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Office Hour: 11:00 AM - 12:00 PM on Tuesday.

Def) (Propositional Logic)

A proposition is a collection of statements
that is either true or false.

e.g.) " $12+9=3-2$ " is false proposition.

" 92 is large number" is not proposition

(Connectives)

Five connectives:

- Or (\vee)
- And (\wedge)
- Negation / Not (\neg)
- Implication / If-then (\rightarrow) *(when this is true,
we use " \Rightarrow " instead of " \rightarrow ")*
- If and only If (\leftrightarrow) \Leftrightarrow

(First)
Mathematical Induction is a Mathematical proof technique.

(It is used to prove that a statement $p(n)$ holds for every $n \in \mathbb{N}$)

Let $p(n)$ be a statement for $n \in \mathbb{N}$.

- ① Show that $p(1)$ is true. (Base case)
- ② Suppose that for $k \in \mathbb{N}$, $p(k)$ is true.
Prove that $p(k+1)$ is true.

Then, the Mathematical Induction gives that

$p(n)$ holds $\forall n \in \mathbb{N}$. $\xrightarrow{\text{for every}}$

e.g) Prove that $1+2+\dots+n = \frac{n(n+1)}{2}$, $\forall n \in \mathbb{N}$.

Pf) Step 1) If $n=1$, (L.H) =

$$(\text{R.H}) = \frac{1 \cdot 2}{2} = 1$$

It holds for $n=1$.

Step 2) Suppose that for $k \in \mathbb{N}$, $1+2+\dots+k = \frac{k(k+1)}{2}$.

Need to show that $1+2+\dots+k+1 = \frac{(k+1)(k+2)}{2}$.

$$1+2+\dots+k+1 = (1+2+\dots+k) + k+1 \quad // \frac{k(k+1)}{2}$$

$$= \frac{k(k+1)}{2} + k+1 \quad \text{because of } \textcircled{1} \text{ assumption}$$

$$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}.$$

$$1+2+3+\cdots+k+1 = \frac{(k+1)(k+2)}{2}.$$

By the Mathematical Induction,

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}, \quad \forall n \in \mathbb{N} \quad \text{④}$$

Strong Induction (Second Mathematical Induction)

Let $p(n)$ be a statement, for $n \in \mathbb{N}$.

① Show that $p(1)$ is true.

② Suppose that for $k \in \mathbb{N}$, $p(1), p(2), \dots, p(k)$ are true.

Prove that $p(k+1)$ is true.

Then, the Strong Induction gives that $p(n)$ holds,
 $\forall n \in \mathbb{N}$.

e.g) Show that every natural number $n > 1$ is the product of prime powers.

pf) Step 1) 2 is already prime. (Base case)

It holds for $n=2$.

Step 2) Suppose that for $k \in \mathbb{N}$, $2, 3, 4, \dots, k$ are the product of prime powers.

Need to show

(N.T.S) $k+1$ is the product of prime powers.

(Case 1) $k+1$ is already prime number.

done!

(Case 2) $k+1$ is composite.

Then, $k+1 = a \cdot b$ for some $1 < a, b < k+1$

By the assumption, a, b are the product of prime powers.

This implies that $a \cdot b$ is the product of prime powers

implies

$\Rightarrow k+1$ is the product of prime powers

By Strong Induction, every natural number $n > 1$

is the product of prime numbers



Well-ordering principle (axiom)

Every non-empty set of positive integers
contains a least element.

Proof by Contradiction

(Mathematical Proof technique)

Suppose that the proposition is false.

If this leads a contradiction, then the proposition
is true.

e.g) Show that there are infinitely many prime numbers.

Pf) Suppose that there are finitely many prime numbers.

Let $P = \{P_1, P_2, \dots, P_n\}$ be a collection of
prime numbers.

Consider $q = (p_1 \cdot p_2 \cdots p_n) + 1$.

Since $q \notin P$, q is not prime number.

This implies that there exists $p' \in P$

such that p' divides q .

Since $p' \in P$, p' divides $p_1 \cdot p_2 \cdots p_n$

So, p' divides q and $p_1 \cdot p_2 \cdots p_n$

Then, p' divides $q - (p_1 \cdot p_2 \cdot p_3 \cdots p_{n-1} \cdot p_n)$

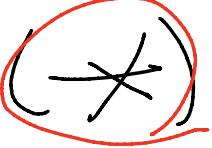
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$(p_1 \cdot p_2 \cdot p_3 \cdots \cancel{p_n \cdot p_n}) + 1 - (p_1 \cdot p_2 \cdot p_3 \cdots \cancel{p_n \cdot p_n})$

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$\Rightarrow p'$ divides 1

$\Rightarrow p' = 1$  \rightarrow Contradiction

Therefore, there are infinitely many prime numbers. 

Set operations.

Let A, B be sets.

$a \in A \Leftrightarrow a$ is an element of A

$A \cup B \Leftrightarrow a \in A \Rightarrow a \in B$

Subset

$A \subset B = \{x : x \in A \text{ or } x \in B\}$

Union

$A \cup B = \{x : x \in A \text{ and } x \in B\}.$

Intersection