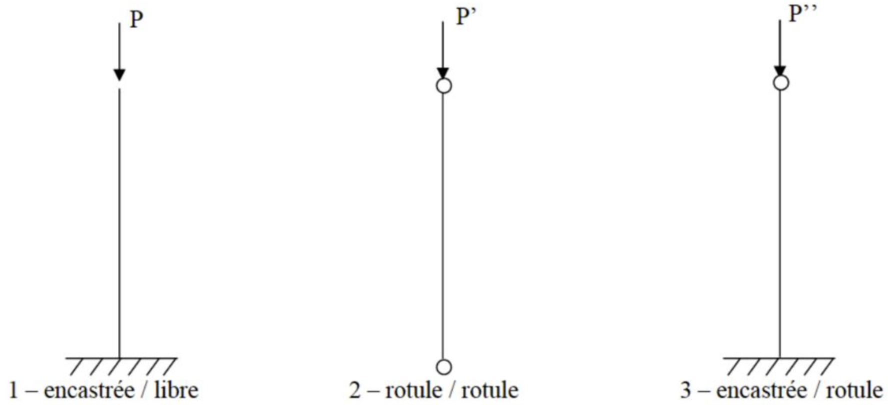
III. Buckling Calculation

A. Beam Buckling Computation

1. Analytical Calculation

In this part, we will calculate the maximum force that the beam could withstand without undergoing plastic deformation and using Euler's formulas, calculate the critical force that can be applied to the structure.

Studied cases:



Given Data

- Beam length , L = 4m
- Diameter of the cross-section ,D = 0.04m
- Young's modulus ,E = 200E9Pa
- Yield strength = 600E6Pa

i. Maximum force

To calculate the maximum force this steel beam can withstand without undergoing plastic deformation, we'll determine the maximum bending moment based on the material's yield strength and then calculate the maximum force that results.

Calculation Steps

-Moment of Inertia (I)

The moment of inertia for a solid circular cross-section is:

$$I = \frac{\pi D^4}{64}$$

This implies that $I = 1.256E - 7m^4$

-Maximum Bending Moment

To ensure the beam does not exceed its elastic limit, the maximum bending moment is calculated as:

$$M_{max} = \sigma_{R0.2} \frac{I}{r}$$

where r is the radius

This implies that $M_{max} = 3768Nm$

-Maximum Force

Assuming a simply supported beam with a central point load, the maximum force is given by:

$$F_{max} = \frac{4M_{max}}{L}$$

This implies that $F_{max} = 3768N$

ii. Critical force

The formula for the critical buckling force is given by:

$$P_{cr} = \frac{\pi^2 EI}{KL^2}$$

For each case, we will calculate the critical force using the values of E, I, L, and the corresponding K.

Given data:

- Young's Modulus (E) = 200E9 Pa

- Moment of Inertia (I) = 3768 Nm

- Length (L) = 4 m

Clamped-Free

K = 2

The critical force for this boundary condition corresponds to a column that is clamped at one end and free at the other.

The critical force is approximately 1.43 MN.

Ball Joint-Ball Joint (Pinned-Pinned)

K = 1

This boundary condition corresponds to a column that is pinned at both ends, allowing rotation but not translation.

The critical force is approximately 5.72 MN.

Clamped-Ball Joint (Clamped-Pinned)

K = 1.41

This boundary condition corresponds to a column that is clamped at one end and pinned (or has a ball joint) at the other.

Clamped-Ball Joint (Clamped-Pinned): The critical force is approximately 11.66 MN.

2. Finite Elements Method Calculation

Clamped-Free

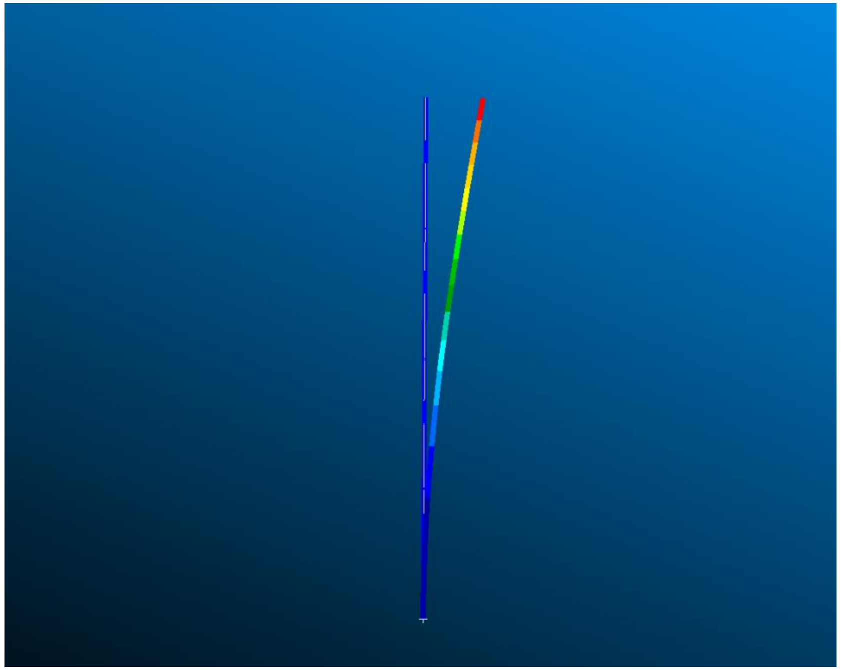


Figure 1: Flexural Buckling

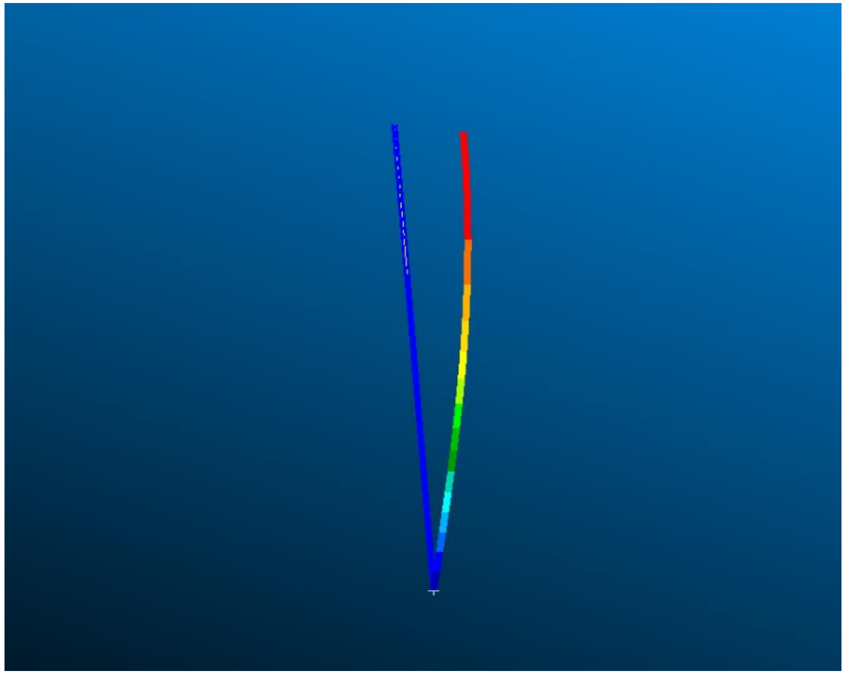


Figure 2: Torsional Buckling

Ball joint-Ball joint

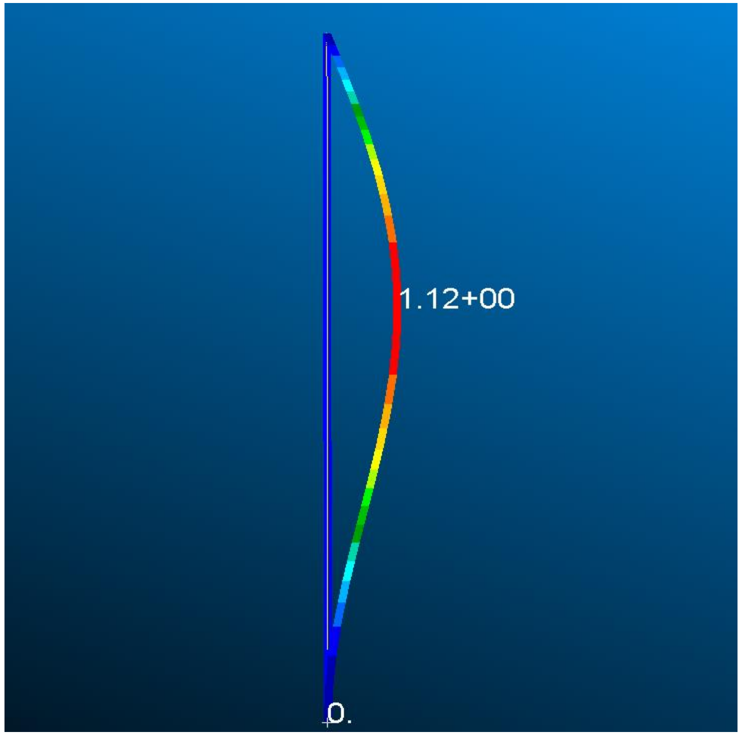


Figure 3: Torsional Buckling

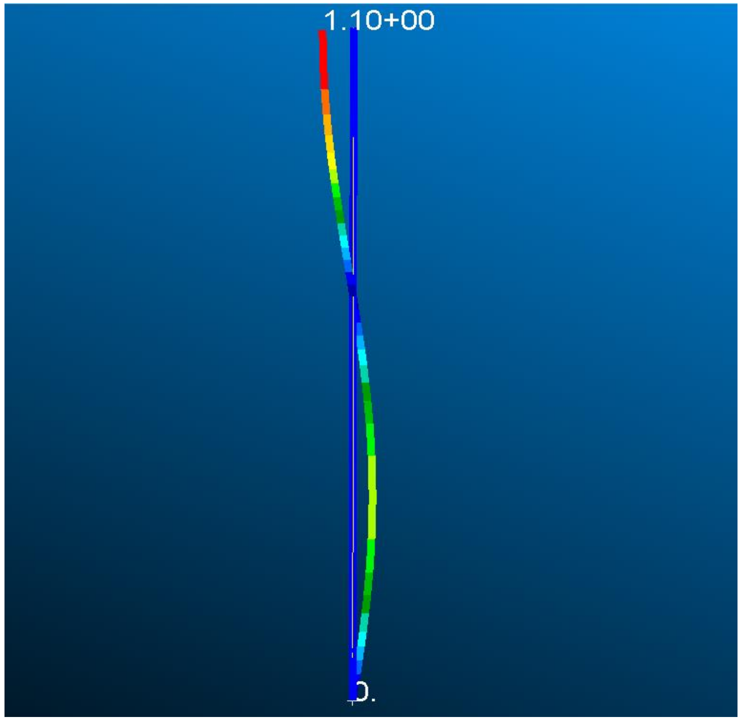


Figure 4: Flexural Buckling

Clamped- Ball joint

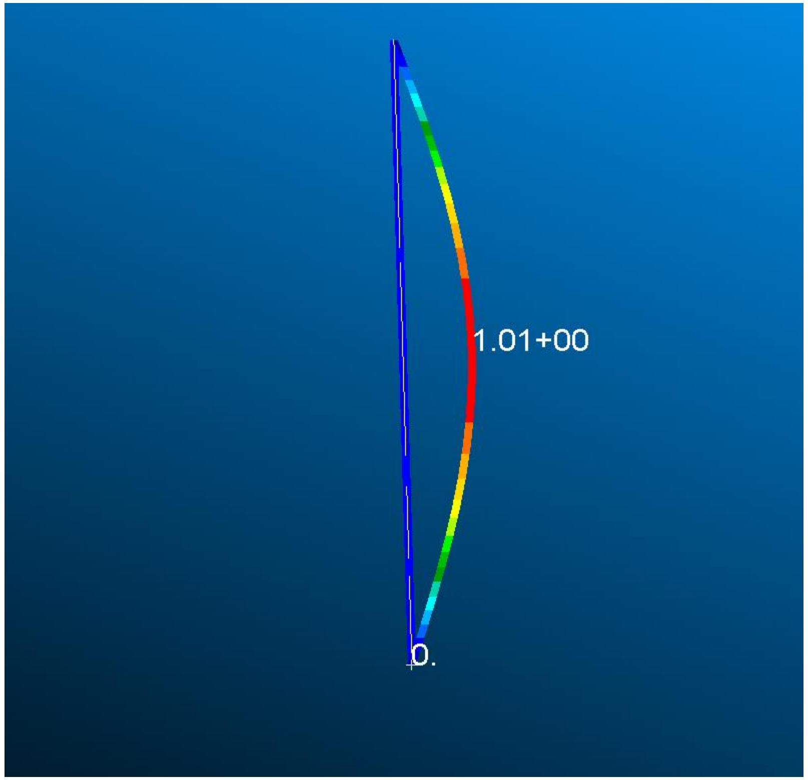


Figure 5: Flexural Buckling

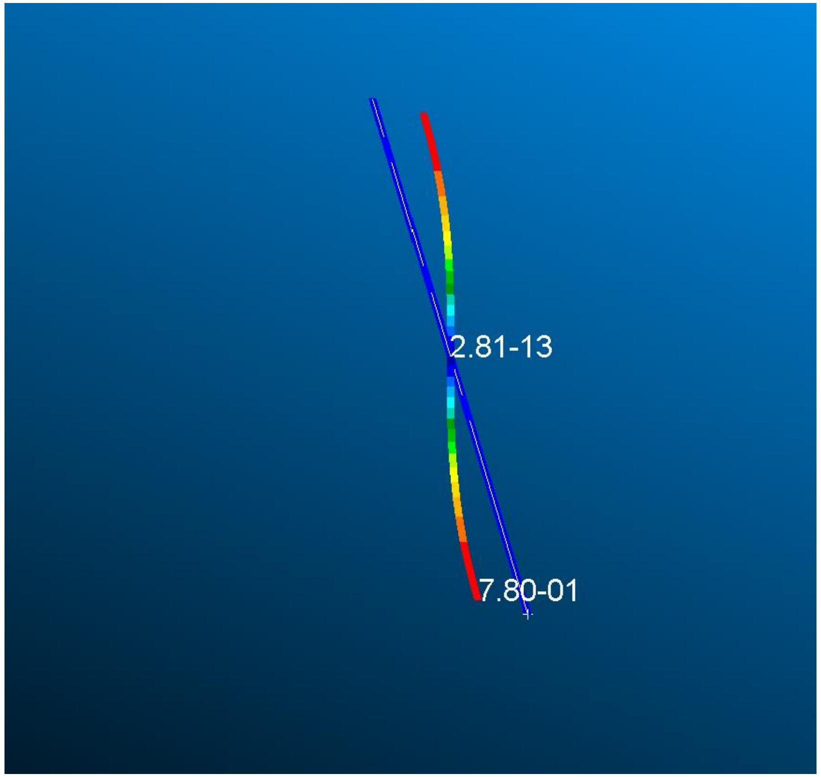


Figure 6: Torsional Buckling

Comparison

Cases	Analytical Results	FEM Results
Clamped-Free	1.43 MN	1.40822
Ball joint-Ball joint	5.72 MN	5.64612e+06
Clamped-Ball joint	11.66 MN	1.13651e+07

Comments

We observe that the critical force remains the same across all cases, regardless of whether we use the analytical method or the finite element method (FEM). This consistency in results is because we are studying linear buckling in simple, idealized configurations. Analytical values for buckling are precise, and numerical methods, like FEM, reproduce them accurately. A noticeable difference between the analytical and numerical methods could arise in more complex cases, such as those involving nonlinear geometry, second-order effects, or non-homogeneous materials. The similarity of results here is attributed to the classic cases studied and the suitability of the simplified assumptions in the analytical approach.

B. Buckling Computation of a Truss Bridge

Geometry

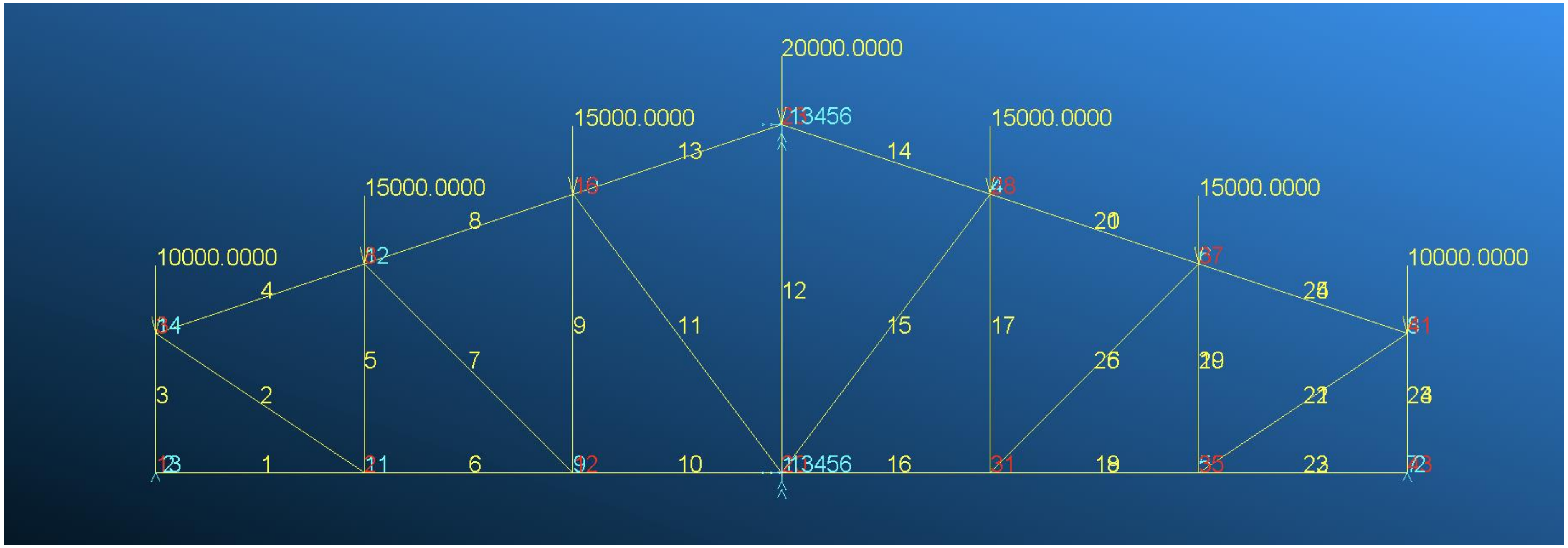


Figure 7: Geometry of the trussbridge

Results

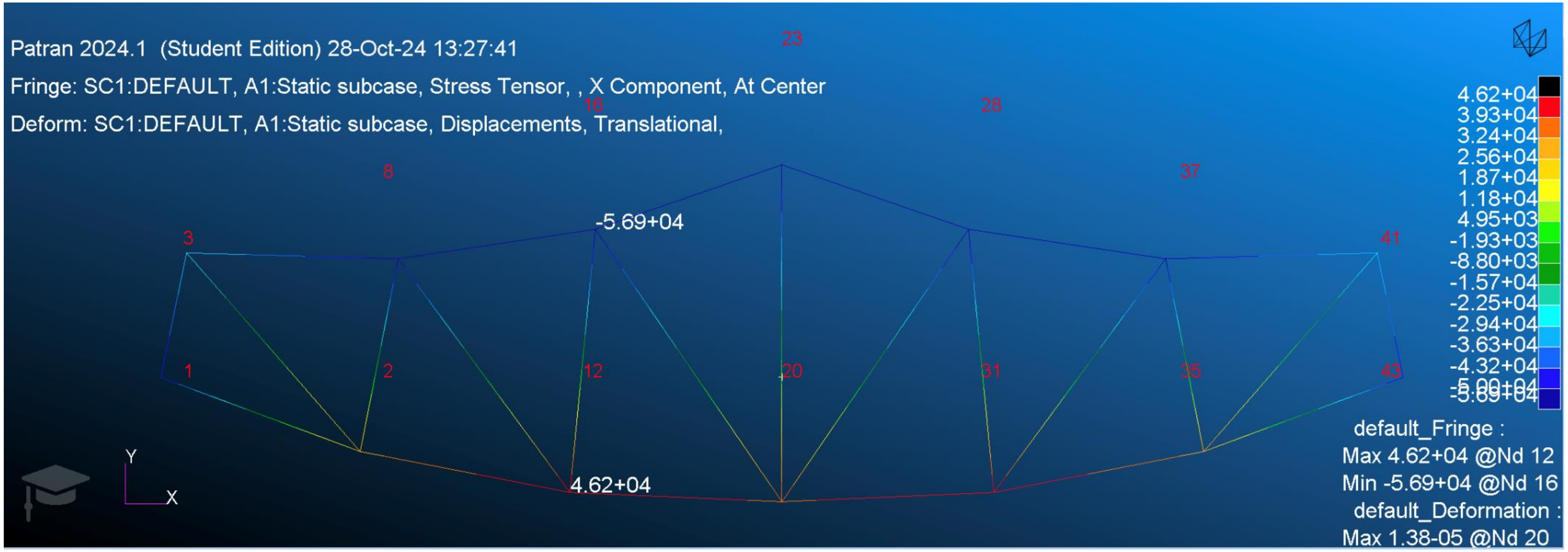


Figure 8: Stress tensor distribution on the X component

Interpretation

The X-component stress distribution in the structure reveals distinct regions of tension and compression, pinpointing critical areas under applied loads. At Node 12, the maximum tensile stress reaches 4.62E4, indicating significant stretching along the X-axis, likely due to bending forces. On the other hand, Node 16 experiences the highest compressive stress, measuring -5.69E4, suggesting that this area is subjected to substantial compressive forces that could lead to buckling if the material's strength limits are exceeded. Bar 13 has been identified as the most compressed element, bearing a maximum normal force of -5.69E4. This considerable compressive load highlights the necessity for meticulous attention to Bar 13's design to ensure it can endure the forces applied without risk of structural failure. The color gradient across the structure smoothly transitions from tension to compression, illustrating a bending pattern with a neutral axis where stress approaches zero. The displacement vectors further show deformation under load, with high-stress regions exhibiting notable movement.

Verification of no Buckling

Length of the most compressed bar	$L_0 = 3.16m$
Buckling length of the most compressed bar	$L_f = 3.16m$
Inertia of the circular section	$I = 2E - 6m^4$
Area of the circular section	$S = 5.03E - 3m^2$
Radius of gyration	$i = \left(\frac{I}{S}\right)^{0.5} = 1.99E - 2m$
Elancement	$\lambda = \frac{L_f}{i} = 158.79$
Critical buckling stress	$\sigma_{cr} = \frac{E\pi^2}{\lambda^2} = 82.2MPa$
Max stress in the part	$\sigma = \frac{N\_max}{S} = 5.69MPa$
Comparison between σ_cr and σ:	$\sigma_{cr} > \sigma$
Conclusion on structure buckling:	*The structure does undergo buckling*

Where;

$L_f = K * L_0$ *and* $K = 1$ *because we have a ball joint − ball joint boundary condition.*

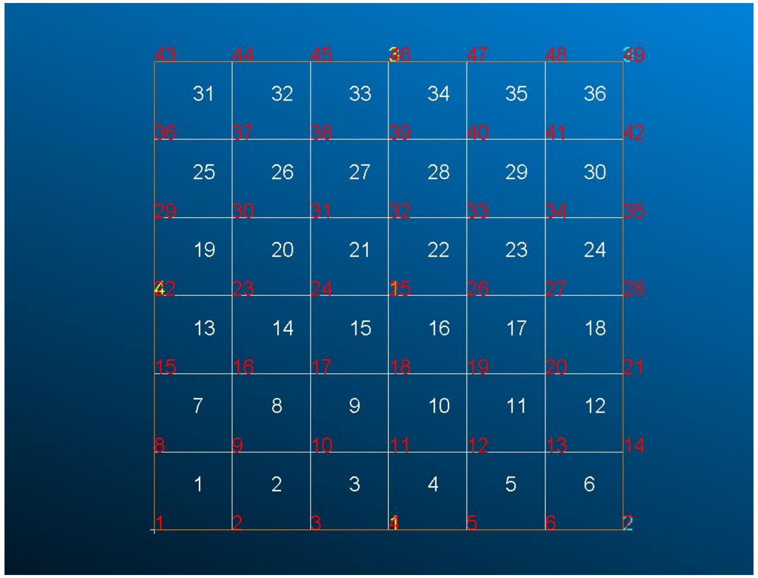
$S = \frac{\pi}{4} * D^2$ *and* D *which is the diameter has a value of* $8E - 2m$

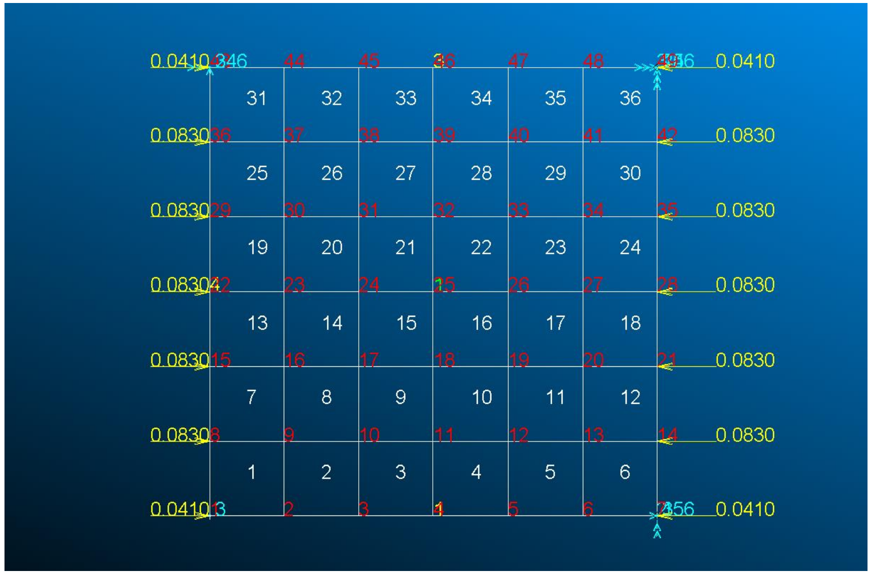
$N_{max} = \sigma_{max} * \pi * \frac{D}{4} = 28601N$

C. Buckling Computation of a Plate

1) FEM

Geometry, meshing and Force distribution (Nodal)





Buckling, eigenvectors

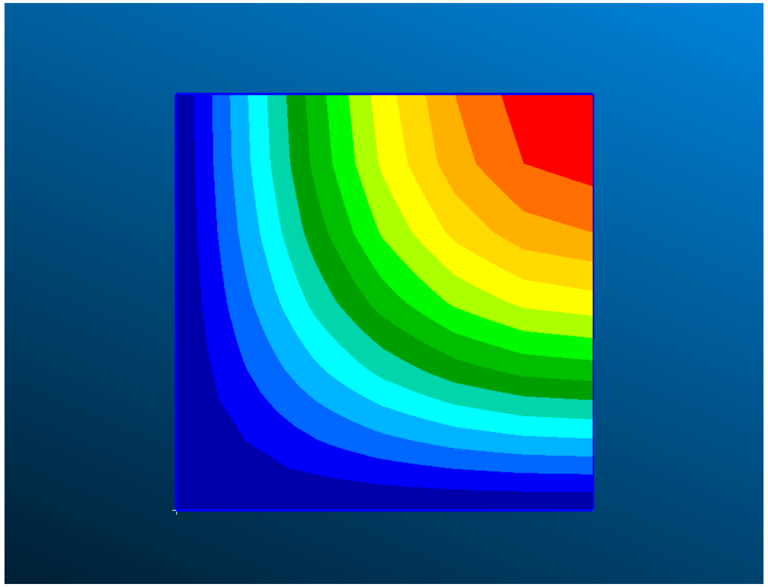


Figure 10: Translational

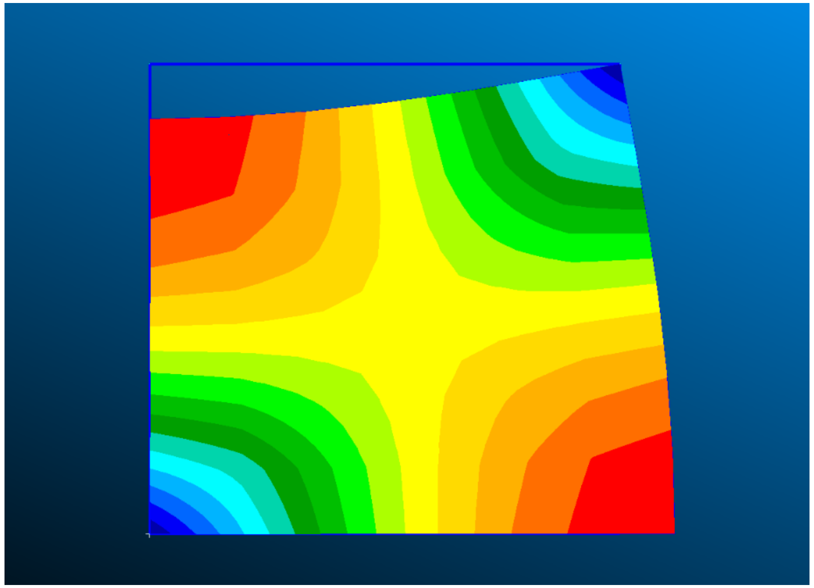


Figure 9: Rotational

Constraints

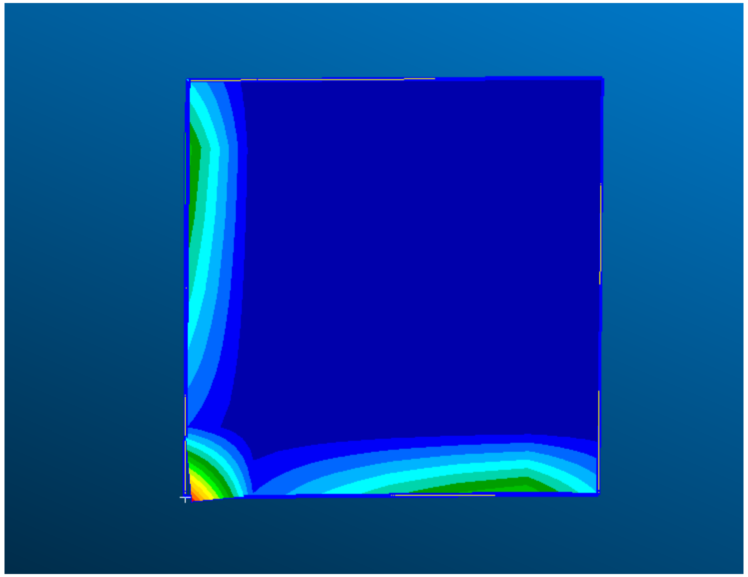


Figure 12: Translational

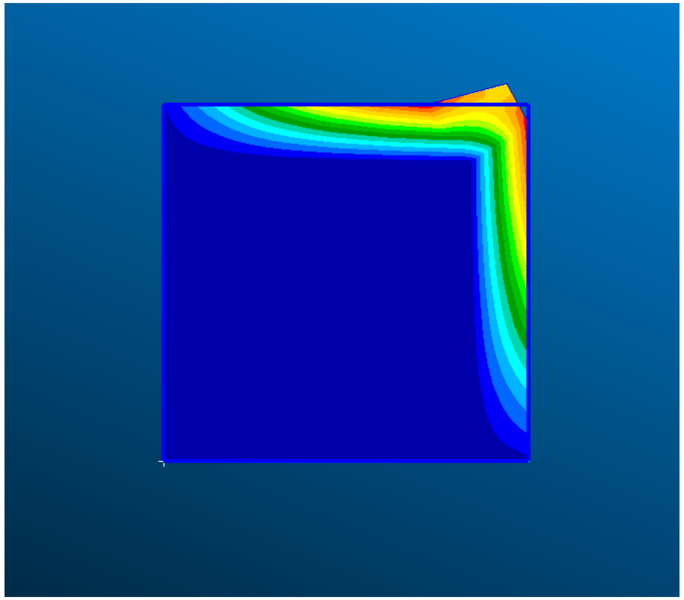


Figure 11: Rotational

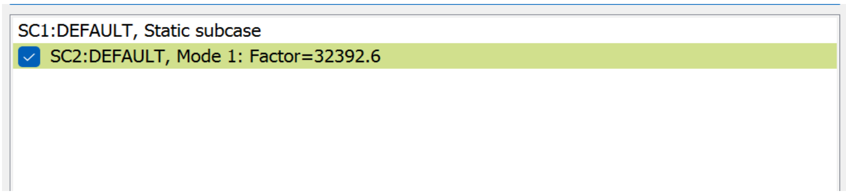


Figure 13: Critical Buckling Load

The Critical Buckling Load with the finite elements methods is: $\boxed{\textbf{Fcr=32392.6 N}}$

2) Analytical Method

Data

Dimensions de la plaque : a=b=1 m

Épaisseur de la plaque : h=5 mm=0.005 m

Module de Young : E=70 GPa

Coefficient de Poisson : ν=0.33

Analytical Formula for Critical Buckling Load

$$F_{cr} = \frac{D\pi^2}{a^2}\left(i + \frac{1}{i}\right)^2$$

$$D = \frac{Eh^3}{12(1-\gamma^2)}$$ the flexural rigidity coefficient of the shell

The critical buckling load for a simply supported plate can be calculated is:

Fcr=32304.28253826054 **N**

3) Computing the error

The error is computed using the following formula: **(Analytical result- FEM result)/ Analytical result**

$\boxed{\textbf{E=0.002734}}$

Once again due the simplified case and simple conditions the analytical result is very accurate and close to the fem results no mater the parameters fem method considered. We expected a small error observing both results but also the studied case.

IV. Conclusion

In this lab assignment, we studied the phenomenon of buckling and the characteristics that define it. *(Buckling refers to the sudden bending or deformation of a structure under compressive stress, influenced by factors such as material properties, length, cross-sectional area, and boundary conditions).

We analysed buckling behaviour specifically for beams under three different boundary conditions: (1) one end fixed and the other free, (2) both ends in pin-joint connection, and (3) one end fixed while the other is in a pin joint. We also examined buckling in a simply supported truss beam. Finally, we studied the buckling of a square plate where the forces were not uniformly distributed across the nodes but were symmetrical.

For each case, we conducted both analytical and numerical studies using the finite element method. We found that in each instance, the analytical and numerical results were closely aligned, with discrepancies within a five percent margin. Specifically, for the square plate case, the error was as low as 0.2%, demonstrating that our analytical simplifications were highly accurate and matched well with our problem setup.

The primary difficulty we encountered was with the truss beam, particularly its modelling and boundary conditions setup. However, overall, this lab assignment allowed us to gain a clear understanding of the buckling phenomenon in various structures. It also underscored the importance of studying buckling to anticipate and prevent potential deformations in structures over time. We feel we have assimilated this knowledge effectively.

V. Index

Critical buckling load computing for exercise 3 (C)

```
% Paramètres de la plaque
a = 1; % longueur du côté de la plaque en mètres
h = 0.005; % épaisseur de la plaque en mètres
E = 70e9; % module de Young en Pascal
nu = 0.33; % coefficient de Poisson
% Calcul de la rigidité en flexion
D = (E * h^3) / (12 * (1 - nu^2));
% Indice de mode
i = 1;
% Calcul de la charge critique de flambage
F_cr = (D * pi^2 / a^2) * (i + 1/i)^2;
% Affichage du résultat
fprintf('La charge critique de flambage est : %.2f N\n', F_cr);
```