PET $\Delta t = 10ns \rightarrow$ locate

440

BME 4400 Fall 2022

Assignment #4

Due: October 20, 2022 9:00 PM

$SA(8hrs) = SA(0) \cdot e^{decay}$

$\rightarrow 90000 \times 37$ MBq

specific activity

$A(t) = \frac{N(t)}{\tau} = \frac{1}{\tau} \cdot N_0 e^{-\alpha t}$

~α ↗ decay fast
~N_0 ↘ inject less

1. A cyclotron provides ^{18}F with specific activity of 90,000 mCi/µg. From this, [^{18}F]-FDG will be produced and shipped to nearby PET imaging facilities. If a 440 MBq dose is to be given to a patient 8 hr after ^{18}F production from the cyclotron, what mass of [^{18}F]-FDG will be injected?

coincidence detections 2 positrons

2. Look at the figure at the bottom of the page, and look up appropriate material characteristics for cortical bone and soft tissue from the NIST site (https://www.nist.gov/pml/x-ray-mass-attenuation-coefficients, tables 2 and 4; use values tabulated at the nearest energy value). A positron emitter (assume ^{18}F) has accumulated in the purple region and is producing an activity of 0.25 MBq along the horizontal path leading to the D+ and D- detectors. Compute the number of coincidence detections expected over a 2 min acquisition.

$A(0) = 0.25MBq$

$N(0,-D) = N(a) e^{-\mu(a) \cdot 6} \cdot e^{-\mu_{bone} 2}$

photon 511keV

$N(a) \sim Activity \cdot t$

$A(\Delta t) = \int_0^t A_0 e^{-\alpha t}$

detector

3. When imaging ^{99m}Tc with a NaI(Tl) scintillation screen with PMT array, photomultiplier
   a. what is a typical photon energy window used to accept detections? 10% ± 140keV
   b. if a scintillation event results in a lower energy deposition that is below this window, why should you not count this event? Since Compton Scattering is a noise factor & scattered photons have lower energy compared to primary photons which can impact image contrast
   c. if scintillation event results in a higher energy deposition that is above this window, why should you not count this event? This correlates to

140 keV

dose & adjacent

4. Compute the radiation dose to the liver for an injection of 100 MBq of ^{99m}Tc sulfur colloid. Assume that 60% of the activity is trapped in the liver, 30% is trapped in the spleen, and 10% in red bone marrow. Assume instantaneous uptake and no biological excretion.

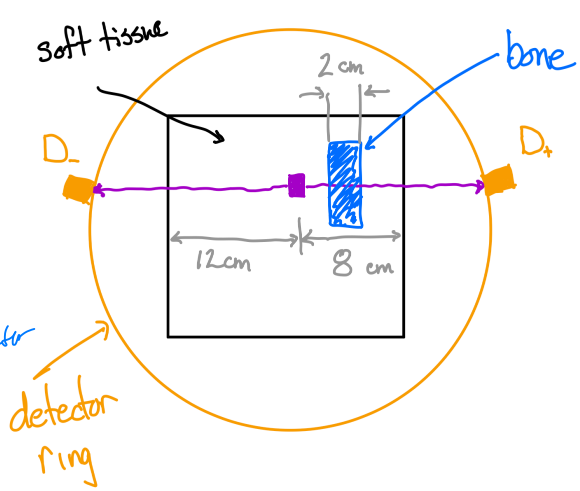
resolution

5. In a ^{18}FDG abdominal scan made with a 90 cm diameter detector ring, how high a resolution are you likely to be able to get and why?

FWHM ~ 4–8mm

spatial resolution 0.0022 D

{Δt, Δx are FWHM of {time, space}



$N(D) = N(a) \cdot e^{-\int_a^d \mu(s) ds}$

Net resolution

$R_{sys}^2 = R_{ring}^2 + R_{pos}^2 + R_{detector}^2$

$= (0.1)^2 + (2.33)^2 + (2.5)^2$

mm

Gamma Emitters

Z	Nuclide	Half-life	Photon Energy (keV)
24	Chromium-51	28d	320
31	Gallium-67	79.2h	92, 184, 296
34	Selenium-75	120d	265
38	Strontium-87m	2.8h	388
43	Technetium-99m	6h	140
49	Indium-111	2.8d	173, 247
	Indium-113m	1.73h	393
53	Iodine-123	13.3h	159
	Iodine-125	60d	35, 27
	Iodine-131	8.04d	364
54	Xenon-133	5.3d	81
80	Mercury-197	2.7d	77
81	Thallium-201	73h	135, 167

Positron Emitters

light atoms

Z	Nuclide	Half-life	Positron Energy (keV)
6	Carbon-11	20.3min	326
7	Nitrogen-13	10.0min	432
8	Oxygen-15	2.1min	696
9	Fluorine-18	110min	202
29	Copper-64	12.7h	656
31	Gallium-68	68min	1900
33	Arsenic-72	26h	3340
35	Bromine-76	16.1h	3600
37	Rubidium-82	1.3min	3150
53	Iodine-122	3.5min	3100

photon E 511 keV?

Source: Wolbarst, 1993.

hydrogen 1 H 1.0079																		helium 2 He 4.0026
lithium 3 Li 6.941	beryllium 4 Be 9.0122												boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998	neon 10 Ne 20.180
sodium 11 Na 22.990	magnesium 12 Mg 24.305												aluminium 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.948
potassium 19 K 39.098	calcium 20 Ca 40.078		scandium 21 Sc 44.956	titanium 22 Ti 47.867	vanadium 23 V 50.942	chromium 24 Cr 51.996	manganese 25 Mn 54.938	iron 26 Fe 55.845	cobalt 27 Co 58.933	nickel 28 Ni 58.693	copper 29 Cu 63.546	zinc 30 Zn 65.39	gallium 31 Ga 69.723	germanium 32 Ge 72.61	arsenic 33 As 74.922	selenium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.80
rubidium 37 Rb 85.468	strontium 38 Sr 87.62		yttrium 39 Y 88.906	zirconium 40 Zr 91.224	niobium 41 Nb 92.906	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.91	palladium 46 Pd 106.42	silver 47 Ag 107.87	cadmium 48 Cd 112.41	indium 49 In 114.82	tin 50 Sn 118.71	antimony 51 Sb 121.76	tellurium 52 Te 127.60	iodine 53 I 126.90	xenon 54 Xe 131.29
caesium 55 Cs 132.91	barium 56 Ba 137.33	57-70 *	lutetium 71 Lu 174.97	hafnium 72 Hf 178.49	tantalum 73 Ta 180.95	tungsten 74 W 183.84	rhenium 75 Re 186.21	osmium 76 Os 190.23	iridium 77 Ir 192.22	platinum 78 Pt 195.08	gold 79 Au 196.97	mercury 80 Hg 200.59	thallium 81 Tl 204.38	lead 82 Pb 207.2	bismuth 83 Bi 208.98	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]
francium 87 Fr [223]	radium 88 Ra [226]	89-102 * *	lawrencium 103 Lr [262]	rutherfordium 104 Rf [261]	dubnium 105 Db [262]	seaborgium 106 Sg [266]	bohrium 107 Bh [264]	hassium 108 Hs [269]	meitnerium 109 Mt [268]	ununnilium 110 Uun [271]	unununium 111 Uuu [272]	ununbium 112 Uub [277]		ununquadium 114 Uuq [289]				

*Lanthanide series	lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
* * Actinide series	actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

Example Problem

0.0025

Your neighborhood radiochemist provides you with 68Ga labelled DOTATOC (molecular weight = 1502.333 g/mol), with a specific activity of 20 MBq/μg at Noon. You will inject 185 MBq intravenously at 1:00 PM.

Question: To what does 68Ga decay? large atomic #

^{68}Ga decays by positron emission — a proton is converted into a neutron + positron. emission

Therefore $^{68}_{31}Ga \longrightarrow {}^{68}_{30}Zn$

Question: What mass of this agent will you inject?

$T_{1/2}$ for ^{68}Ga = 68 min

activity — specific activity (exp decay) — mass

$$A(t=60\,min) = 185\ MBq = SA(t=60\,min) \times m$$

$$SA(t=60\,min) = SA(t=0\,min)\exp\left(-\frac{60\,min}{\tau}\right)$$

$S(t) = S(0)\exp\left(\frac{-t}{\tau}\right)$ where $\tau = \frac{T_{1/2}}{\log(2)}$

$$\boxed{\tau = \frac{T_{1/2}}{\log(2)} = \frac{68\,min}{0.693} = 98.1\,min}$$

Depending on the isotope the half-life varies between fractions of seconds and billions of years.

Note that the presence of radioactivity in the body depends not only on the radioactive decay but also on biological excretion. Assuming a biological half-life T_B, the effective half-life T_E can be calculated as

$$\frac{1}{T_E} = \frac{1}{T_B} + \frac{1}{T_{1/2}}. \qquad (5.9)$$

Currently the preferred unit of radioactivity is the becquerel (Bq). The curie (Ci) is the older unit.* One Bq means one expected event per second and 1 mCi = 37 MBq. Typical doses in imaging are on the order of 10^2 MBq.

It can be shown that the probability of measuring n photons when r photons are expected, equals

$$p_r(n) = \frac{e^{-r} r^n}{n!}. \qquad (5.10)$$

$$SA(t=0\,min) = 20\ \frac{MBq}{\mu g}$$

log (natural log)
$\log_{10}$

$$\therefore SA(t=60\,min) = 20\ \frac{MBq}{\mu g}\exp\left(\frac{-60}{98.1}\right) = 10.85\ \frac{MBq}{\mu g}$$

$$\therefore m = \frac{185\ MBq}{10.85\ MBq/\mu g} = \underline{\underline{17.1\ \mu g}}$$

$T_{1/2}$ for ^{18}F = 110 min

$$m = \frac{A_{cAtivity}}{SA} = \frac{A_{inject}}{S_0 \cdot e^{\left(\frac{-8hrs\times 60}{\tau}\right)}}$$

$$\tau = \frac{T_{1/2}}{\log(2)}$$

$$\tau = \frac{110}{\ln(2)} = 158.69$$

$$m = \frac{440}{90{,}000 \times 37 \times e^{\left(\frac{-8\times 60}{158.69}\right)}}$$

$= 0.00272\ \mu g$

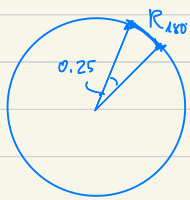
$= 2.72 \times 10^{-3}\ \mu g$

Q.3

a) NaI (Tl) scintillators has a ±10% energy resolution.
Since ^{99m}Tc produces photon at 140keV.
Typical energy window for ^{99m}Tc with NaI(Tl) scintillator and PMT array is 126-154keV.

b) Events with lower energy deposition can stem from Compton Scattering. The scattered photon got deflected from its original direction and has a lower energy. compared to primary photons comming straight into the detector.

c) This corresponds to an event where two or more photon arrive at the detector at the same time, and the culmulated energy is computed as 1 event.



Q5 PET resolution depends on:

1.) Positron range: ^{18}F : $R_{range} = 0.1$ mm

2) annihilation photons' angle deviates from 180°, $R_{180} = \frac{0.25}{360} \times 2\pi \times \left(\frac{90 \times 10}{2}\right) = 1.9625$ mm

3) detector characteristic : $R_{detector} = 25$ mm

Thus $R_{sys}^2 = R_{range}^2 + R_{180}^2 + R_{detector}^2$

$= (0.1)^2 + (1.9625)^2 + (2.5)^2$

$= 10.11$

$R_{sys} = \sqrt{10.11} = 3.18$ mm

Q1

$$m = \frac{A(8hrs)}{SA(8hrs)} = \frac{A(8hrs)}{SA(0).e^{\left(\frac{-8\times 60}{\tau}\right)}}$$

We have $T_{1/2}$ for $^{18}F = 110$ min.

$$\tau = \frac{T_{1/2}}{\log(2)} = \frac{110}{\log(2)} = 158.69 \text{ min}$$

$A(8hrs) = 440$ MBq

$SA(0) = 90{,}000 \times 37 = 3.33 \times 10^6$ MBq

$$\text{Thus, } m = \frac{440}{3{,}33 \times 10^6 \times e^{\left(\frac{-8\times 60}{158.69}\right)}} = 2.72 \times 10^{-3} \mu g$$

Q4

60% in liver $\qquad S_{liver-liver} = 3.23 \times 10^{-6}$

30% in spleen $\qquad S_{spleen-liver} = 7.2 \times 10^{-8}$

10% in red bone marrow $\qquad S_{marrow-liver} = 8.93 \times 10^{-8}$

Biological excretion is ignored so cumulated activity is $\tilde{A} = 1.44 T_{1/2} A_0$

$$= 1.44 \times (6 \times 60 \times 60) \times 100$$

$$= 3{,}110{,}400 \text{ MBq.sec}$$

Thus dose is:

$$D = D_{spleen \to liver} + D_{marrow \to liver} + D_{liver \to liver}$$

$$= \tilde{A}(30\% \times S_{spleen \to liver} + 10\%.S_{marrow \to liver} + 60\%.S_{liver \to liver})$$

$$= 3{,}110{,}400 \times (0.3 \times 7.2 \times 10^{-8} + 0.1 \times 8.93 \times 10^{-8} + 0.6 \times 3.23 \times 10^{-6}) = 6.123 \text{ mGy}$$

For the liver:

$$E = W_{liver}.D = 0.04 \times 6.123 = 0.245 \text{ mGy}$$

$\boxed{Q2}$

511 keV

	ρ (g/cm3)	μ/ρ (cm²/g)	μ (cm⁻¹)
Bone	1.920	9.022×10^{-2}	0.1732
Tissue	1.060	9.598×10^{-2}	0.102

$$N(+D,-D) = N(a) \cdot e^{-\mu_{tissue}(12+6)} \cdot e^{-\mu_{bone}(2)}$$

$$A(t=0) = 0.25 MBq$$

$$A(t) = A(t=0) \cdot e^{-\alpha t}$$

where $\alpha = \frac{1}{\tau} = \frac{\log(2)}{T_{1/2}} = \frac{\log(2)}{110\,(min) \cdot 60 \frac{sec}{min}} = 1.05 \times 10^{-4}\ sec^{-1}$

Let TA be total activity or number of decays over 2min or 120 sec

$$TA = \int_0^{t=120sec} A(t=0) \cdot e^{-\alpha t} \cdot dt$$

$$= \frac{A(t=0)}{-\alpha} \cdot e^{-\alpha t} \Big|_0^{t=120}$$

$$= \frac{A(t=0)}{-\alpha} \left(e^{-\alpha \cdot 120} - 1 \right)$$

$$= \frac{A(t=0)}{\alpha} \left(1 - e^{-\alpha \cdot 120} \right) = 29811791.31 \text{ decays}$$

with $A(t=0) = 0.25 MBq = 0.25 \times 10^6 Bq$

$\alpha = 1.05 \times 10^{-4}$

Thus $N(+D, D) = (TA) \cdot e^{-0.102 \times 18} \times e^{-0.1732 \times 2}$

$\simeq 3361892$ coincidence detections