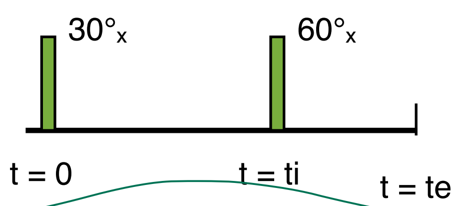
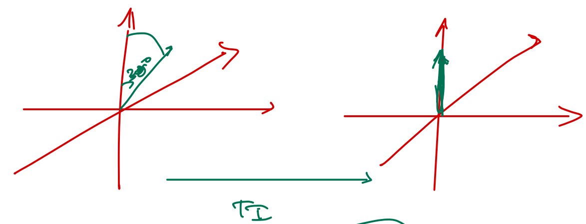
BME 4400 Fall 2022
Assignment #6
Due: November 6, 2022 11:59 PM

$\theta = 90°$
$t = 2ms$

1.
   a. What is the Larmor Frequency (in Hz) of ¹H at 4.7T?
   b. Calculate B_1 (in μT) required to rotate a magnetization vector through 90° in 2 ms.
   c. If Gx = 10 mT/m, Gy = 10 mT/m, and Gz = 0 mT/m, what is the resonance frequency (in the rotating frame) at location (x,y,z) = (5 cm, 16 cm, -5 cm)? ω

2. Given pulse sequence below, determine $M_\perp(t_e)$ as a function of M_0, T_1, and , T_2^*, assuming $M_z(t=0^-) = M_0$ and $M_\perp(t = t_i^-) = 0$.





3. Given the pulse sequence diagram in Fig 4.16a (spin echo) in your text and assuming the following scan parameters—Slice Thickness FWHM = 10 mm, 2.5 kHz BW RF pulses, FOV = 25.6 cm x 25.6 cm, Receiver BW = 100 kHz, nominal resolution in plane = 2 mm x 2 mm—solve for read gradient amplitude (G_R), increment amplitude of the phase encode gradient (ΔG_y) and the amplitude of the slice selection gradients (G_S). Assume that the phase encode period = Tacq/2, where Tacq is the time to acquire the the spin echo signal. Treat all gradients pulses as rectangular (that is, ignore the finite rise and fall times).

Gz 2.5kHz

slice thickness $= \frac{2\pi BW_{RF}}{\gamma G_S}$

$\frac{1}{FOV_x} = \frac{\gamma G_x}{2\pi BW_{rec}}$

$FOV_y = \frac{2\pi}{\gamma \Delta G_{PE} \cdot T_{ph}}$

$T_{ph} = T_{acq}/2$

Tacq for Gx
Tph for ΔGy

FOV/resolution = # samples = Nr? = N_{PE}

Tacq Gx on

4. For a standard spin echo sequence (Fig 4.16 and Eq 4.114) and tissue characteristics listed in Table 1 below, determine the relative image intensities of white matter, gray matter, and CSF if TE = 10ms and TR = 3000 ms

proportional to $\rho(1-e^{-TR/T_1})e^{-TE/T_2}$

5. For a standard spoiled gradient-echo pulse sequence (Fig 4.20 and Eq 4.115) and tissue characteristics listed in Table 1 below,
   a. determine the relative image intensities of white matter, gray matter, and CSF if TR = 100 ms, TE = 5 ms, and flip angle α = 30°.
   b. use MATLAB to plot the image intensity of white and gray matter versus excitation flip angle (θ) for TR = 1000 ms and identify the optimal θ for white/gray contrast. Is this equal to the Ernst angle? $\arccos(e^{-TR/T_1})$ T2 weighted long TR
   c. For TR = 50 ms, what flip angle will provide the greatest contrast between white and gray matter?

white/gray

$$\rho e^{-\frac{TE}{T_2^*}} \frac{(1-e^{-\frac{TR}{T_1}})\sin\alpha}{1-e^{-\frac{TR}{T_1}} \times \cos\alpha}$$

Table 1

Tissue	density (g/ml)	T1 (ms)	T2 (ms)	T2* (ms)
CSF	1	4500	2200	200
WM	0.65	600	80	55
GM	0.8	950	100	70

1a) For ^1H, $\gamma = 2\pi \times 42.576$ MHz/T

$$\omega = \gamma . B = \left(2\pi \times 42.576 \times 10^6\right) \times 4.7 \simeq 1.256 \times 10^9 \text{ Hz}$$

or $f = \frac{\omega}{2\pi} = 200 \times 10^6$ Hz

b) $\theta = \gamma B_1 t$

$B_1 = \frac{\theta}{\gamma t}$ $\qquad \theta = \frac{\pi}{2}$; $t = 2$ms

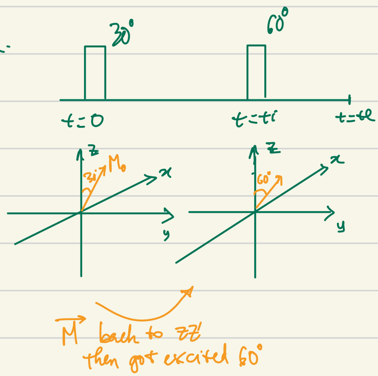
$$= \frac{\pi}{2} \times \frac{1}{2\pi \times 42.576 \times 2 \times 10^{-3}} = 2.94 \mu T$$

c)
$$\omega = \gamma B_z$$
$$= \gamma \vec{G} . \vec{r}$$
$$= \left(2\pi \times 42.576 \times 10^6\right) \times \left[10^{-3} \times (10, 10, 0) \cdot 10^{-2} \times (5, 16, -5)\right]$$
$$= 0.561 \text{ MHz}$$

or

$$f = \frac{\omega}{2\pi} = 0.089 \text{ Hz}$$

2.



After $\vec{M}$ got flipped 30°, $0 \le t \le ti$:

$$M_z(t) = M_0 - \left(M_0 - M_{z(t=0)}\right) e^{-t/T_1}$$
$$= M_0 - \left(M_0 - M_0 \cos 30°\right) e^{-t/T_1}$$
$$M_z(ti) = M_0 - M_0\left(1 - \frac{\sqrt{3}}{2}\right) e^{-TI/T_1} \qquad (1)$$

Flip $\vec{M}$ 60°, $\quad ti < t \le te$

$$M_\perp(t) = M_z(ti) . \sin 60° . e^{-t/T_2^*}$$
$$= M_z(ti) . \frac{\sqrt{3}}{2} . e^{-t/T_2^*} \qquad (2)$$

From (1)(2): $M_\perp(te) = \frac{M_0\sqrt{3}}{2}\left[1 - \left(1 - \frac{\sqrt{3}}{2}\right) . e^{-TI/T_1}\right] . e^{-te/T_2^*}$

3. $\gamma =$

G_z:

$$\Delta z = 10 \text{ mm}$$

$$\Delta \omega = BW_{RF} = 2.5 \text{kHz}$$

$$G_s = \frac{2\pi BW_{RF}}{\gamma \Delta z} = \frac{2\pi}{2\pi \times 42.576 \times 10^6} \cdot \frac{2.5 \times 10^3}{10 \times 10^{-3}} = 5.87 \times 10^{-3} T = 5.87 \text{ mT}$$

G_x:

$$FOV = 256\text{mm} \times 256\text{mm}$$

$$BW_{rec} = 100\text{kHz}$$

$\Delta x = 2\text{mm}$ (nominal resolution $2\text{mm} \times 2\text{mm}$)

$$N_x = \frac{FOV}{\Delta x}$$

$$G_x = \frac{2\pi}{\gamma} \cdot \frac{BW_{rec}}{FOV_x} = \frac{2\pi}{2\pi \times 42.576 \times 10^6} \cdot \frac{100 \times 10^3}{25.6 \times 10^{-2}} = 9.17 \times 10^{-3} T = 9.17 \text{ mT}$$

ΔG_y: T_{acq} is time to acquire spin echo signal. G_x is on during this time, so

$T_{acq} = N_x \times \Delta t$ (Δt : time step between samples)

$$= \frac{FOV_x}{\Delta x} \times \frac{1}{BW_{rec}} = 1.28 \times 10^{-3} \text{(sec)}$$

$$T_{ph} = \frac{1}{2} T_{acq}$$

$$\Delta k_y = \frac{\gamma}{2\pi} \Delta G_y T_{ph} = \frac{1}{FOV_y}$$

So $\Delta G_y = \frac{2\pi}{\gamma} \cdot \frac{1}{FOV_y T_{ph}}$

$$= \frac{2\pi}{2\pi \times 42.576 \times 10^6} \cdot \frac{1}{256 \times 10^{-3} \times \left(\frac{1}{2} \times 1.28 \times 10^{-3}\right)}$$

$$= 0.143 \times 10^{-3} T = 0.143 \text{mT}$$

$\boxed{4}$ For SE sequence, signal is relative to $\rho(1-e^{-TR/T_1})e^{-TE/T_2}$

$$S_{CSF} \sim 1 \cdot \left(1 - e^{-\frac{3000}{4500}}\right) \cdot e^{-\frac{10}{2200}} = 0.484$$

$$S_{WM} \sim 0.65 \cdot \left(1 - e^{-\frac{3000}{600}}\right) \cdot e^{-\frac{10}{80}} = 0.57$$

$$S_{GM} \sim 0.8 \left(1 - e^{-\frac{3000}{950}}\right) \cdot e^{-\frac{10}{100}} = 0.693$$

Thus $S_{WM} : S_{GM} : S_{CSF} = 0.57 : 0.693 : 0.484$

$\boxed{5}$ Below in MATLAB