

Autoregressive (AR) Signal Model

also known as

Linear Prediction
or All-pole Model

The difference equation is (P^{th} order model)

$$x[n] = a_1 x[n-1] + a_2 x[n-2] + \dots + a_p x[n-p] + w[n]$$

↑
modelling
error,
prediction
error

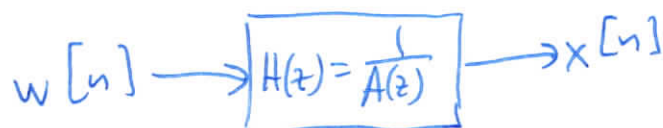
$x[n]$ is prediction by a linear combination of its P past values, i.e., Linear Prediction.

Take the z -Transform

$$X(z) = a_1 z^{-1} X(z) + \dots + a_p z^{-p} X(z) + W(z)$$

$$H(z) = \frac{X(z)}{W(z)} = \frac{1}{1 - a_1 z^{-1} - \dots - a_p z^{-p}} = \frac{1}{A(z)}$$

All-pole model



We observe $x[n]$ over $n=0, 1, \dots, N-1$.

Starting at $n=P$ we get

$$x[P] = a_1 x[P-1] + \dots + a_p x[0] + w[P]$$

$$x[P+1] = a_1 x[P] + \dots + a_p x[1] + w[P+1]$$

$$\vdots$$

$$x[N-1] = a_1 x[N-2]$$

$$\begin{bmatrix} x[P] \\ x[P+1] \\ \vdots \\ x[N-1] \end{bmatrix} = \begin{bmatrix} x[P-1] & x[P-2] & \dots & x[0] \\ x[P] & x[P-1] & & \vdots \\ \vdots & & \ddots & \\ x[N-2] & & & \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_p \end{bmatrix} + \begin{bmatrix} w[P] \\ \vdots \\ w[N-1] \end{bmatrix}$$

$$\underline{xvec} = X \underline{a} + \underline{w}$$

$$\underline{w} = \underline{xvec} - X \underline{a}$$

Sum of squared errors

$$E = \underline{w}^T \underline{w} \quad \leftarrow \text{choose } \underline{a} \text{ to minimize } E.$$

This is the same as the overdetermined LS solution.

In Matlab, make sure \underline{x} is an $N \times 1$ column vector.

$$xvec = x((P+1):N);$$

$$Xfirstcolumn = x(P:(N-1));$$

$$Xfirstrow = x(P:(-1):1);$$

$$X = \text{toeplitz}(Xfirstcolumn, Xfirstrow);$$

$$ahatls = \text{pinv}(X) * xvec;$$

$$A = [1 \\ -ahatls];$$

A is the vector of $A(z)$ coefficients.