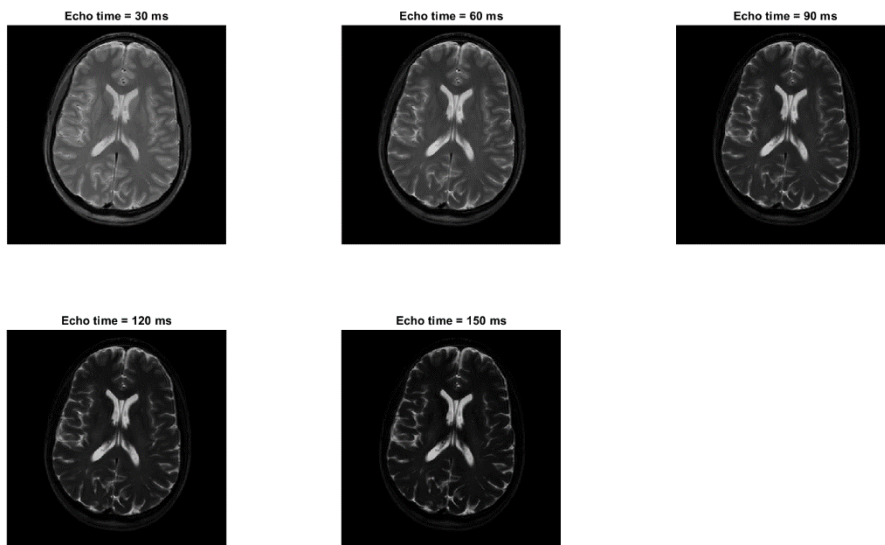


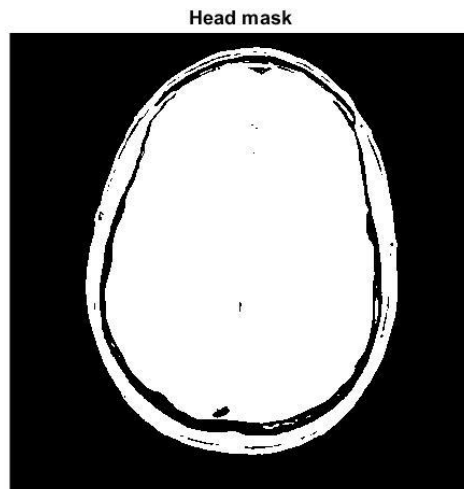
## Mapping $T_1$ and $T_2$ relaxation times in MRI

### Part 1: Mapping $T_2$ relaxation times

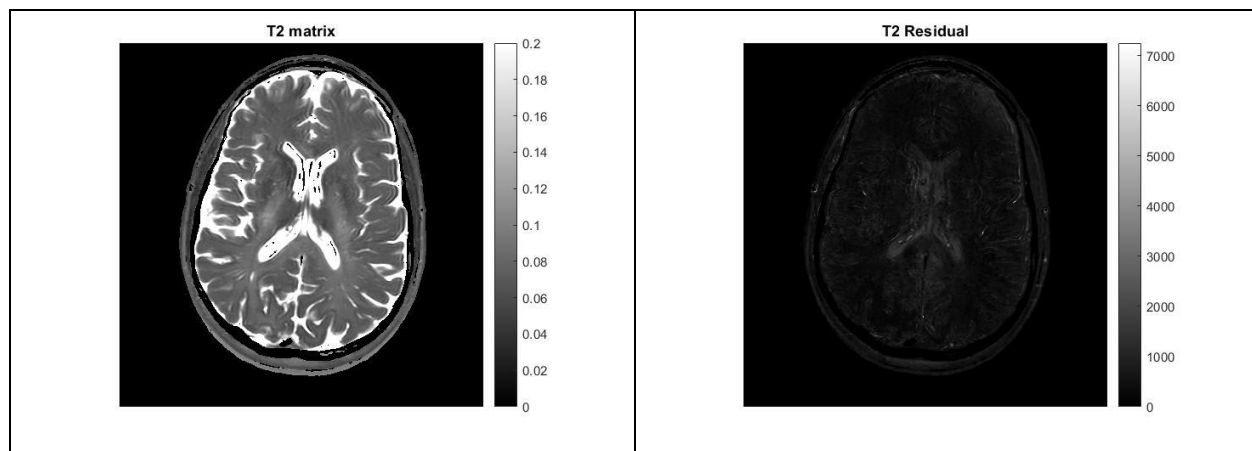
1. Display of all five images in one plot with calculated echo times



2. Create head mask

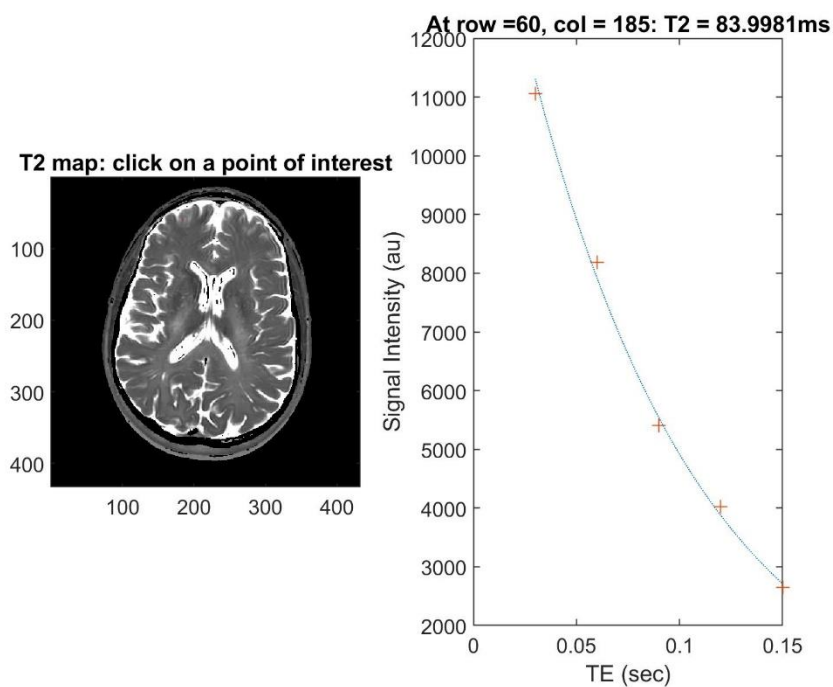


3. Fitted a line for each pixel and create a residual plot

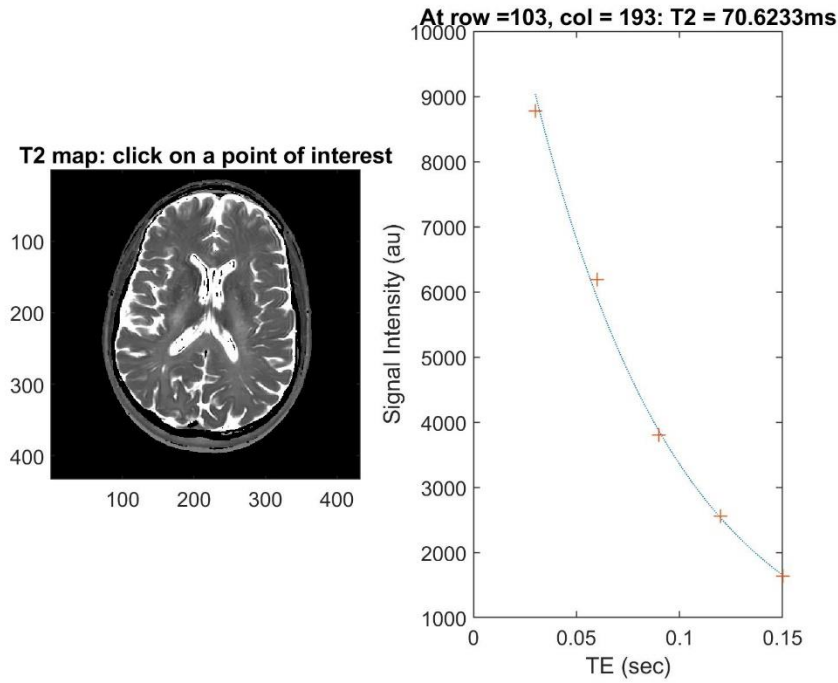


4. Make two plots which show the T2 map and TE with a best fit line

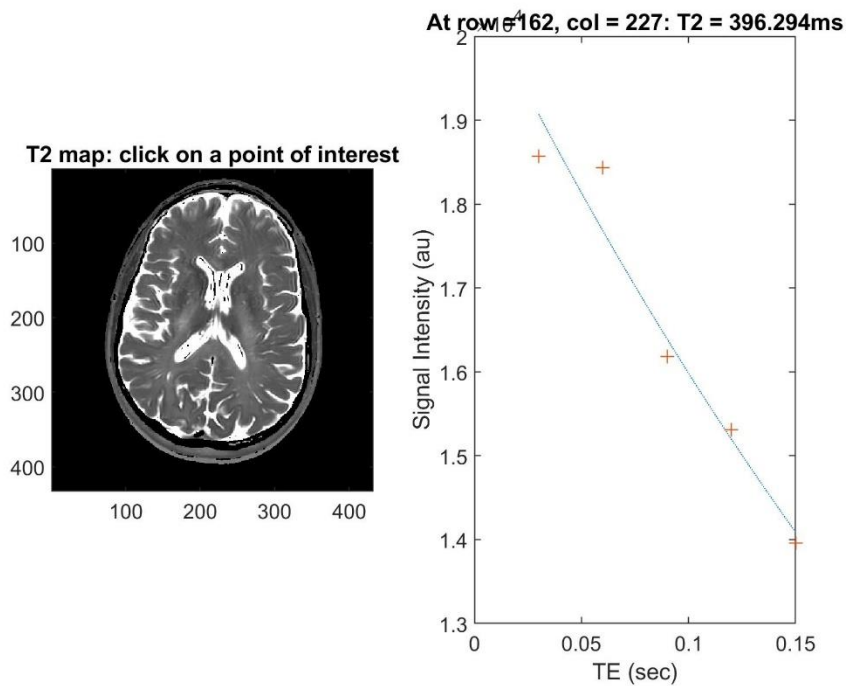
Example for a pixel from gray matter:



Example for a pixel from white matter:



Example for a pixel from CSF:



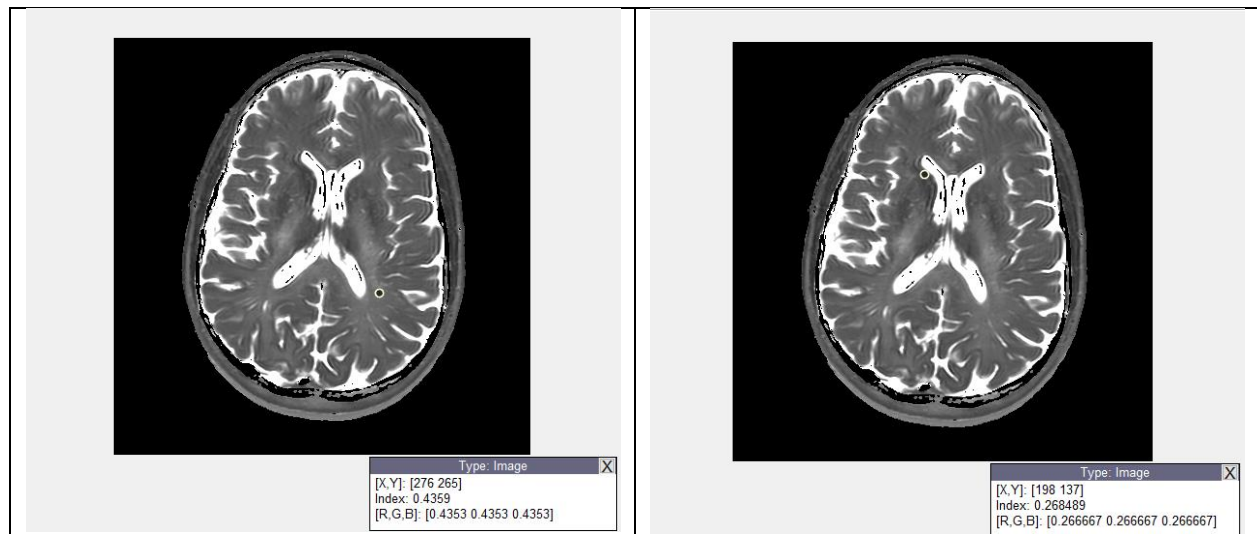
How well do they agree? Compare  $T_2$  values in the gray matter, white matter and cerebrospinal fluid (CSF)—how different are these?

- The model (shown by the curve) agrees closely with the data as shown in the above graphs.

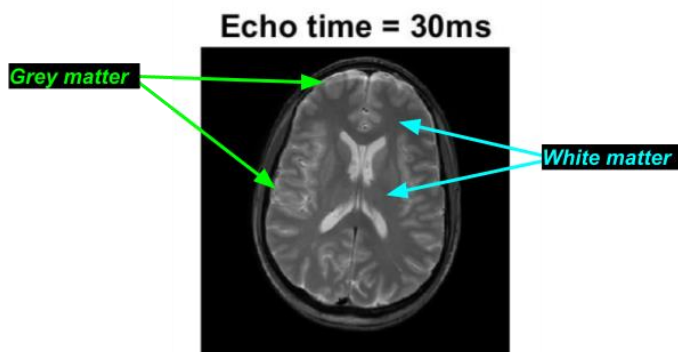
- The T2 values in the white matter are smaller than in the gray matter (ex: 70.6 ms versus 84.0 ms).
- However, T2 values in the CSF are much larger than in either the gray or white matter (ex: 396.3 ms).

##### 5. T2 data as a gray scale

T2 value of pixel is denoted as 'index', with range in gray scale from [0 – 1].

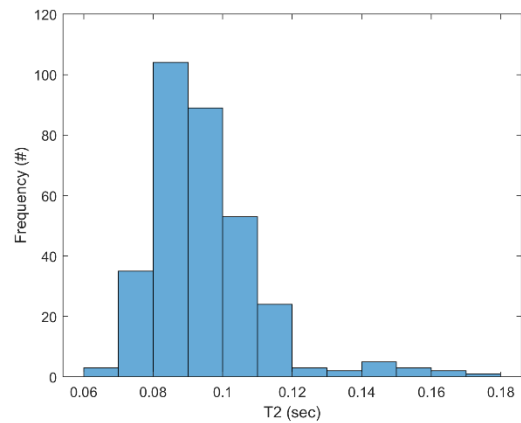
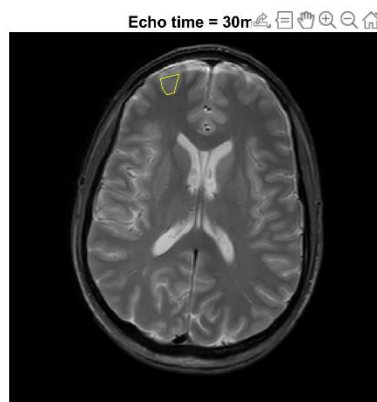


##### 6. Identify the gray matter and white matter in first image



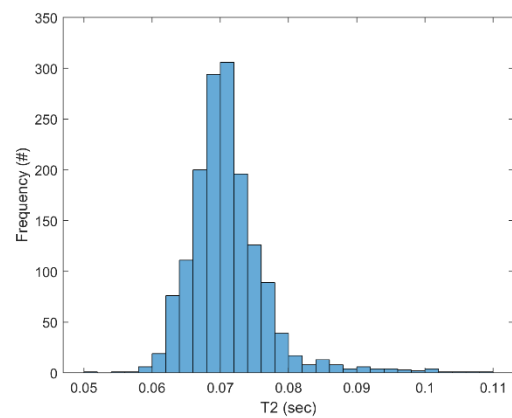
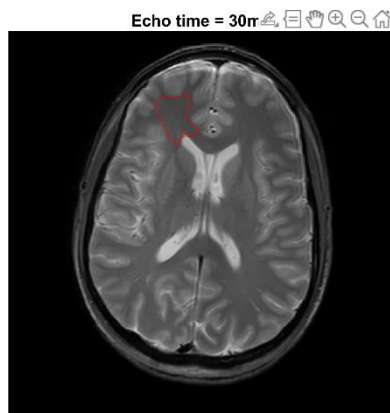
##### 7. Identify region of interest for gray and white matter, and show histograms

Gray Matter:



Gray matter  $T_2$  mean:  $94.2 \pm 12.5$  ms (converted)

White Matter:

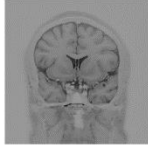


White matter  $T_2$  mean:  $70.1 \pm 5.2$  ms (converted)

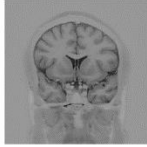
## Part II: Mapping $T_1$ relaxation times

1. Inversion recovery images with positive and negative  $M_z$  values

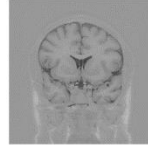
**Inversion time = 100ms**



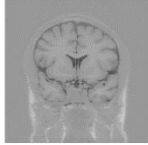
**Inversion time = 300ms**



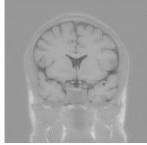
**Inversion time = 500ms**



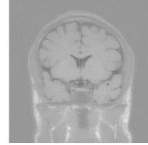
**Inversion time = 700ms**



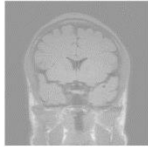
**Inversion time = 1000ms**



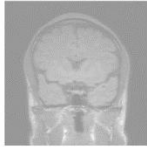
**Inversion time = 1500ms**



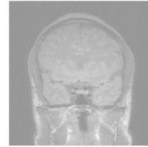
**Inversion time = 2000ms**



**Inversion time = 2500ms**



**Inversion time = 3000ms**

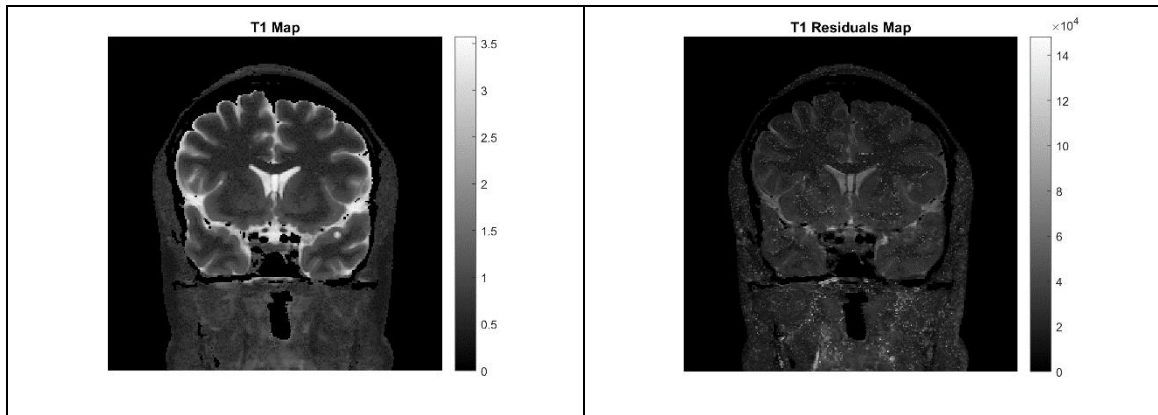


2.  $T_1$  map and calculate the residual of the fit

Create head mask

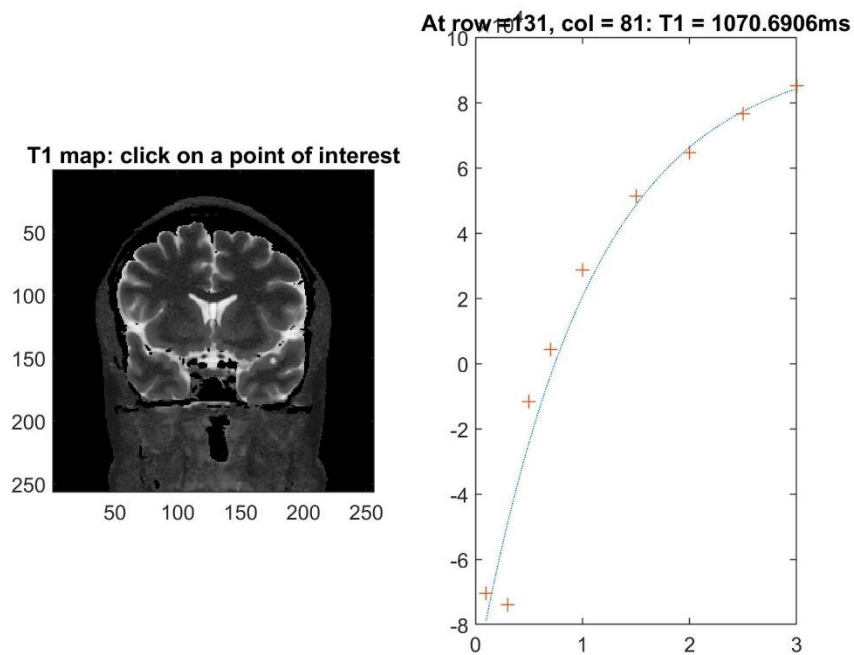
Head mask T1



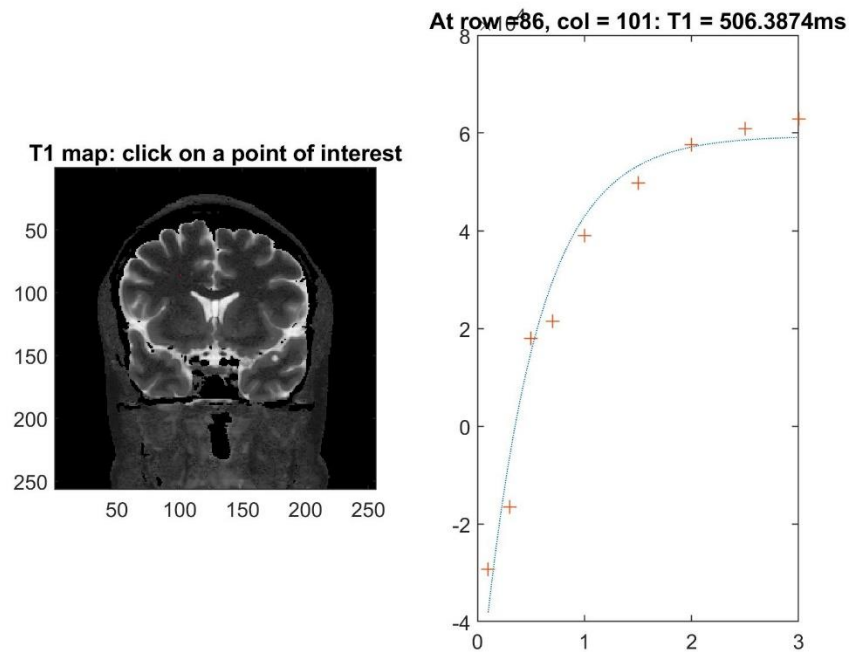


3. Plot Mz and T1 with modeled curve

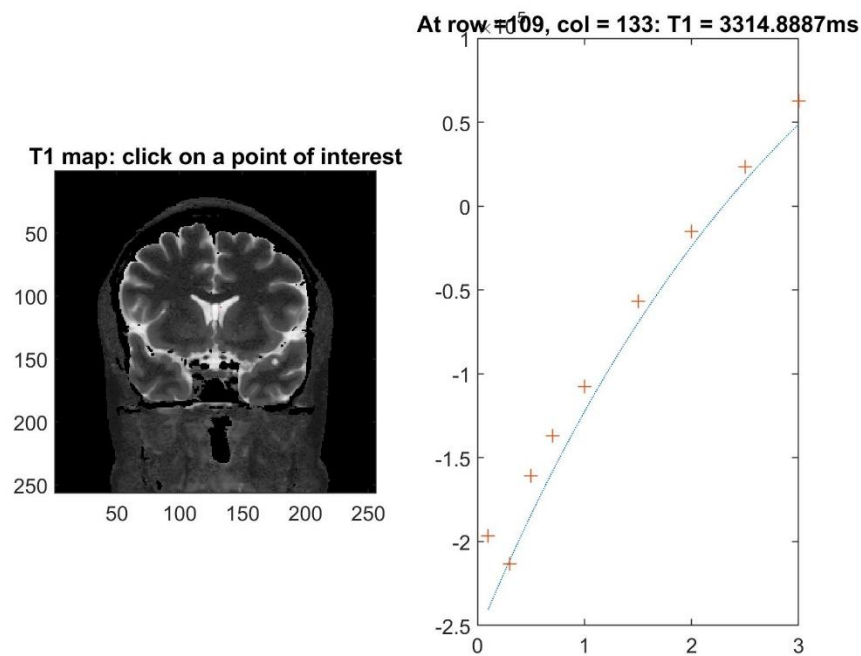
Example for a gray matter pixel:



Example for a white matter pixel:



Example for a CSF pixel:

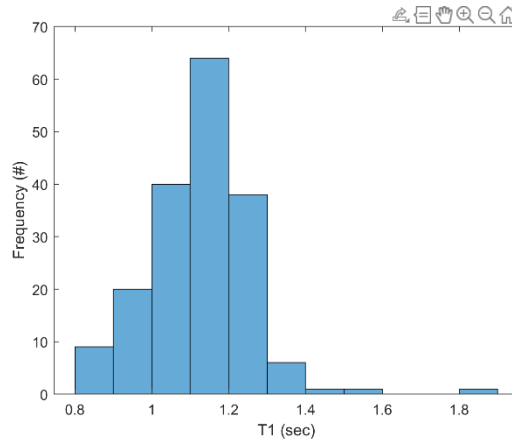
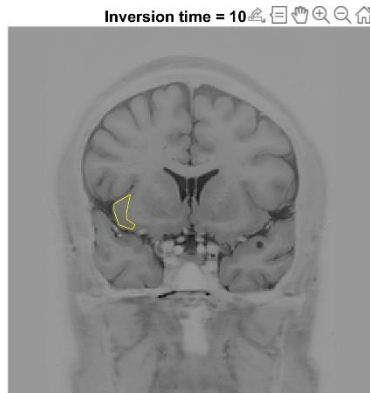


T1 fit is a bit better for gray matter.

4. Identify region of interest for gray and white matter, and show histograms

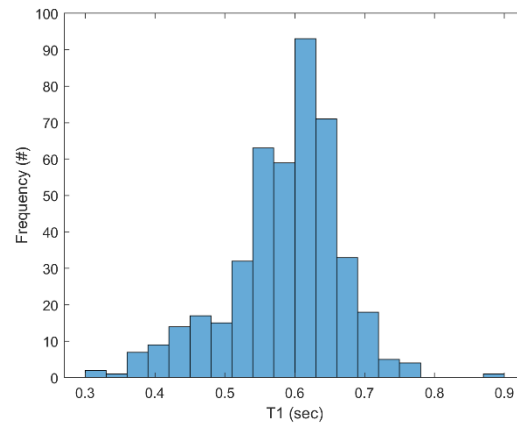
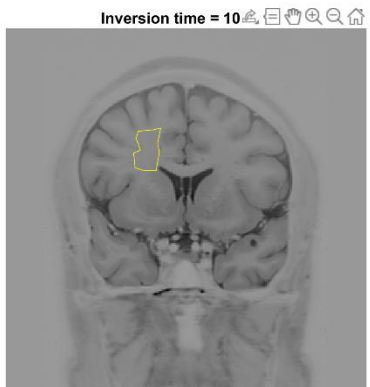


## Gray Matter:



Gray matter  $T_1$  mean:  $1070.6 \pm 170.7$  ms (converted)

## White Matter:

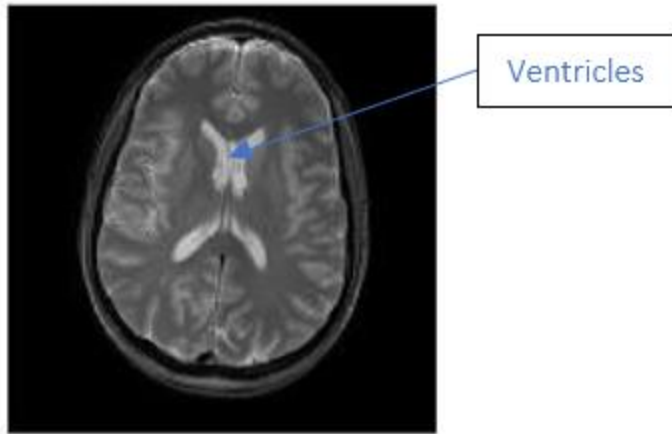


White matter  $T_2$  mean:  $588.5 \pm 73.4$  ms (converted)

## Questions

1. The large, bright “X” shaped structure in the center of the brain in part 1 is part of the ventricular system. The ventricles are cavities in the brain filled with cerebral spinal fluid (CSF). Based on the signal decay in the five  $T_2$ -weighted images, does the CSF have longer or shorter  $T_2$  relaxation time than brain tissue? (Explain using qualitative observations, not your fitting results).

Based on the signal decay in the five  $T_2$ -weighted images, we see that CSF has a longer  $T_2$  relaxation time than brain. This part of the brain is mainly water (a small molecule), thus slow decay and longer  $T_2$  time. This is why CSF (or the ventricles) appear brighter than tissue in the image: the signal intensity is higher in CSF than in tissue.



2. What is a typical  $T_2$  value for brain tissue? How much variation do you see over the brain?

Wansapura et al. reported average  $T_2$  values for white matter in various locations between 74 and 84 ms. We saw an average of about 71 ms in the example provided. The same article showed  $T_2$  values for gray matter in various locations between 98-132 ms. We saw an average of about 94 ms. We saw standard deviations of 5.2 and 12.1 ms in our examples of gray and white matter, respectively.

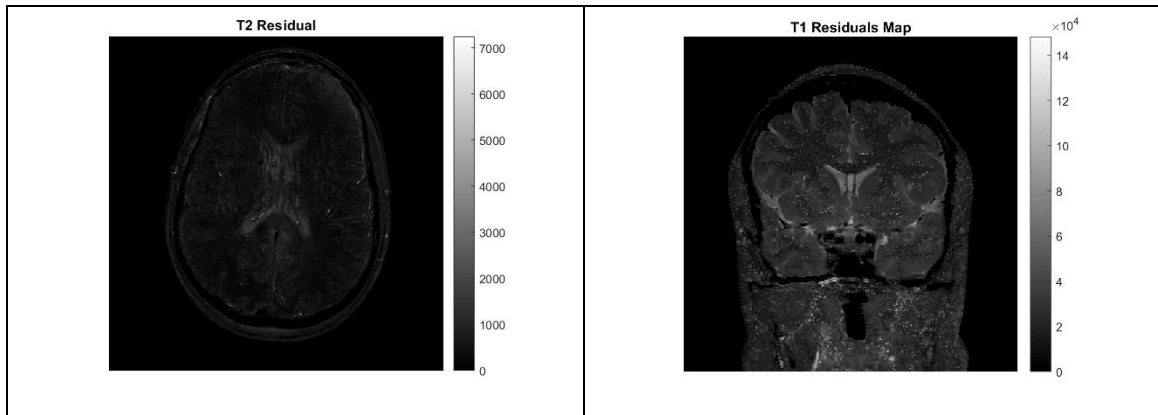
There is some variation in the brain depending on how the ROI was drawn as well as the location picked. Note that MRIs are shift invariant, so this may contribute to variations in  $T_2$ . Additionally, even though the same machine was used, different models or operators may affect the results.

Wansapura JP, Holland SK, Dunn RS, Ball WS Jr. NMR relaxation times in the human brain at 3.0 tesla. *J Magn Reson Imaging*. 1999 Apr;9(4):531-8. doi: 10.1002/(sici)1522-2586(199904)9:4<531::aid-jmri4>3.0.co;2-l. PMID: 10232510.

3. Are the residuals of the fit uniform in the head? If not, where are they larger and where are they smaller and what might cause the variation?

There are some signals which are significantly higher than others. These are noisy signals that come from the machine itself and are not physiologically relevant.

Physiologically speaking there are different residuals based on tissue. For example, in the  $T_2$  residual map, we can clearly see the ventricles (CSF fluid). Similarly, we see this same distinction in the  $T_1$  residual map. This difference is because of the different signals being propagated within the regions of interest. When you take the norm of each pixel, you find the magnitude of the signal which is dependent on the tissue which corresponds to the pixel.



4. How well can you distinguish between gray matter and white matter on the basis of  $T_2$  values in this map? What is the contrast in  $T_2$  values between gray and white matter (i.e., mean  $T_2$  for the gray matter ROI minus the mean  $T_2$  for the white matter ROI)? If the 'noise' in this measurement is the standard deviation of the difference between  $T_2$  values,  $\sigma_{T_2(GM)-T_2(WM)}$ , what is the contrast-to-noise ratio, CNR, between gray and white matter in the  $T_2$  map? Use the propagation of errors to find an expression for  $\sigma_{T_2(GM)-T_2(WM)}$  in terms of  $\sigma_{T_2(GM)}$  and  $\sigma_{T_2(WM)}$ .
- Gray and white matter seem to be relatively distinguishable using  $T_2$  values. There is some variation. However, the averages are more than 3 standard deviations away from each other, meaning they are probably statistically significantly different.
  - The contrast in  $T_2$  values is  $(94.2\text{ms} - 70.7\text{ ms}) = 23.5\text{ ms}$ .
  - The noise (standard deviation of the difference between  $T_2$  values) is 13.17 ms. The following equation was used to calculate this:  $\sigma_f = \sqrt{\sigma_g^2 + \sigma_w^2}$ , assuming that covariance is zero.
  - Knowing that  $SNR = \frac{S}{\sigma}$ , the contrast to noise ratio (CNR) would be  $CNR = \frac{23.5}{13.17} = 1.7844$

$$\sigma^2(f) = \sigma^2(s_1) \left( \frac{\partial f}{\partial s_1} \right)^2 + \sigma^2(s_2) \left( \frac{\partial f}{\partial s_2} \right)^2$$

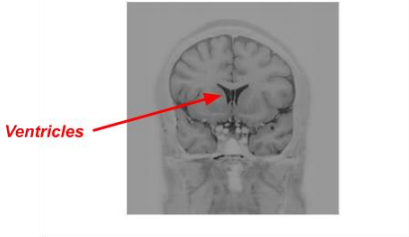
$$\text{Let } f = s_1 - s_2 \quad (\text{Contrast eq})$$

$$\sigma^2(f) = \sigma^2(s_1) \cdot 1^2 + \sigma^2(s_2) \cdot 1^2$$

$$\sigma(f) = \sqrt{\sigma^2(s_1) + \sigma^2(s_2)}$$

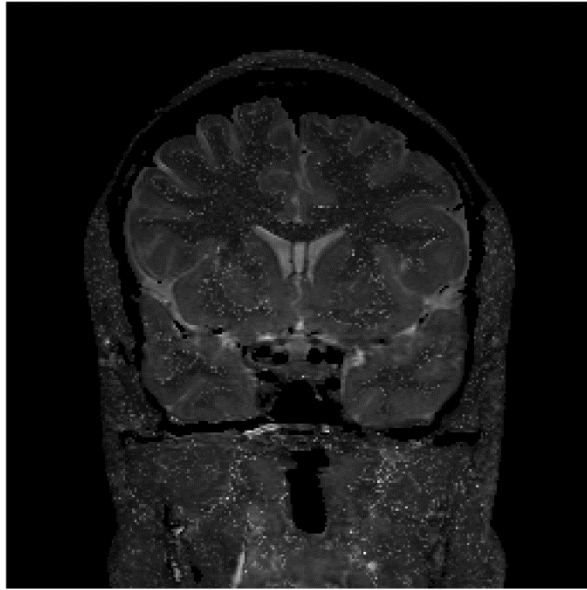
5. Find the lateral ventricles in the inversion recovery images. Based on the  $M_z$  recovery as a function of TI, does CSF have longer or shorter  $T_1$  relaxation time than brain tissue? (Again, explain using your qualitative observations, not the fitting results).

**Inversion time = 100ms**



- The CSF appears “dark” in the inversion recovery images, meaning that the signal in the CSF is taking longer to recover towards  $M_0$  than other tissue in the brain.
  - If  $T_1$  is longer, then  $M_z$  recovers more slowly towards  $M_0$ . Thus,  $T_1$  in the CSF is longer than in other brain tissues.
6. What is a typical  $T_1$  value for brain tissue? How much variation do you observe over the brain?
- A typical  $T_1$  value depends on the type of brain tissue. For instance, a typical  $T_1$  value for white matter is around 500 ms, a typical  $T_1$  value for gray matter is around 1000 ms, and a typical  $T_1$  value for the CSF is around 3000 ms. Thus, there is significant variation in  $T_1$  across the brain depending on the tissue type.
  - Specifically, based on the ROIs created for regions of gray and white matter, the mean  $T_1$  for gray matter was 1.0776 seconds and the standard deviation was 0.1707 seconds. For white matter, the mean  $T_1$  was 0.5885 seconds and the standard deviation was 0.0734 seconds.
  - Across the whole brain, the standard deviation was calculated to be 0.6274 seconds (or a variance of 0.3936 seconds<sup>2</sup>) which is a large amount of variation due to the different compositions of different tissue types in the brain.
7. Do the estimated (modeled) recovery curves match the measured data well? Are the residuals similar for different tissue types?
- The modeled recovery curves appear to match the measured data well based on visual inspection of the curves.
  - The average norm between the model and the measured data (across the whole brain) was  $1.28e4$ .
  - The average norm for the gray matter ROI was  $2.9655e4$  whereas the average norm for the white matter ROI was  $1.7367e4$ . Therefore, the residuals were somewhat larger for the gray matter than for the white matter. This is supported by the map of the residuals below. As shown, the gray matter appears brighter than the white matter regions, indicating higher residual values in the gray matter. However, the CSF appears the brightest on the residual map, indicating the largest residual values.

**T1 Residuals Map**



8. How well can you distinguish between gray matter and white matter on the basis of  $T_1$ ? What is the contrast in  $T_1$  values between gray and white matter? If the 'noise' in this measurement is the standard deviation of  $T_1$  values around the means, what is the contrast-to-noise ratio, CNR, between gray and white matter in the  $T_1$  map?

White and grey matter can be distinctively distinguished using  $T_1$  contrast. Contrast in  $T_1$  values between gray and white matter (i.e. mean  $T_1$  for the gray matter ROI minus the mean  $T_2$  for the white matter ROI) is around  $1.0776 - 0.5885 = 0.4891$  sec or 489.1ms.

To calculate the noise (standard deviation of the difference between  $T_1$  values of gray and white matter), we use equation  $\sigma_f = \sqrt{\sigma_g^2 + \sigma_w^2}$  similar to  $T_2$ . This gives  $\sigma_f = 113.6$  ms.

$$\text{Thus the } CNR = \frac{S}{\sigma_f} = \frac{489.1}{113.6} = 4.30$$

9. Which relaxation time,  $T_1$  or  $T_2$ , provides the highest CNR between gray and white matter? If you had to classify each pixel in the brain as either gray or white matter (ignoring CSF), which relaxation time map would you use?

It is clear CNR between gray and white matter using  $T_1$  ( $CNR = 4.30$ ) is higher than  $T_2$  ( $CNR = 1.78$ ). Thus  $T_1$  relaxation time map is more preferable for distinguishing between white and gray matter quantitatively.

10. Ideally, the residual maps reflect only random noise. Do you see evidence for non-random errors in the  $T_1$  or  $T_2$  fits?

In both T1 & T2 map, there are regions of high residual signals which can be caused by spins being impacted by its neighbors.

Aside, T1 residual map exhibits more noises than T2, which can be caused by the popular imperfect 180 degree inversion of spins. Optimized pulse design with improved inversion efficiency can reduce this error.

Matlab code: