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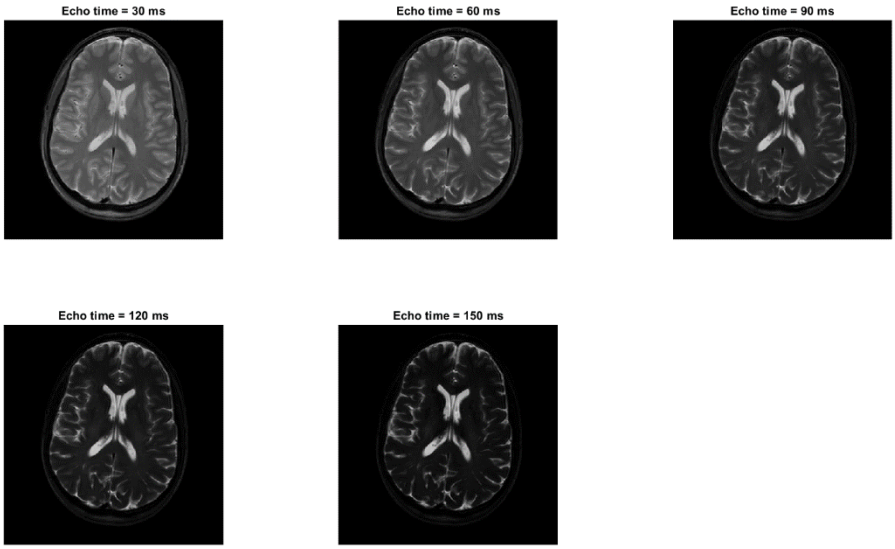
BME 4420/7450 - Quantitative and Functional Imaging

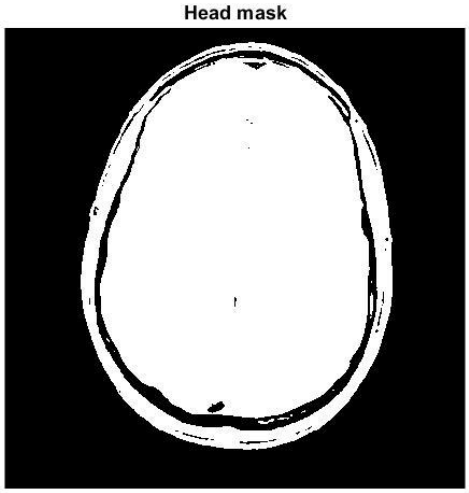
Mapping T_1 and T_2 relaxation times in MRI

Part 1: Mapping T_2 relaxation times

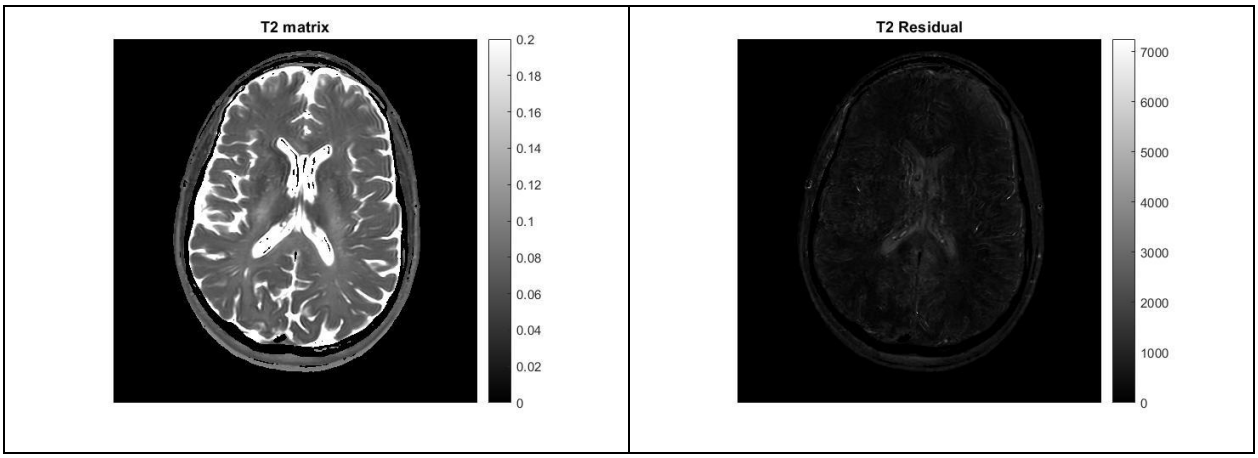
1. Display of all five images in one plot with calculated echo times



2. Create head mask

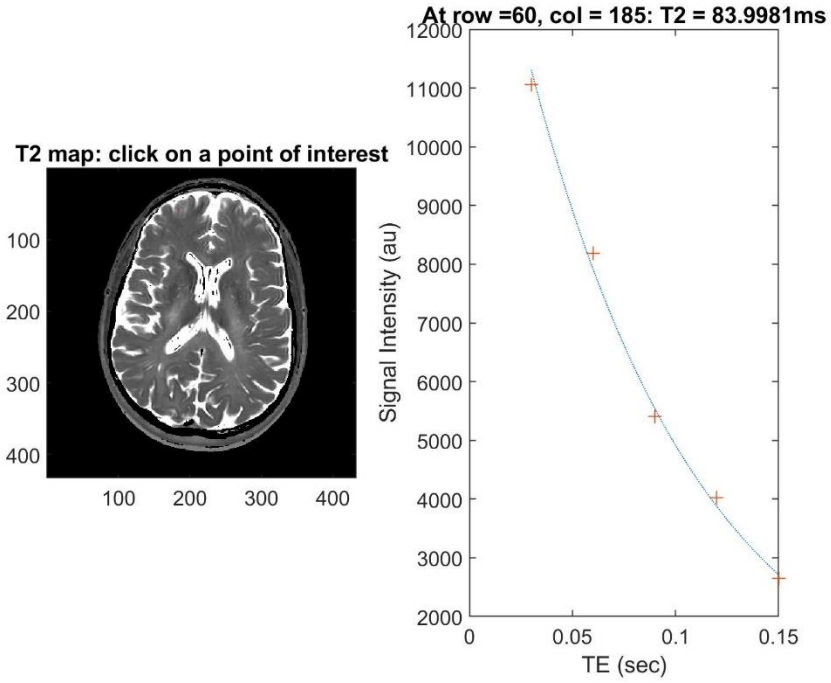


3. Fitted a line for each pixel and create a residual plot

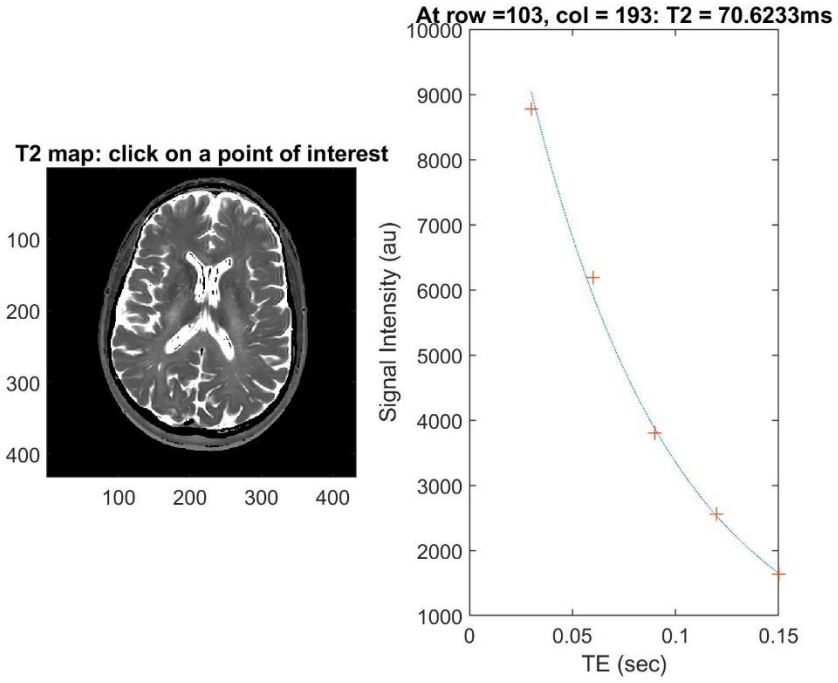


4. Make two plots which show the T2 map and TE with a best fit line

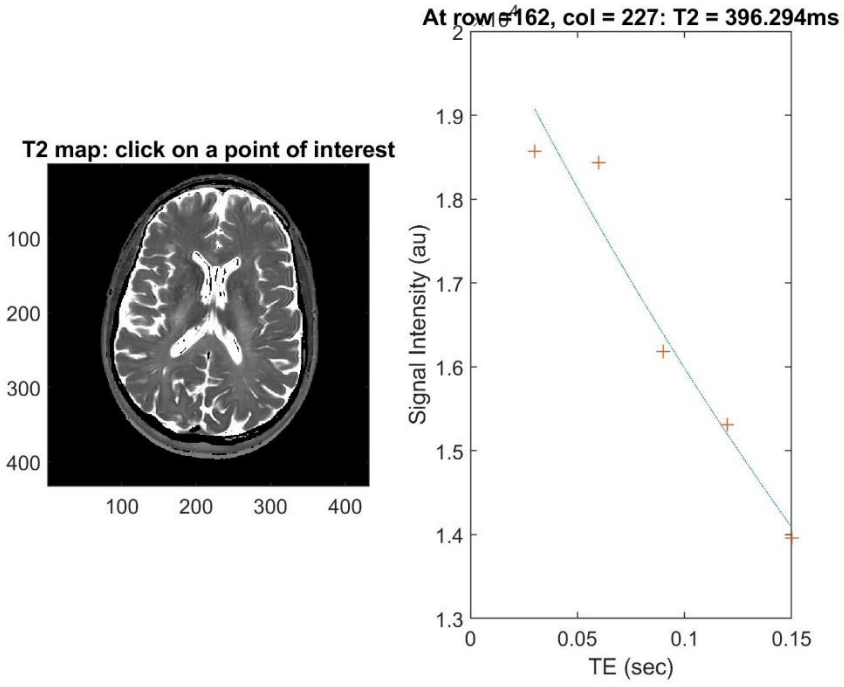
Example for a pixel from gray matter:



Example for a pixel from white matter:



Example for a pixel from CSF:



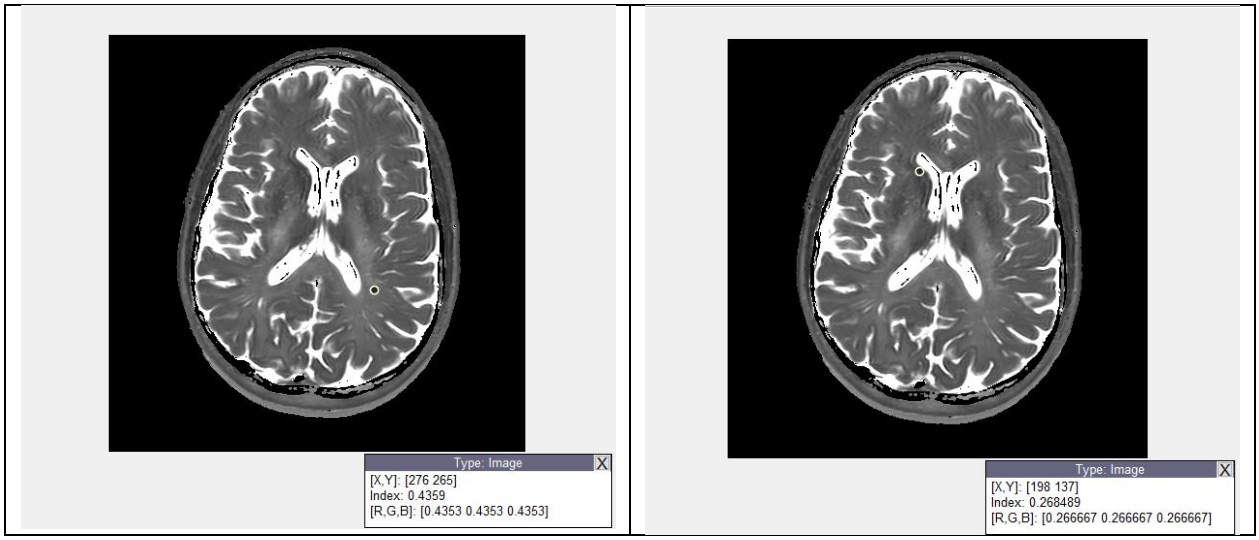
How well do they agree? Compare T_2 values in the gray matter, white matter and cerebrospinal fluid (CSF)—how different are these?

- The model (shown by the curve) agrees closely with the data as shown in the above graphs.

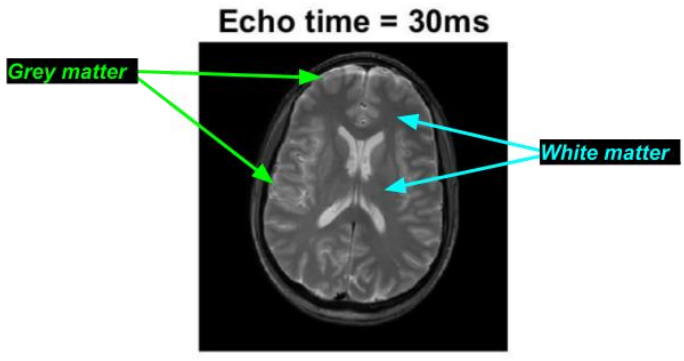
- The T2 values in the white matter are smaller than in the gray matter (ex: 70.6 ms versus 84.0 ms).
- However, T2 values in the CSF are much larger than in either the gray or white matter (ex: 396.3 ms).

5. T2 data as a gray scale

T2 value of pixel is denoted as 'index', with range in gray scale from [0 – 1].

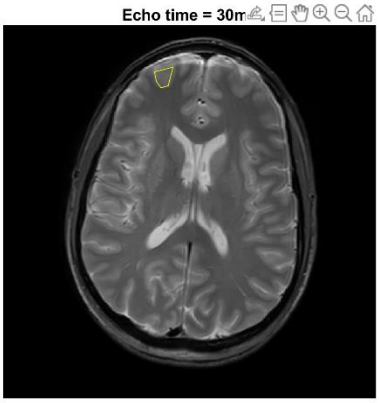


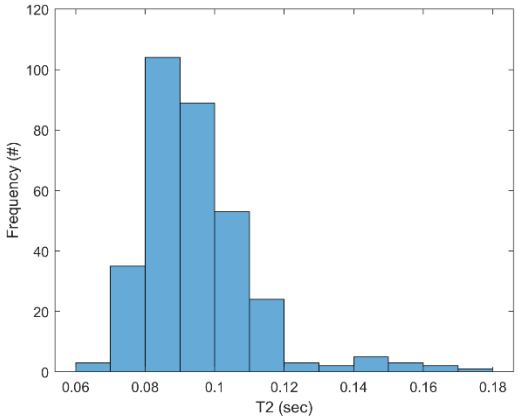
6. Identify the gray matter and white matter in first image



7. Identify region of interest for gray and white matter, and show histograms

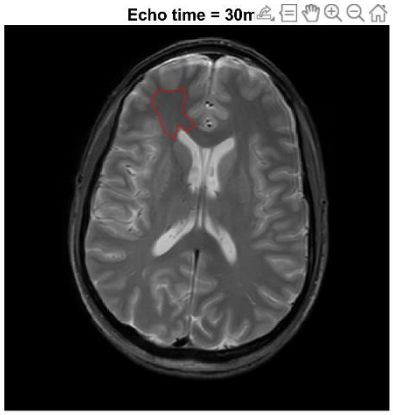
Gray Matter:

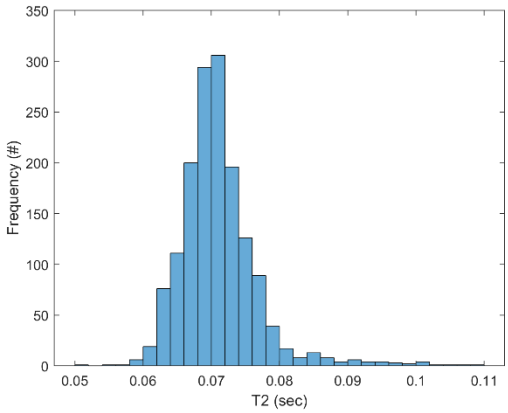




$Gray\ matter\ T2\ mean$: $94.2 \pm 12.5\ ms$ (converted)

White Matter:

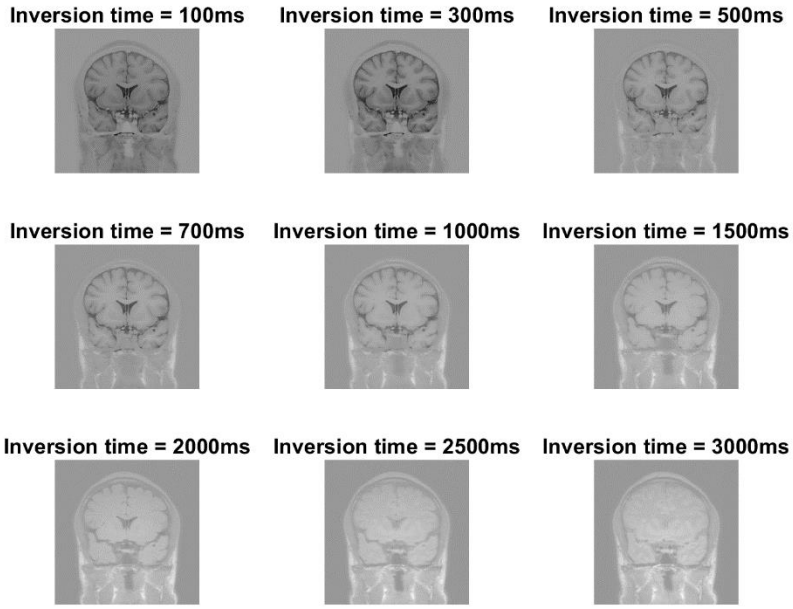




$White\ matter\ T2\ mean$: $70.1 \pm 5.2\ ms$ (converted)

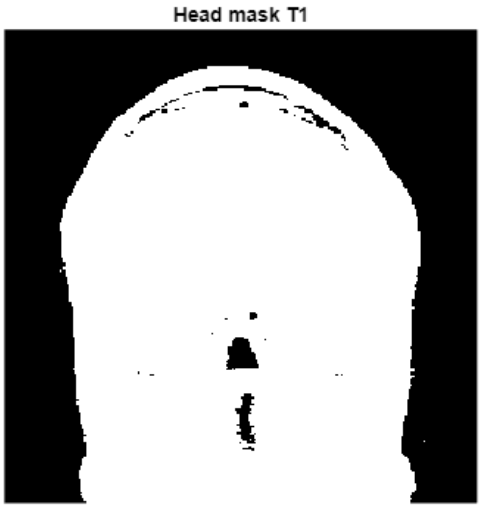
Part II: Mapping T_1 relaxation times

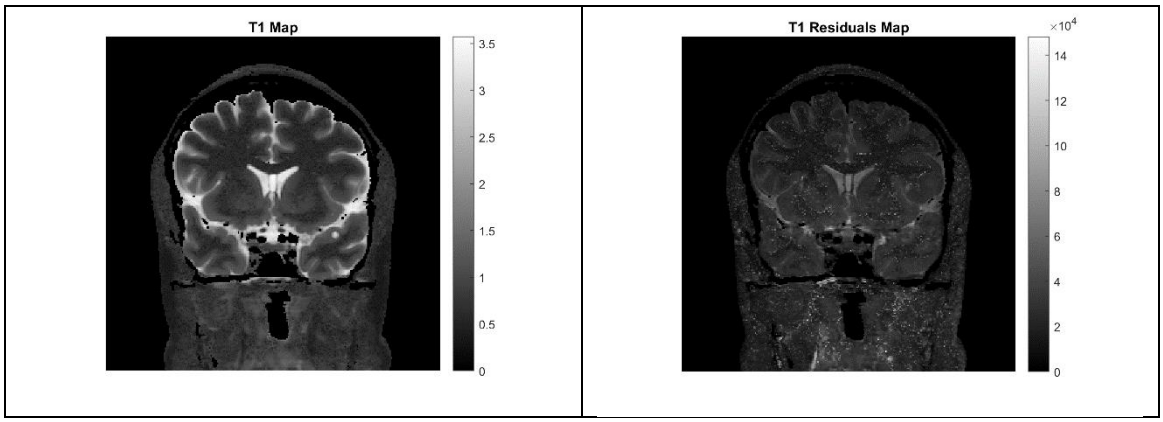
1. Inversion recovery images with positive and negative Mz values



2. T_1 map and calculate the residual of the fit

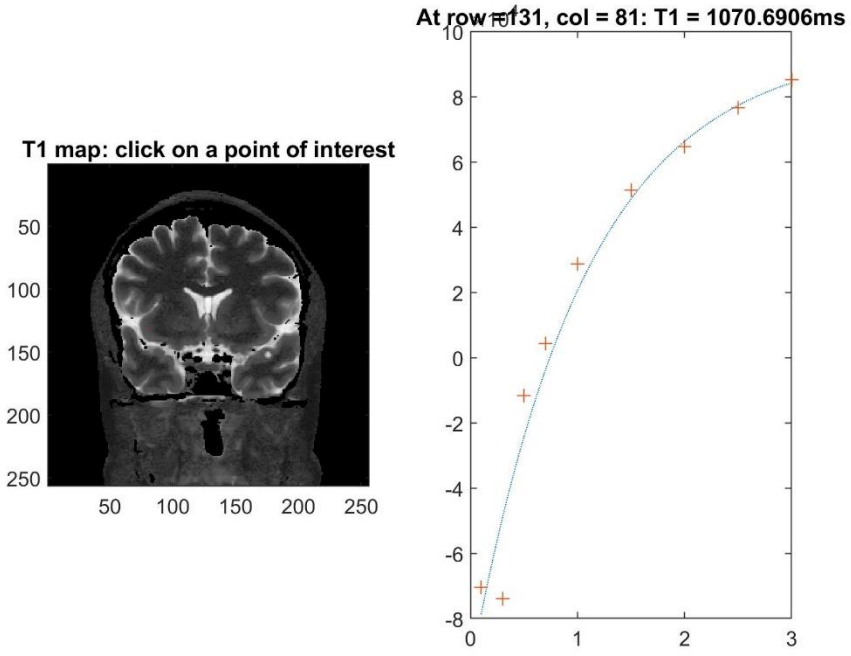
Create head mask



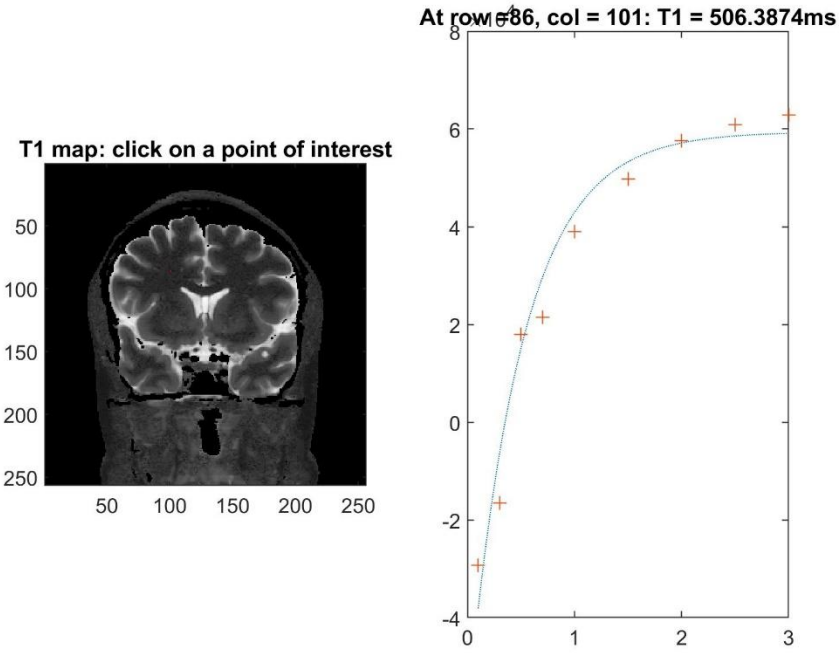


3. Plot Mz and T1 with modeled curve

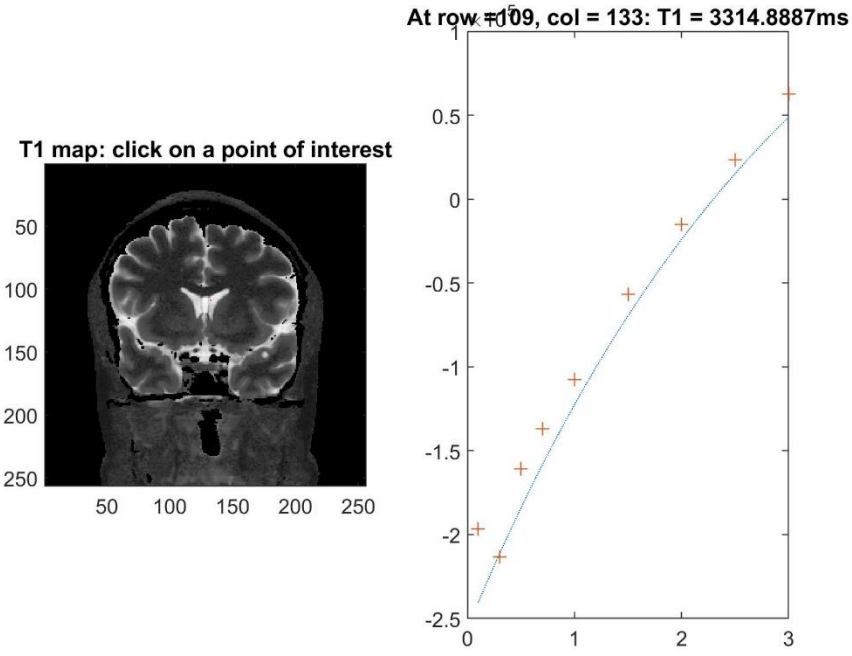
Example for a gray matter pixel:



Example for a white matter pixel:



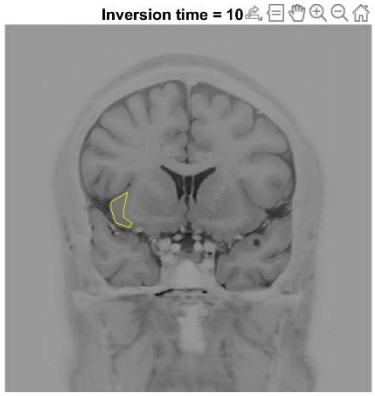
Example for a CSF pixel:

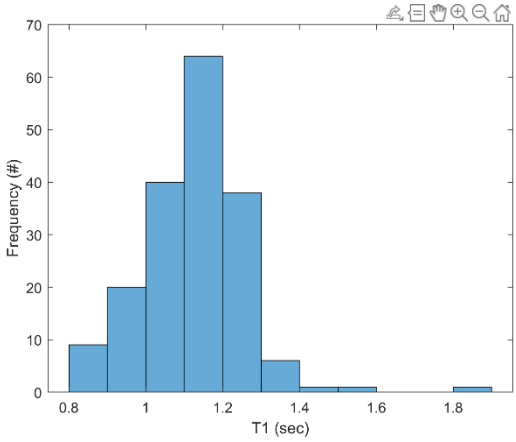


T1 fit is a bit better for gray matter.

4. Identify region of interest for gray and white matter, and show histograms

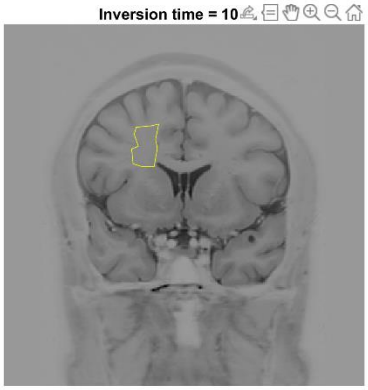
Gray Matter:

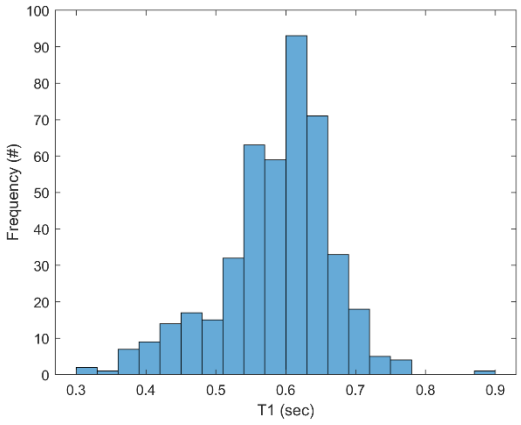




$Gray\ matter\ T1\ mean$: $1070.6 \pm 170.7\ ms$ (converted)

White Matter:



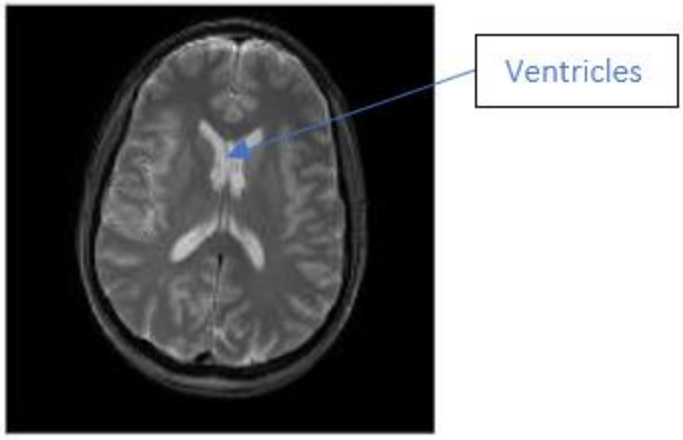


$White\ matter\ T2\ mean$: $588.5 \pm 73.4\ ms$ (converted)

Questions

1. The large, bright "X" shaped structure in the center of the brain in part 1 is part of the ventricular system. The ventricles are cavities in the brain filled with cerebral spinal fluid (CSF). Based on the signal decay in the five T_2–weighted images, does the CSF have longer or shorter T_2 relaxation time than brain tissue? (Explain using qualitative observations, not your fitting results).

   Based on the signal decay in the five T2-weighted images, we see that CSF has a longer T2 relaxation time than brain. This part of the brain is mainly water (a small molecule), thus slow decay and longer T2 time. This is why CSF (or the ventricles) appear brighter than tissue in the image: the signal intensity is higher in CSF than in tissue.



2. What is a typical T_2 value for brain tissue? How much variation do you see over the brain?

   Wansapura et al. reported average T2 values for white matter in various locations between 74 and 84 ms. We saw an average of about 71 ms is the example provided. The same article showed T2 values for gray matter in various locations between 98-132 ms. We saw an average of about 94 ms. We saw standard deviations of 5.2 and 12.1 ms in our examples of gray and white matter, respectively.

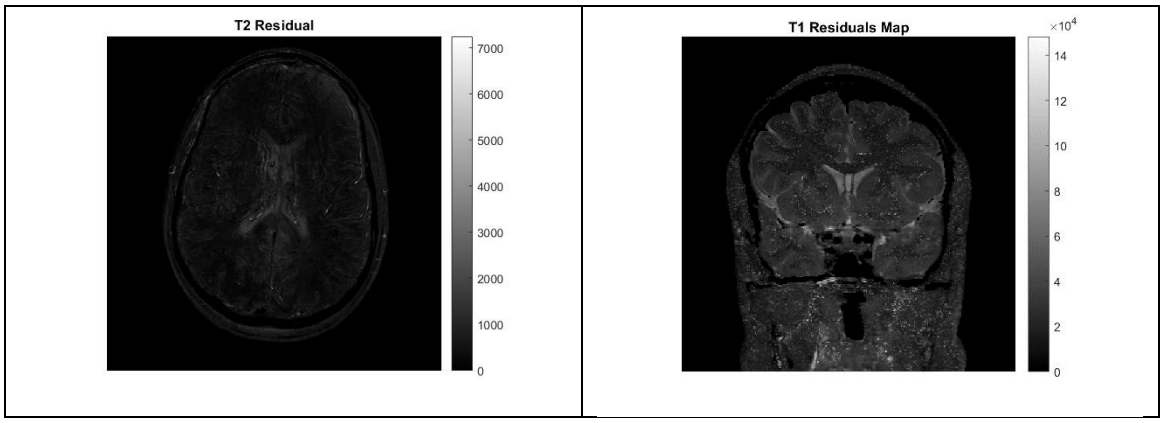
   There is some variation in the brain depending on how the ROI was drawn as well as the location picked. Note that MRIs are shift invariant, so this may contribute to variations in T2. Additionally, even though the same machine was used, different models or operators may affect the results.

   Wansapura JP, Holland SK, Dunn RS, Ball WS Jr. NMR relaxation times in the human brain at 3.0 tesla. J Magn Reson Imaging. 1999 Apr;9(4):531-8. doi: 10.1002/(sici)1522-2586(199904)9:4<531::aid-jmri4>3.0.co;2-l. PMID: 10232510.

3. Are the residuals of the fit uniform in the head? If not, where are they larger and where are they smaller and what might cause the variation?

   There are some signals which are significantly higher than others. These are noisy signals that come from the machine itself and are not physiologically relevant.

   Physiologically speaking there are different residuals based on tissue. For example, in the T2 residual map, we can clearly see the ventricles (CSF fluid). Similarly, we see this same distinction in the T1 residual map. This difference is because of the different signals being propagated within the regions of interest. When you take the norm of each pixel, you find the magnitude of the signal which is dependent on the tissue which corresponds to the pixel.



4. How well can you distinguish between gray matter and white matter on the basis of T_2 values in this map? What is the contrast in T_2 values between gray and white matter (i.e., mean T_2 for the gray matter ROI minus the mean T_2 for the white matter ROI)? If the 'noise' in this measurement is the standard deviation of the difference between T_2 values, $\sigma_{T2(GM)-T2(WM)}$, what is the contrast-to-noise ratio, CNR, between gray and white matter in the T_2 map? Use the propagation of errors to find an expression for $\sigma_{T2(GM)-T2(WM)}$ in terms of $\sigma_{T2(GM)}$ and $\sigma_{T2(WM)}$.

- Gray and white matter seem to be relatively distinguishable using T2 values. There is some variation. However, the averages are more than 3 standard deviations away from each other, meaning they are probably statistically significantly different.
- The contrast in T2 values is (94.2ms - 70.7 ms) = 23.5 ms .
- The noise (standard deviation of the difference between T2 values) is 13.17 ms. The following equation was used to calculate this: $\sigma_f = \sqrt{\sigma_g^2 + \sigma_w^2}$, assuming that covariance is zero.
- Knowing that $SNR = \frac{S}{\sigma}$, the contrast to noise ratio (CNR) would be $CNR = \frac{23.5}{13.17} = 1.7844$

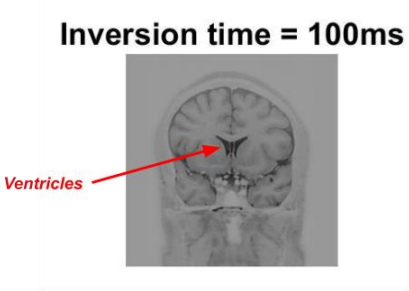
$$\theta^2(f) = \theta^2(s_1)\left(\frac{\partial f}{\partial s_1}\right)^2 + \theta^2(s_2)\left(\frac{\partial f}{\partial s_2}\right)^2$$

Let $f = s_1 - s_2$ (Contrast eq)

$$\theta^2(f) = \theta^2(s_1)\cdot 1^2 + \theta^2(s_2)\cdot 1^2$$

$$\theta(f) = \sqrt{\theta^2(s_1) + \theta^2(s_2)}$$

5. Find the lateral ventricles in the inversion recovery images. Based on the M_z recovery as a function of TI, does CSF have longer or shorter T_1 relaxation time than brain tissue? (Again, explain using your qualitative observations, not the fitting results).



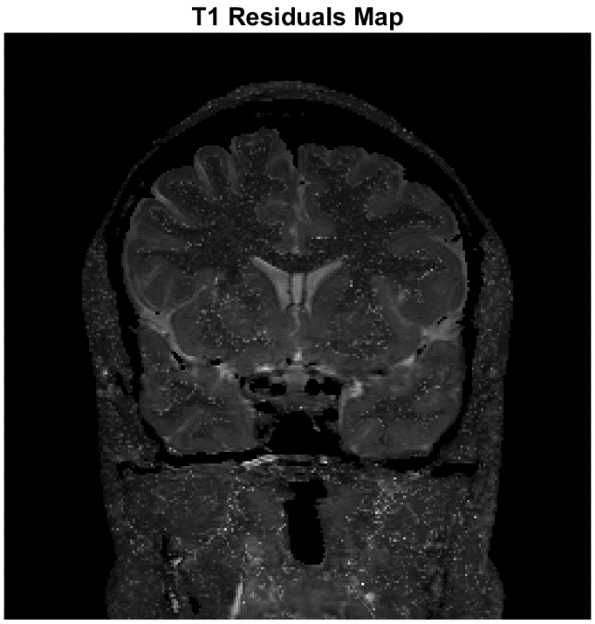
- The CSF appears "dark" in the inversion recovery images, meaning that the signal in the CSF is taking longer to recover towards M0 than other tissue in the brain.
- If T1 is longer, then Mz recovers more slowly towards M0. Thus, T1 in the CSF is longer than in other brain tissues.

6. What is a typical T_1 value for brain tissue? How much variation do you observe over the brain?

- A typical T1 value depends on the type of brain tissue. For instance, a typical T1 value for white matter is around 500 ms, a typical T1 value for gray matter is around 1000 ms, and a typical T1 value for the CSF is around 3000 ms. Thus, there is significant variation in T1 across the brain depending on the tissue type.
- Specifically, based on the ROIs created for regions of gray and white matter, the mean T1 for gray matter was 1.0776 seconds and the standard deviation was 0.1707 seconds. For white matter, the mean T1 was 0.5885 seconds and the standard deviation was 0.0734 seconds.
- Across the whole brain, the standard deviation was calculated to be 0.6274 seconds (or a variance of 0.3936 seconds^2) which is a large amount of variation due to the different compositions of different tissue types in the brain.

7. Do the estimated (modeled) recovery curves match the measured data well? Are the residuals similar for different tissue types?

- The modeled recovery curves appear to match the measured data well based on visual inspection of the curves.
- The average norm between the model and the measured data (across the whole brain) was 1.28e4.
- The average norm for the gray matter ROI was 2.9655e4 whereas the average norm for the white matter ROI was 1.7367e4. Therefore, the residuals were somewhat larger for the gray matter than for the white matter. This is supported by the map of the residuals below. As shown, the gray matter appears brighter than the white matter regions, indicating higher residual values in the gray matter. However, the CSF appears the brightest on the residual map, indicating the largest residual values.



8. How well can you distinguish between gray matter and white matter on the basis of T_1? What is the contrast in T_1 values between gray and white matter? If the 'noise' in this measurement is the standard deviation of T_1 values around the means, what is the contrast-to-noise ratio, CNR, between gray and white matter in the T_1 map?

   White and grey matter can be distinctively distinguished using T1 contrast. Contrast in T1 values between gray and white matter (i.e. mean T1 for the gray matter ROI minus the mean T2 for the white matter ROI) is around $1.0776 - 0.5885 = 0.4891$ sec or $489.1ms$.

   To calculate the noise (standard deviation of the difference between T1 values of gray and white matter), we use equation $\sigma_f = \sqrt{\sigma_g^2 + \sigma_w^2}$ similar to T2. This gives $\sigma_f = 113.6\, ms$.

   Thus the $CNR = \frac{S}{\sigma_f} = \frac{489.1}{113.6} = 4.30$

9. Which relaxation time, T_1 or T_2, provides the highest CNR between gray and white matter? If you had to classify each pixel in the brain as either gray or white matter (ignoring CSF), which relaxation time map would you use?

   It is clear CNR between gray and white matter using T1 (CNR = 4.30) is higher than T2 (CNR = 1.78). Thus T1 relaxation time map is more preferrable for distinguishing between white and gray matter quantitatively.

10. Ideally, the residual maps reflect only random noise. Do you see evidence for non-random errors in the T_1 or T_2 fits?

In both T1 & T2 map, there are regions of high residual signals which can be caused by spins being impacted by its neighbors.

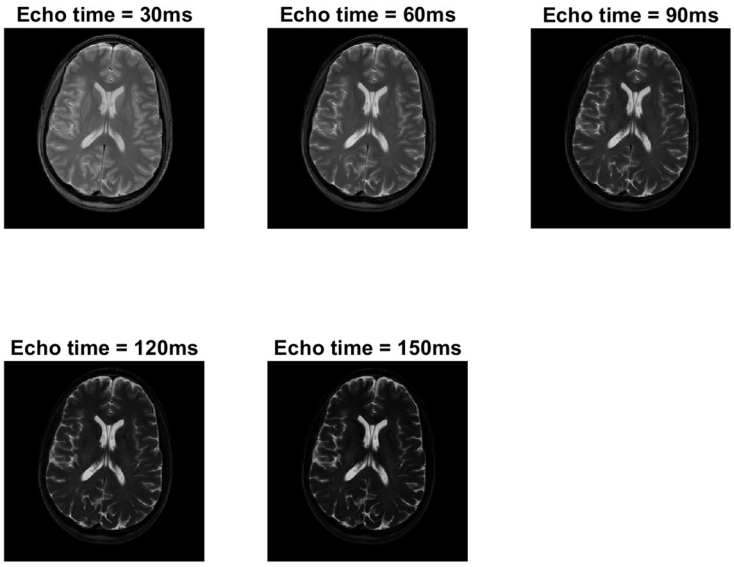
Aside, T1 residual map exhibits more noises than T2, which can be caused by the popular imperfect 180 degree inversion of spins. Optimized pulse design with improved inversion efficiency can reduce this error.

Matlab code:

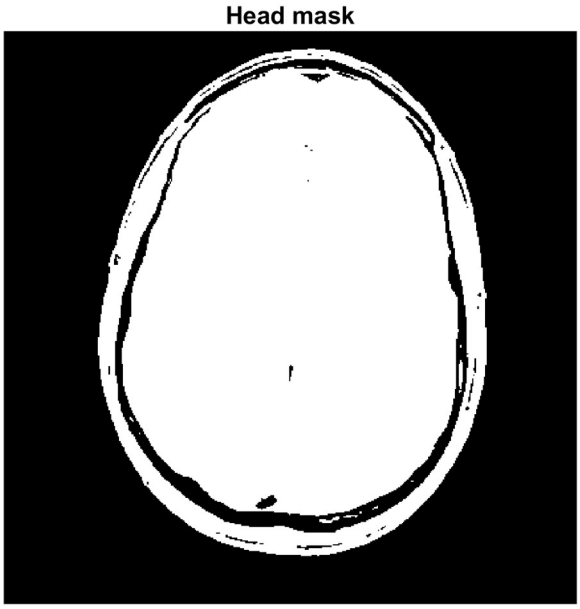
Project 1

Part I: Mapping T2 Relaxation Times

```
clc
clear
load('proj1aData_QFI')

[nRows, nCols, nTe] = size(image_3d) ;

figure
subplot(2,3,1)
imagesc(squeeze(image_3d(:,:,1)))
intLimits_v = get(gca,"CLim");
axis image
axis off
colormap(gray)
title(['Echo time = ', num2str(te_v(1)*1000),'ms'])
for index = 2:nTe
    subplot(2,3,index)
    imagesc(squeeze(image_3d(:,:,index)), intLimits_v)
    axis image
    axis off
    title(['Echo time = ', num2str(te_v(index)*1000),'ms'])
end
```

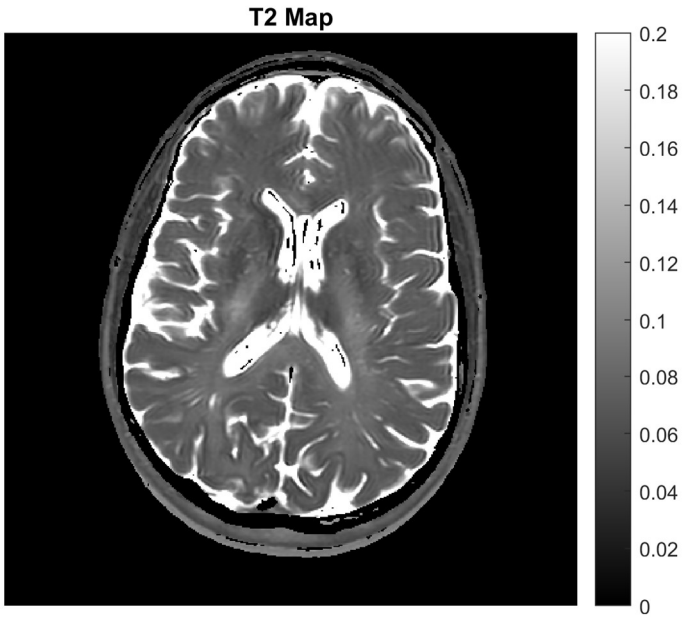



```
% 2. Use the magnitude of the first image to form a "binary mask" matrix

image_1 = squeeze(image_3d(:,:,1)) ;

mask_1 = (image_1 > 0.1 * max(image_1(:))) ;
figure
imagesc(mask_1)
colormap(gray)
axis image
axis off
title("Head mask")
```

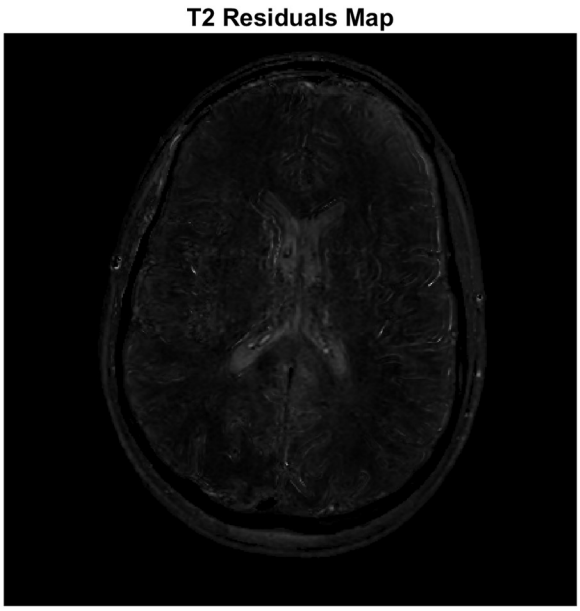



```
% 3. For each pixel in the head, calculate the time constant of the
% decaying signal.

t2_m = zeros(nRows, nCols) ;
s0_m = zeros(nRows, nCols) ;
norm_m = zeros(nRows, nCols) ;

for row = 1:nRows
    for col = 1:nCols
        if (mask_1(row,col)==1)
            signal_v = squeeze(image_3d(row,col,:)) ;
            coeff_v = polyfit(te_v, log(signal_v), 1) ;
            slope = coeff_v(1) ;
            logS0 = coeff_v(2) ;    % intercept
            S0 = exp(logS0) ;
            t2 = -1/slope ;
            % store results:
            t2_m(row,col) = min(t2,5) ;         % from class
            t2_m(row,col) = t2 ;
            s0_m(row,col) = S0 ;
            % calculate norm for each pixel:
            norm_m(row,col) = norm(signal_v-S0.*exp(-te_v./t2)) ;
        end
    end
end
```

```
% display the T2 map with values up to 0.2 seconds
imagesc(t2_m, [0,0.2])
title('T2 Map')
axis image
axis off
colorbar
```



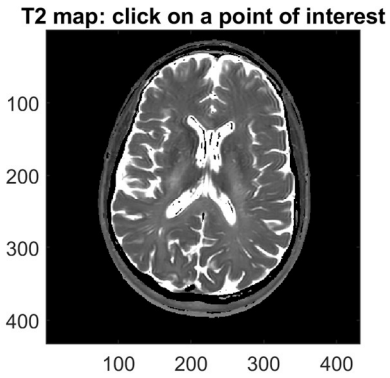

```
% t2 residual map:

figure
imagesc(norm_m)
colormap(gray)
axis image
axis off
title("T2 Residuals Map")
```




```
% 4. Make a new figure with two subplots. In the first show the T2 map:

% View fit:
figure
subplot(1, 2, 1)
t2Max = 0.2;                          % 0.2 s maximum value for display.
t2ceil_m = min(t2_m, t2Max*ones(nRows, nCols));
red0_m = t2ceil_m/t2Max;
green0_m = t2ceil_m/t2Max;
blue0_m = t2ceil_m/t2Max;
color_3d = cat(3, red0_m, green0_m, blue0_m);
image(color_3d)
axis image
title('T2 map: click on a point of interest')

% Construct a loop that allows the user of your program to select an
% arbitrary number of pixels to examine. For each selected pixel, plot the
% signal intensity versus TE along with the best-fit curve.

nPoints = 50;
contFlag = 1;
while(contFlag==1)

    % Get position of mouse-click on image:
    [x,y] = ginput(1);
    row = round(y);
```

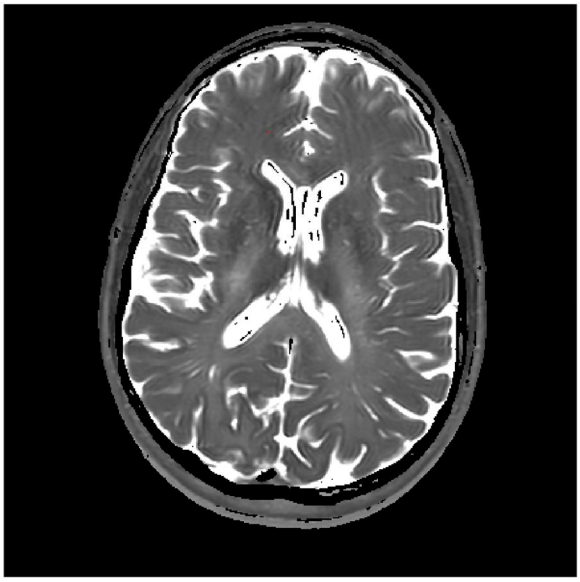
```
    col = round(x);
    % Exit loop if the mouse click is outside the image:
    if (row<1 | row>nRows | col<1 | col>nCols)
        contFlag = 0;
        continue
    end
    red_m = red0_m;
    green_m = green0_m;
    blue_m=blue0_m;
    % Show the position of the pixel in red:
    red_m(row,col)=1;
    green_m(row,col)=0;
    blue_m(row,col)=0;
    color_3d=cat(3,red_m,green_m,blue_m);
    image(color_3d)
    axis image
    title('T2 map: click on a point of interest')

    % Show fit:
    t2 = t2_m(row,col);
    S0 = s0_m(row,col);
    s_v = squeeze(image_3d(row,col,:));
    % create an array of nPoints TE values from the minimum to maximum TE:
    tiFit_v = linspace(min(te_v),max(te_v),nPoints);
    % find corresponding signal at each TE using estimates of T2 and s0:
    sFit_v = S0.*exp(-tiFit_v./t2) ;
    S0
    subplot(1,2,2)
    plot(tiFit_v,sFit_v,':',te_v,s_v,'+')
    title(['At row =', num2str(row),', col = ',num2str(col),': T2 = ', ...
        num2str(t2*1000),'ms'])
end
```

```
S0 = 1.2594e+04
```

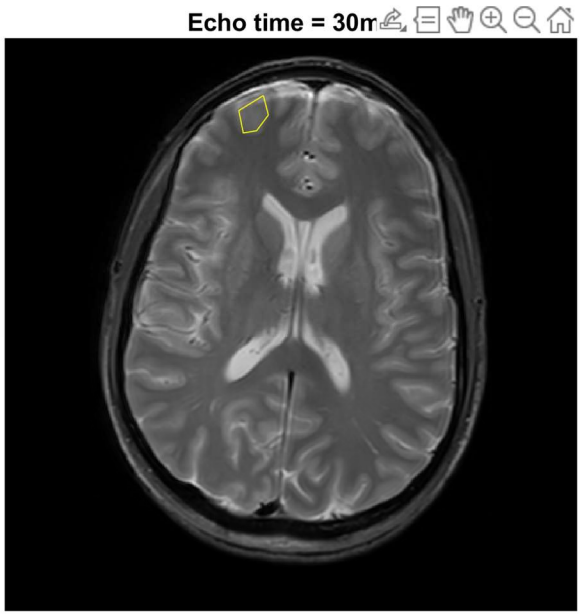



```
Error using ginput (line 84)
Interrupted by figure deletion
```

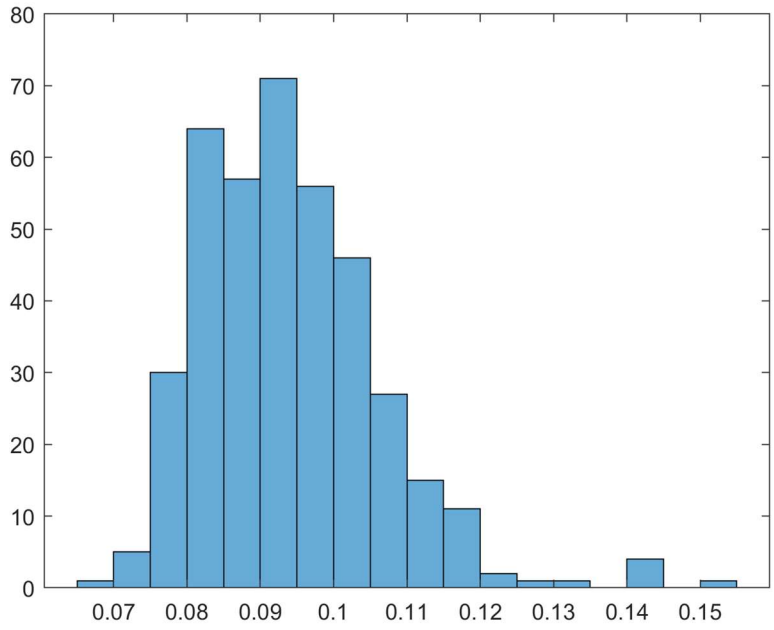
```
% 5. Display the T2 data as a gray scale map in a new figure.

image(color_3d)
axis image
axis off
colormap(gray)
datacursormode('on')
```



```
% 6. Identify the gray matter and white matter in the first image (shortest
% TE image). Look at examples on Google Images.

% 7. Ise the roipoly command to draw a region of interest (ROI) in each
% tissue (one ROI for gray matter, another for white matter). Make the
% regions as large as you can without including the other tissue type.

% plot first image again & draw polygon around gray matter

[nRows, nCols, nTe] = size(image_3d) ;

figure
image1 = imagesc(squeeze(image_3d(:,:,1))) ;
intLimits_v = get(gca,"CLim");
axis image
axis off
colormap(gray)
title(['Echo time = ', num2str(te_v(1)*1000),'ms'])

[mask_gray, x_v, y_v] = roipoly ;
line(x_v, y_v, 'Color', 'y') ;
```

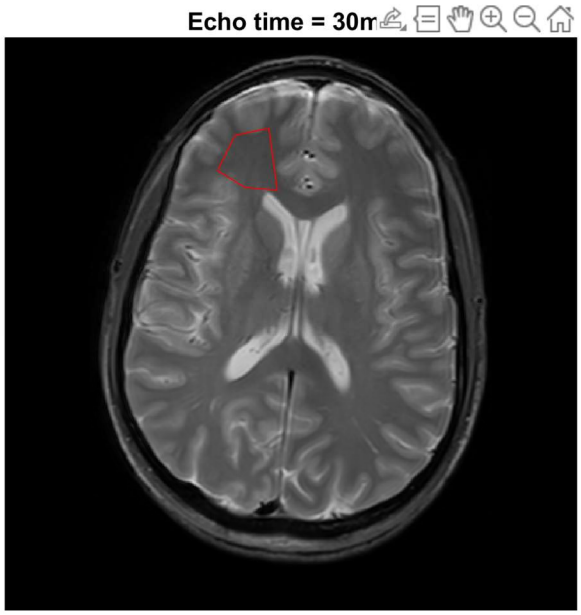



```
% Make a histogram of T2 values in each region and draw the ROI that the
% histogram represents. Find the mean and standard deviation of T2 for gray
% and white matter.
histogram(t2_m(mask_gray))
```

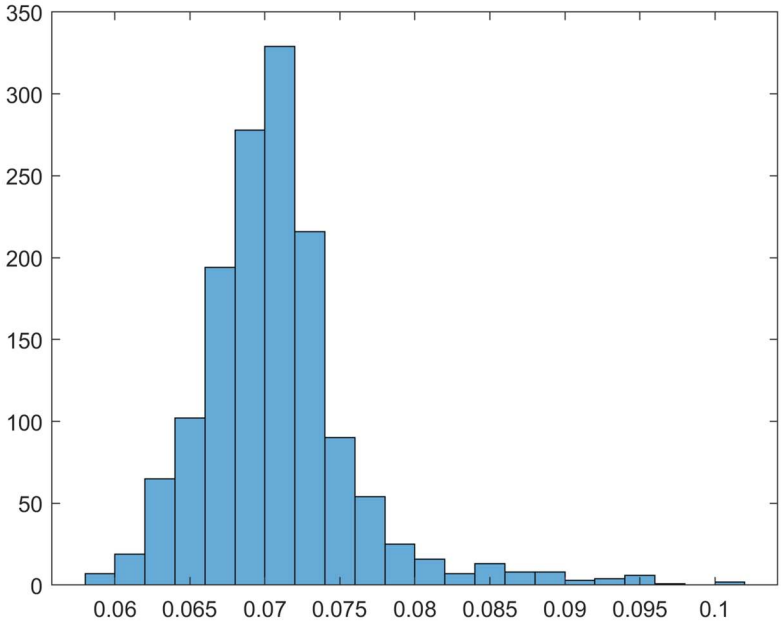



```
% plot first image again & draw polygon around white matter
```

```
[nRows, nCols, nTe] = size(image_3d) ;

figure
image1 = imagesc(squeeze(image_3d(:,:,1))) ;
intLimits_v = get(gca,"CLim");
axis image
axis off
colormap(gray)
title(['Echo time = ', num2str(te_v(1)*1000),'ms'])
datacursormode('on')

[mask_white, x_v, y_v] = roipoly ;
line(x_v, y_v, 'Color', 'r') ;
```




```
% Make a histogram of T2 values in each region and draw the ROI that the
% histogram represents.
histogram(t2_m(mask_white))
```




```
% Find the mean and standard deviation of T2 for gray
% and white matter.

t2_gray = t2_m(mask_gray) ;
mean_gray = mean(t2_gray(:))
```

```
mean_gray = 0.0942
```

```
std_gray = std(t2_gray(:))
```

```
std_gray = 0.0121
```

```

t2_white = t2_m(mask_white) ;
mean_white = mean(t2_white(:))
```

```
mean_white = 0.0707
```

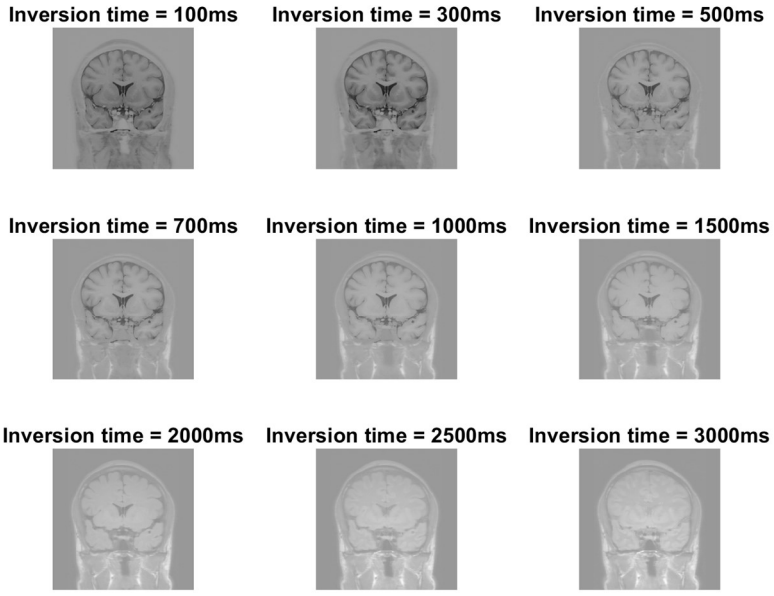
```
std_white = std(t2_white(:))
```

```
std_white = 0.0052
```

Part II: Mapping T1 Relaxation Times

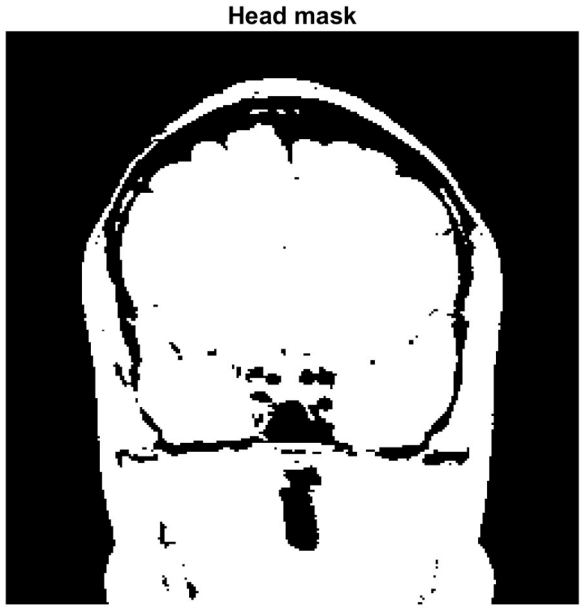
```
load('proj1bData_QFI')

[nRows, nCols, nTi] = size(irImage_3d) ;

% Display all inversion recovery images in one figure
```

```
figure
subplot(3,3,1)
intLimits_v = [min(irImage_3d(:)), max(irImage_3d(:))];
imagesc(squeeze(irImage_3d(:,:,1)), intLimits_v)
axis image
axis off
colormap(gray)
title(['Inversion time = ', num2str(ti_v(1)*1000),'ms'])
for index = 2:nTi
    subplot(3,3,index)
    imagesc(squeeze(irImage_3d(:,:,index)), intLimits_v)
    axis image
    axis off
    title(['Inversion time = ', num2str(ti_v(index)*1000),'ms'])
end
```



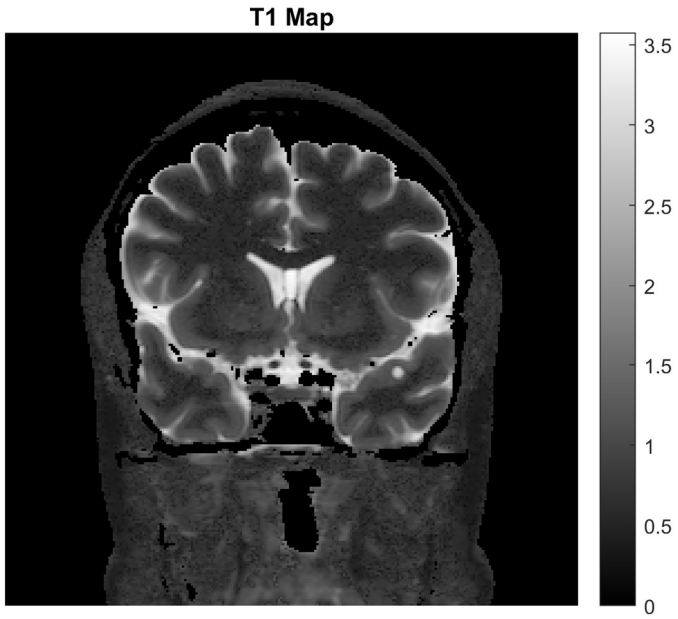

```
% 3. Estimate the T1 value for each pixel in the head and display the T1
% map for the entire slice. Also, calculate the residual of the fit (norm).
% Display your T1 and residual maps.

% first, make mask:

image_1 = squeeze(irImage_3d(:,:,end)) ;
```

```
mask_1 = (image_1 > 0.2 * max(image_1(:))) ;
figure
imagesc(mask_1)
colormap(gray)
axis image
axis off
title("Head mask")
```



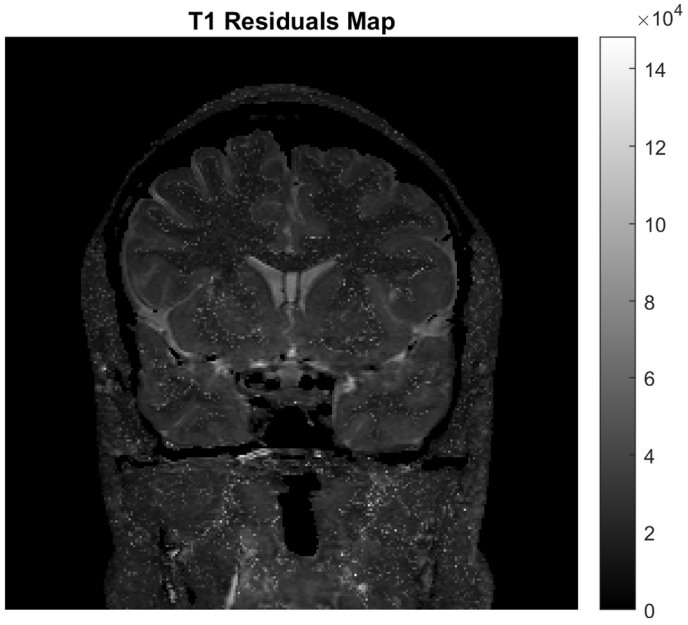

```
% calculate T1 for each pixel:

t1_m = zeros(nRows, nCols) ;
norm_m = zeros(nRows, nCols) ;

for row = 1:nRows
    for col = 1:nCols
        if (mask_1(row,col)==1)
            mz_v = squeeze(irImage_3d(row,col,:)) ;
            m0 = m0_m(row,col);
            inds =  find(mz_v < m0) ;
            coeff_v = polyfit(ti_v(inds), -log(0.5.*(1-(mz_v(inds)./m0))), 1) ;
            slope = coeff_v(1) ;
            t1 = 1/slope ;
```

```
            % store results:
            t1_m(row,col) = t1 ;

            % calculate norm for each pixel:
            norm_m(row,col) = norm(mz_v-m0.*(1-2.*exp(-ti_v./t1))) ;
        end
    end
end

% display the T1 for the entire slice
figure
imagesc(t1_m)
title('T1 Map')
axis image
axis off
colorbar
colormap(gray)
```




```
% display the residuals map
figure
imagesc(norm_m)
title('T1 Residuals Map')
axis image
```

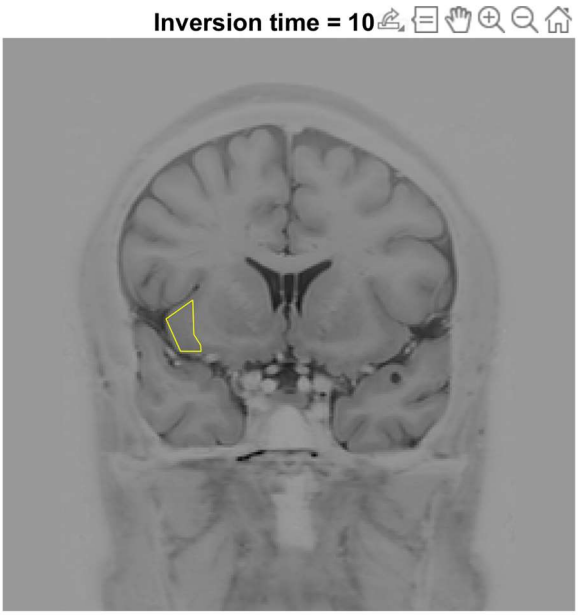
```
axis off
colorbar
colormap(gray)
```



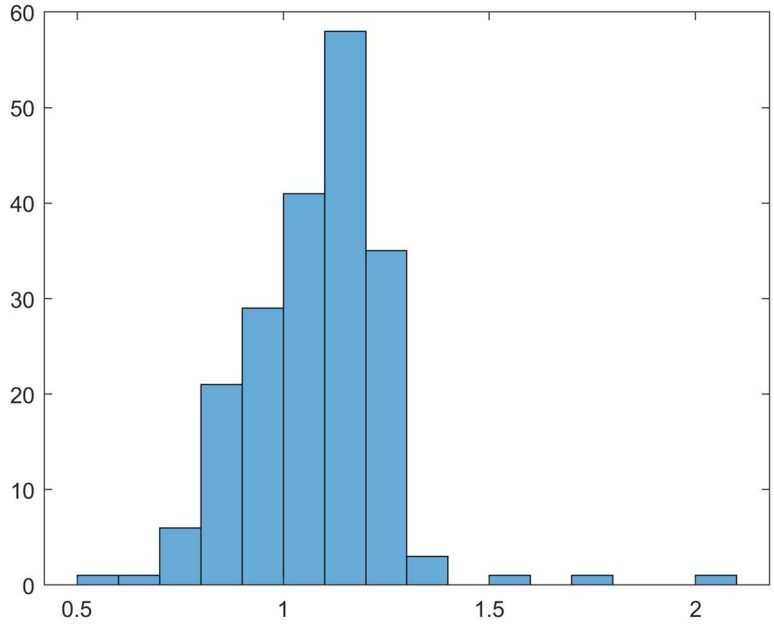

```
% 4. Add a loop to allow the user to show the T1 fit for an arbitrary
% number of pixels. For each selected pixel, plot the measured Mz versus T1
% along with the modeled curve using estaimate of T1.

% View fit:
figure
subplot(1, 2, 1)
t1Max = max(max(t1_m));
t1ceil_m = min(t1_m, t1Max*ones(nRows, nCols));
red0_m = t1ceil_m/t1Max;
green0_m = t1ceil_m/t1Max;
blue0_m = t1ceil_m/t1Max;
color_3d = cat(3, red0_m, green0_m, blue0_m);
image(color_3d)
axis image
title('T1 map: click on a point of interest')

% Construct a loop that allows the user of your program to select an
% arbitrary number of pixels to examine. For each selected pixel, plot the
% signal intensity versus TE along with the best-fit curve.
```

```
nPoints = 50;
contFlag = 1;
while(contFlag==1)

    % Get position of mouse-click on image:
    [x,y] = ginput(1);
    row = round(y);
    col = round(x);
    % Exit loop if the mouse click is outside the image:
    if (row<1 | row>nRows | col<1 | col>nCols)
        contFlag = 0;
        continue
    end
    red_m = red0_m;
    green_m = green0_m;
    blue_m=blue0_m;
    % Show the position of the pixel in red:
    red_m(row,col)=1;
    green_m(row,col)=0;
    blue_m(row,col)=0;
    color_3d=cat(3,red_m,green_m,blue_m);
    image(color_3d)
    axis image
    title('T1 map: click on a point of interest')

    % Show fit:
    t1 = t1_m(row,col);
    m0 = m0_m(row,col);
    mz_v = squeeze(irImage_3d(row,col,:));
    % create an array of nPoints Ti values from the minimum to maximum Ti:
    tiFit_v = linspace(min(ti_v),max(ti_v),nPoints);
    % find corresponding signal at each Ti using estimate of T1:
    mzFit_v = m0 .* (1-2.*exp(-tiFit_v./t1)) ;
    subplot(1,2,2)
    plot(tiFit_v,mzFit_v,':',ti_v,mz_v,'+')
    title(['At row =', num2str(row),', col = ',num2str(col),': T1 = ', ...
        num2str(t1*1000),'ms'])
end
% 5. Display the T1 map in a new figure. Use roipoly to define regions of
% interest in the gray matter and white matter. Plot histograms of the T1
% distributions in gray and white matter and show the ROIs used to extract
% the histogram data. Find the mean and standard deviation of T1 for gray
% and white matter.

figure
```

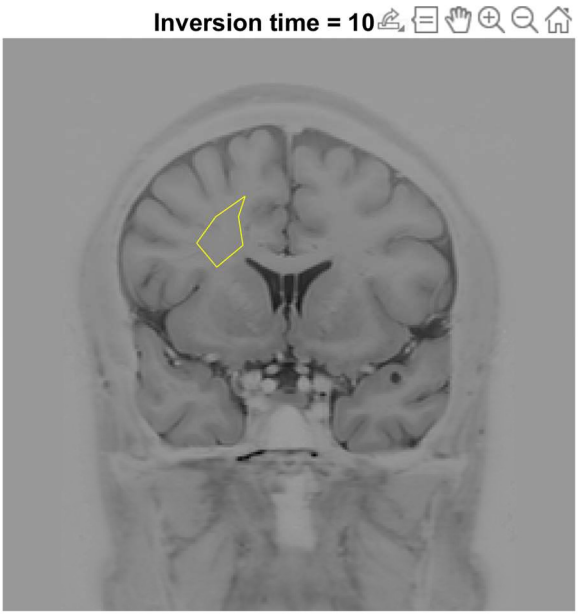
```
intLimits_v = [min(irImage_3d(:)), max(irImage_3d(:))];
imagesc(squeeze(irImage_3d(:,:,1)), intLimits_v)
axis image
axis off
colormap(gray)
title(['Inversion time = ', num2str(ti_v(1)*1000),'ms'])

[mask_gray, x_v, y_v] = roipoly ;
line(x_v, y_v, 'Color', 'y') ;
```



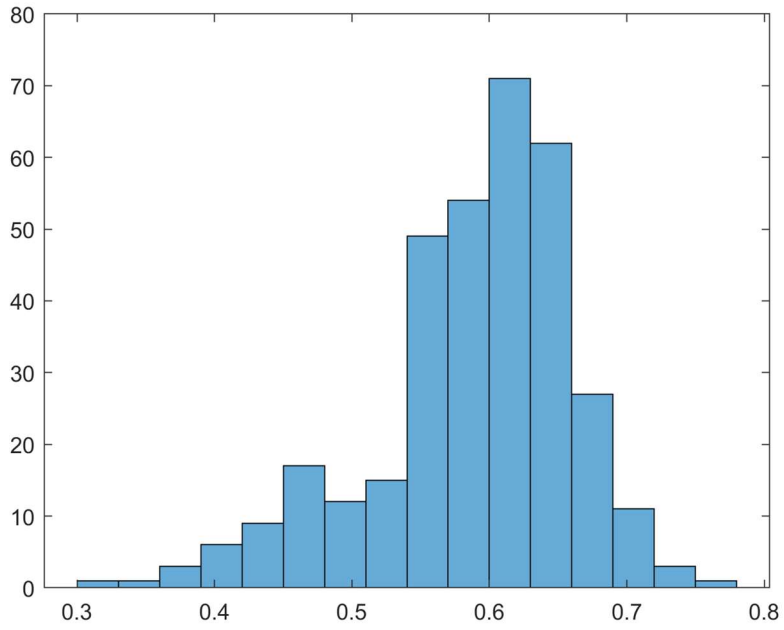

```
histogram(t1_m(mask_gray))
```




```
% define region of interest in white matter

figure
intLimits_v = [min(irImage_3d(:)), max(irImage_3d(:))];
imagesc(squeeze(irImage_3d(:,:,1)), intLimits_v)
axis image
axis off
colormap(gray)
title(['Inversion time = ', num2str(ti_v(1)*1000),'ms'])

[mask_white, x_v, y_v] = roipoly ;
line(x_v, y_v, 'Color', 'y') ;
```




```
histogram(t1_m(mask_white))
```




```
% Find the mean and standard deviation of T1 for gray
% and white matter.

t1_gray = t1_m(mask_gray) ;
```

```
mean_gray = mean(t1_gray(:))
```

```
mean_gray = 1.0776
```

```
std_gray = std(t1_gray(:))
```

```
std_gray = 0.1707
```

```
t1_white = t1_m(mask_white) ;
mean_white = mean(t1_white(:))
```

```
mean_white = 0.5885
```

```
std_white = std(t1_white(:))
```

```
std_white = 0.0734
```