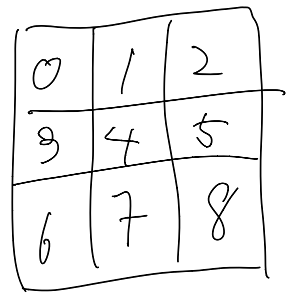
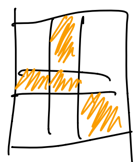
We prepare 9 bits for light config, 9 bits for solution candidates, and 1 bit for the phase flip ($|-\rangle$)



$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ 012 345 678

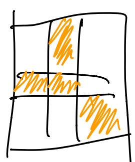
If light config is $|010\ 110\ 001\rangle$, then the initial state is

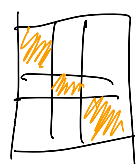


Let U_k represent pushing button k

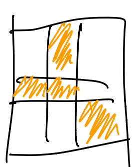
e.g.

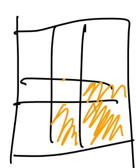
$\rightarrow$ $U_0 |010\ 110\ 001\rangle = |100\ 010\ 001\rangle$





$U_4 |010\ 110\ 001\rangle = |000\ 001\ 011\rangle$





lemma

Λ

$\Gamma := U_j U_k - U_k U_j$

$\Rightarrow \forall j,k \in \{0,1,\cdots,8\} \quad [U_j, U_k] = 0$

$\forall j \in \Lambda \quad U_j^2 = 1$

$\{U_j\}_{j\in\Lambda}$: Generating set of solution space.

The order of pushing buttons does not affect the final lights state.

Pushing the same button twice is equivalent to pushing the button 0 times.

$\Rightarrow$ The solution, if exists, can be represented by 9-bit string.

The state prepared before the Grover's loop is written as below

$$|\psi^{(0)}\rangle = \underbrace{|\text{light config}\rangle_c}_{\text{e.g. } |\text{light config}\rangle = |010\ 110\ 001\rangle_c \ \Rightarrow\ = X_1 X_3 X_4 X_8 |0\rangle_c} \otimes \underbrace{H^{\otimes 9}|0\rangle_s}_{= \frac{1}{\sqrt{2^9}} \sum_{j=0}^{2^9-1} |j\rangle_s} \otimes |-\rangle_f$$

To realise U_j in a quantum circuit, we use $\tilde{U}_j$

$$\tilde{U}_0 := CX_{\underbrace{s_0}_{\text{0th bit of } |\ \rangle_s} c_3}\, CX_{s_0 \underbrace{c_1}_{\text{1st bit of } |\ \rangle_c}}\, CX_{s_0 c_0}$$

$$\tilde{U}_1 := CX_{s_1 c_4}\, CX_{s_1 c_2}\, CX_{s_1 c_1}\, CX_{s_1 c_0}$$

$$\vdots$$

$$\tilde{U}_8 := CX_{s_8 c_5}\, CX_{s_8 c_7}\, CX_{s_8 c_8}$$

$$\longrightarrow \quad \forall j,k \in \Lambda \quad [\hat{U}_j, \hat{U}_k] = 0 \ \wedge \ \hat{U}_j^2 = 1$$

$$\tilde{U} := \prod_{j \in \Lambda} \hat{U}_j$$

$$\tilde{U} |\psi^{(0)}\rangle = \frac{1}{\sqrt{2^9}} \left\{ \underbrace{|0\rangle_c}_{\text{all the bits are } 0} |\text{Solution}\rangle_s + \sum_{j \neq \text{Solution}} \underbrace{|\neg 0_j\rangle_c}_{\text{1 or more bits are 1.}} |j\rangle_s \right\} |-\rangle_f$$

$MCX_{\{c\}f}$: Multi-Controlled X whose controls are $|\ \rangle_c$ and target is $|-\rangle_f$

$$\overline{MCX_{\{c\}f}} := X_c^{\otimes 9} \, MCX_{\{c\}f} \, X_c^{\otimes 9}$$

$$\Rightarrow \underbrace{\tilde{U} \, \overline{MCX_{\{c\}f}} \, \tilde{U}}_{\text{The oracle !!}} |\psi^{(0)}\rangle = \frac{1}{\sqrt{2^9}} |\text{big con}\rangle_c \left\{ - |\text{sol}\rangle_s + \sum_{j \neq \text{sol}} |j\rangle_s \right\} |-\rangle_f$$