

Math 3137 - Homework 1

1. Rudin, Ch 1, # 1, 4, and 5.
2. Let \mathbb{F} be a field and x, y and elements of \mathbb{F} . Prove the following using only the field axioms and the property of cancellation.
 - (i) If $x + y = x$ then $y = 0$ (The additive identity is unique)
 - (ii) If $x + y = 0$ then $y = -x$ (The additive inverse is unique)
 - (iii) If $x \neq 0$ and $xy = x$ then $y = 1$ (The multiplicative identity is unique)
 - (iv) If $x \neq 0$ and $xy = 1$ then $y = 1/x$ (The multiplicative inverse is unique)
3. Let \mathbb{F} be an ordered field and $x, y, z \in \mathbb{F}$ arbitrary. Prove the following cancellation laws
 - (a) If $x + y < x + z$ then $y < z$.
 - (b) If $xy < xz$ and $x > 0$, then $y < z$.