1a) For ^1H, $\gamma = 2\pi \times 42.576\ MHz/T$

$$\omega = \gamma \cdot B = \left(2\pi \times 42.576 \times 10^6\right) \times 4.7 \simeq 1.256 \times 10^9\ Hz$$

or $f = \frac{\omega}{2\pi} = 200 \times 10^6\ Hz$

b) $\theta = \gamma B_1 t$

$$B_1 = \frac{\theta}{\gamma t} \qquad \theta = \frac{\pi}{2}\ ;\ t = 2ms$$

$$= \frac{\pi}{2} \times \frac{1}{2\pi \times 42.576 \times 2 \times 10^{-3}} = 2.94\mu T$$

c) $\omega = \gamma B_z$

$$= \gamma \vec{G} \cdot \vec{r}$$

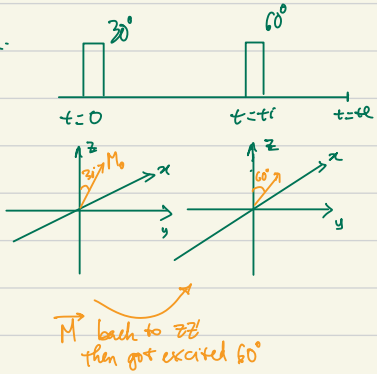
$$= \left(2\pi \times 42.576 \times 10^6\right) \times \left[10^{-3} \times (10, 10, 0) \cdot 10^{-2} \times (5, 16, -5)\right]$$

$$= 0.561\ MHz$$

or

$$f = \frac{\omega}{2\pi} = 0.089\ Hz$$

2.



$\vec{M}$ back to zz then got excited 60°

After $\vec{M}$ got flipped 30°, $0 \le t \le ti$:

$$M_z(t) = M_0 - \left(M_0 - M_z(t=0)\right) e^{-t/T_1}$$

$$= M_0 - \left(M_0 - M_0 \cos 30^\circ\right) e^{-t/T_1}$$

$$M_z(ti) = M_0 - M_0\left(1 - \frac{\sqrt{3}}{2}\right) e^{-TI/T_1} \qquad (1)$$

Flip $\vec{M}$ 60°, $ti < t \le te$

$$M_\perp(t) = M_z(ti) \cdot \sin 60^\circ \cdot e^{-t/T_2^*}$$

$$= M_z(ti) \cdot \frac{\sqrt{3}}{2} \cdot e^{-t/T_2^*} \qquad (2)$$

From (1)(2): $M_\perp(te) = \frac{M_0\sqrt{3}}{2}\left[1 - \left(1 - \frac{\sqrt{3}}{2}\right) \cdot e^{-TI/T_1}\right] \cdot e^{-te/T_2^*}$

3. $\gamma =$

G_z: $\Delta z = 10\ mm$

$\Delta\omega = BW_{RF} = 25 kHz$

$$G_S = \frac{2\pi BW_{RF}}{\gamma \Delta z} = \frac{2\pi}{2\pi \times 42.576 \times 10^6} \cdot \frac{2.5 \times 10^3}{10 \times 10^{-3}} = 5.87 \times 10^{-3} T = 5.87\ mT$$

G_x: $FOV = 256mm \times 256mm$

$BW_{rec} = 100 kHz$

$\Delta x = 2mm$ (nominal resolution 2mm × 2mm)

$$N_x = \frac{FOV}{\Delta x}$$

$$G_x = \frac{2\pi}{\gamma} \cdot \frac{BW_{rec}}{FOV_x} = \frac{2\pi}{2\pi \times 42.576 \times 10^6} \cdot \frac{100 \times 10^3}{256 \times 10^{-3}} = 9.17 \times 10^{-3} T = 9.17\ mT$$

ΔG_y: T_{acq} is time to acquire spin-echo signal. G_x is on during this time, so

$$T_{acq} = N_x \times \Delta t \quad (\Delta t : \text{time step between samples})$$
$$= \frac{FOV_x}{\Delta x} \times \frac{1}{BW_{rec}} = 1.28 \times 10^{-3} (sec)$$

$$T_{ph} = \frac{1}{2} T_{acq}$$

$$\Delta k_y = \frac{\gamma}{2\pi} \Delta G_y T_{ph} = \frac{1}{FOV_y}$$

So $$\Delta G_y = \frac{2\pi}{\gamma} \cdot \frac{1}{FOV_y T_{ph}}$$

$$= \frac{2\pi}{2\pi \times 42.576 \times 10^6} \cdot \frac{1}{256 \times 10^{-3} \times \left(\frac{1}{2} \times 1.28 \times 10^{-3}\right)}$$

$$= 0.143 \times 10^{-3} T = 0.143 mT$$

4 For SE sequence, signal is relative to $\rho(1-e^{-TR/T_1})e^{-TE/T_2}$

$S_{CSF} \sim 1.\left(1-e^{-\frac{3000}{4500}}\right).e^{-\frac{10}{2200}} = 0.484$

$S_{WM} \sim 0.65.\left(1-e^{-\frac{3000}{600}}\right).e^{-\frac{10}{60}} = 0.57$

$S_{GM} \sim 0.8\left(1-e^{-\frac{3000}{850}}\right).e^{-\frac{10}{100}} = 0.693$

Thus $S_{WM} : S_{GM} : S_{CSF} = 0.57 : 0.693 : 0.484$

5 Below in MATLAB

```
S_csf = 1*(exp(-5/200))*(1-exp(-100/4500))*sin(pi/6)/(1-exp(-100/4500)*cos(pi/6))
```

```
S_csf = 0.0700
```

```
S_wm = 0.65*(exp(-5/55))*(1-exp(-100/600))*sin(pi/6)/(1-exp(-100/600)*cos(pi/6))
```

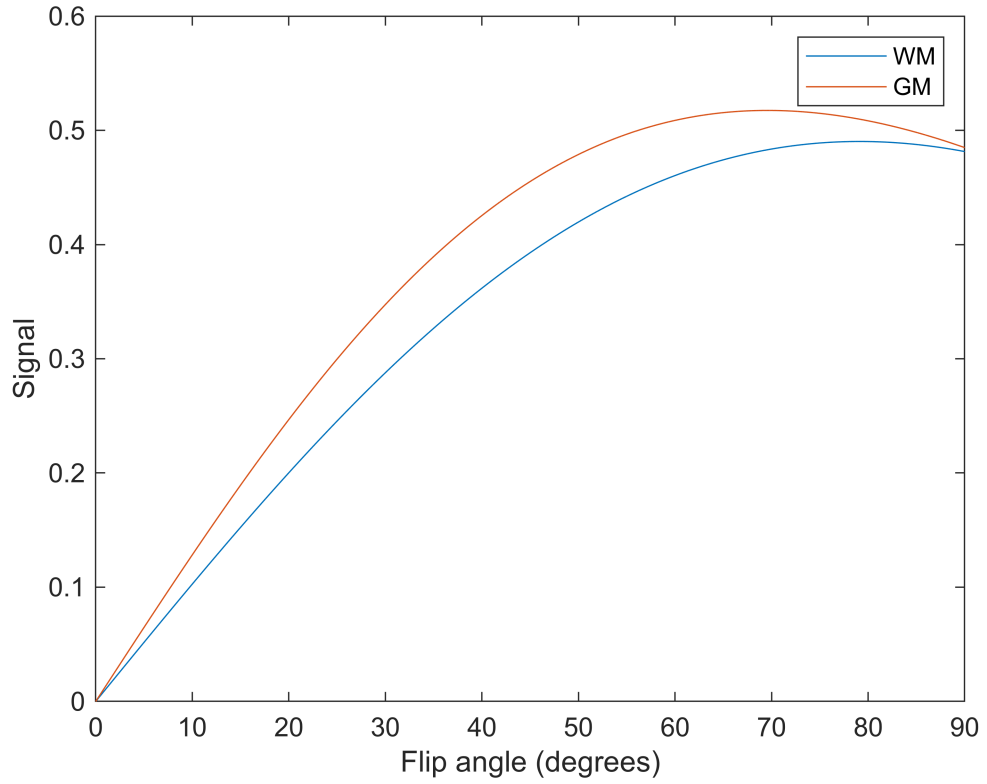
```
S_wm = 0.1707
```

```
S_gm = 0.8*(exp(-5/70))*(1-exp(-100/950))*sin(pi/6)/(1-exp(-100/950)*cos(pi/6))
```

```
S_gm = 0.1688
```

```
theta = linspace(0,pi/2,10000);
S_TR1000_wm = 0.65*(exp(-5/55))*(1-exp(-1000/600))*sin(theta)./ ...
                                (1-exp(-1000/600)*cos(theta));
S_TR1000_gm = 0.8*(exp(-5/70))*(1-exp(-1000/950))*sin(theta)./ ...
                                (1-exp(-1000/950)*cos(theta));
```

```
plot(theta/pi*180, S_TR1000_wm, theta/pi*180, S_TR1000_gm)
legend("WM", "GM")
ylabel('Signal')
xlabel('Flip angle (degrees)')
```




```
[maxContrast, index1000] = max(abs(S_TR1000_gm - S_TR1000_wm));
optimalTheta1000 = theta(index1000)/pi*180
```

```
optimalTheta1000 = 39.3699
```

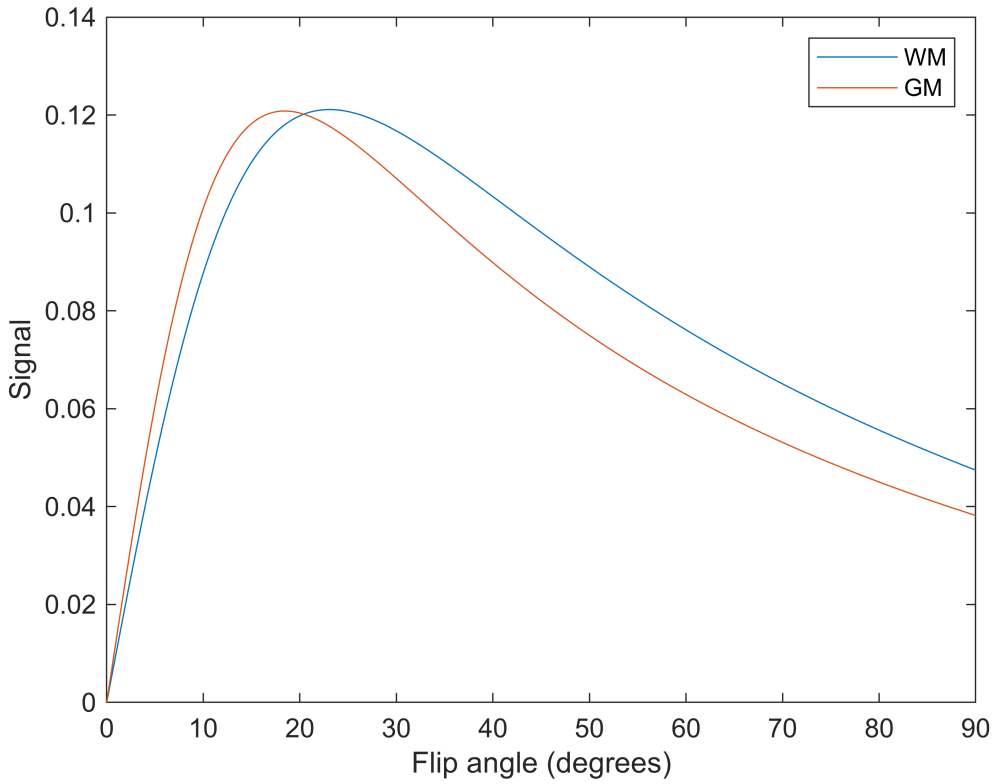
```
ernstAngle_WM_GM = acos(exp(-1000./[600 950]))/pi*180
```

```
ernstAngle_WM_GM = 1×2
   79.1128   69.5727
```

```
"Opitmal theta different from Ernst angles"
```

```
ans =
"Opitmal theta different from Ernst angles"
```

```
% ------ TR = 50ms ------
S_TR50_wm = 0.65*(exp(-5/55)).*((1.-exp(-50/600)).*sin(theta)./ ...
                                (1.-exp(-50/600).*cos(theta)));
S_TR50_gm = 0.8*(exp(-5/70)).*((1.-exp(-50/950)).*sin(theta)./ ...
                                (1.-exp(-50/950).*cos(theta)));
plot(theta/pi*180, S_TR50_wm, theta/pi*180, S_TR50_gm)
legend("WM", "GM")
ylabel('Signal')
xlabel('Flip angle (degrees)')
```




```
[maxContrast50, index50] = max(abs(S_TR50_gm - S_TR50_wm));
optimalTheta50 = theta(index50)/pi*180
```

```
optimalTheta50 = 47.3987
```