Thanh Chau Nguyen_HW1

1) $y[n] = x[n] * h[n]$

```
>> h=[2,6,2]

h =

     2     6     2

>> x=[2,-1,1,3,5]

x =

     2    -1     1     3     5

>> conv(x,h)

ans =

     4    10     0    10    30    36    10
```

$x[m] = 2 \quad -1 \quad 1 \quad 3 \quad 5$

$n=0$, $h[n-m]$

$= 2 \quad 6 \quad 2$

$n=1$, $h[n-m]$

$= \quad 2 \quad 6 \quad 2$

$n=2$, $h[n-m]$

$= \quad\quad 2 \quad 6 \quad 2$

$n=3$, $h[n-m]$

$= \quad\quad\quad 2 \quad 6 \quad 2$

$n=4$, $h[n-m]$

$= \quad\quad\quad\quad 2 \quad 6 \quad 2$

$n=5$, $h[n-m]$

$= \quad\quad\quad\quad\quad 2 \quad 6 \quad 2$

$n=6$, $h[n-m]$

$= \quad\quad\quad\quad\quad\quad 2 \quad 6 \quad 2$

$y[n] = 4 \quad 10 \quad 0 \quad 10 \quad 30 \quad 36 \quad 10$

Thus $y[n] = \begin{cases} 0 & (n<0 \ \& \ n>6) \\ [4, 10, 0, 10, 30, 36, 10] & \text{else} \end{cases}$

$\boxed{2}$ $$H(e^{j\omega}) = \frac{1+e^{-j\omega}}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{3}e^{-j\omega}\right)}$$

$$= \frac{A}{1-\frac{1}{2}e^{-j\omega}} + \frac{B}{1-\frac{1}{3}e^{-j\omega}}$$

So $$1+e^{-j\omega} = A\left(1-\frac{1}{3}e^{-j\omega}\right) + B\left(1-\frac{1}{2}e^{-j\omega}\right)$$

Then $$\begin{cases} 1 = A + B \\ 1 = \frac{-A}{3} + \frac{-B}{2} \end{cases} \Leftrightarrow \begin{cases} 2 = 2A + 2B \\ 6 = -2A - 3B \end{cases} \Leftrightarrow \begin{cases} B = -8 \\ A = 9 \end{cases}$$

Thus, $$H(e^{j\omega}) = \frac{9}{1-\frac{1}{2}e^{-j\omega}} - \frac{8}{1-\frac{1}{3}e^{-j\omega}}$$

And $$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) \cdot e^{j\omega n} d\omega$$ (since $H(e^{-j\omega})$ is periodic)

$$= \frac{1}{2\pi}\left(\int_{-\pi}^{\pi} \frac{9}{1-\frac{1}{2}e^{-j\omega}} e^{j\omega n} d\omega - \int_{-\pi}^{\pi} \frac{8}{1-\frac{1}{3}e^{-j\omega}} e^{j\omega n} d\omega\right)$$

$$= 9 \cdot \left(\frac{1}{2}\right)^n \cdot u[n] - 8\left(\frac{1}{3}\right)^n u[n]$$

$\boxed{4}$ $y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = x[n]$

FT:
$$Y(e^{j\omega}) - \frac{3}{4}(e^{-j\omega}) \cdot Y(e^{j\omega}) = X(e^{j\omega})$$ (time shift property)

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - \frac{3}{4} e^{-j\omega}}$$

Thus, $h[n] = \left(\frac{3}{4}\right)^n u[n]$ by formular $\sum a^n u[n] \longleftrightarrow \frac{1}{1 - ae^{-j\omega}}$, $|a| < 1$

$\boxed{5}$ $x[n] = 3 \left(\frac{1}{2}\right)^n u[n] + 4 \left(\frac{1}{3}\right)^n u[n]$

$$X(e^{j\omega}) = 3 \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n] e^{-j\omega n} + 4 \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n u[n] e^{-j\omega n}$$

$$= 3 \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n + 4 \sum_{n=0}^{\infty} \left(\frac{1}{3} e^{-j\omega}\right)^n$$

By summation identity $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$ if $|\alpha| < 1$, we have

$$X(e^{j\omega}) = \frac{3}{1 - \frac{1}{2} e^{-j\omega}} + \frac{4}{1 - \frac{1}{3} e^{-j\omega}}$$

$\boxed{3}$ $x[n] = e^{j\pi/3\, n}$ $\quad -\infty < n < +\infty$

$$H(e^{j\pi/3}) = \frac{1}{1 - 0.9e^{j\pi/3}}$$

We have $e^{j\pi/3} = \cos\frac{\pi}{3} + j\sin\frac{\pi}{3}$

$$= \frac{1}{2} + j\frac{\sqrt{3}}{2}$$

So $H(e^{j\pi/3}) = \dfrac{1}{1 - \frac{9}{10}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)} = \dfrac{1}{\frac{11}{20} - \frac{9\sqrt{3}}{20}j}$

Thus $|H(e^{j\pi/3})| = \left|\dfrac{1}{\frac{11}{20} - \frac{9\sqrt{3}}{20}j}\right| = \dfrac{|1|}{\left|\frac{11}{20} - \frac{9\sqrt{3}}{20}j\right|} = \dfrac{1}{\sqrt{\left(\frac{11}{20}\right)^2 + \left(\frac{9\sqrt{3}}{20}\right)^2}} = \dfrac{10\sqrt{91}}{91}$

$$\simeq 1.0483$$

Also, since $\angle H = \left(\dfrac{a+bj}{c+dj}\right) = \tan^{-1}\left(\dfrac{b}{a}\right) - \tan^{-1}\left(\dfrac{d}{c}\right)$

$$\angle H(e^{j\pi/3}) = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{-9\sqrt{3}}{11}\right) = 0.9563$$

For $y[n]$, let $\omega_0 = \pi/3$, we have:

$$y[n] = x[n] * h[n]$$

$$= \sum_{m=-\infty}^{\infty} h[m] \cdot x[n-m]$$

$$= \sum_{m=-\infty}^{\infty} h[m] \cdot e^{j\omega_0(n-m)}$$

$$= e^{j\omega_0 n} \sum_{m=-\infty}^{\infty} h[m] \cdot e^{-j\omega_0 m}$$

$$= e^{j\omega_0 n} \cdot H(e^{j\omega_0})$$

So,

$$y[n] = (e^{j\omega_0 n}) \cdot H(e^{j\omega_0})$$

$$= (e^{j\omega_0 n}) \cdot |H(e^{j\omega_0})| \cdot e^{j\angle H(e^{j\omega_0})}$$

$$= |H(e^{j\omega_0})|\, e^{j(\omega_0 n + \angle H(e^{j\omega_0}))}$$

$$= \frac{10\sqrt{91}}{91} \cdot e^{j(\pi/3\, n + 0.9563)}$$

(special case of convolution when $x[n]$ is a sinusoid)

Matlab

```
temp = 1-0.9*exp(1i*pi/3)
1i
H = 1/temp
```

```
temp = 0.5500 - 0.7794i
ans = 0.0000 + 1.0000i
H = 0.6044 + 0.8565i
```

```
H_Mag = abs(H)
H_angle=angle(H)
```

```
H_Mag = 1.0483
H_angle = 0.9563
```