

AR signal model applied to sinusoids

$$x[n] = \cos(\hat{\omega}_0 n) \quad n=0, 1, \dots, N-1$$

A single real sinusoid has an AR model order of 2, one for each of the two complex sinusoids that make it.

$$x[n] = \cos(\hat{\omega}_0 n) = \frac{1}{2} e^{j\hat{\omega}_0 n} + \frac{1}{2} e^{-j\hat{\omega}_0 n}$$

So if the signal contains K real sinusoids the true AR model would be $P = 2K$.

Sinusoids are perfectly predictable so the prediction error, $w[n]$, ideally would be zero.

Also the vector x_{vec} is perfectly predicted from the X matrix. Thus x_{vec} lies in the range space, spanned by the columns of X .

Also, the roots of the resulting $A(z)$ polynomial will have roots lying exactly on the unit circle.

So if

$$x[n] = \cos(\hat{\omega}_0 n) \quad n=0, 1, \dots, N-1$$

The resulting $A(z)$ will have poles at $e^{+j\hat{\omega}_0}$ and $e^{-j\hat{\omega}_0}$.

The frequency response of

$$H(z) = \frac{1}{A(z)}$$

will have a tall narrow peak at $\hat{\omega} = \hat{\omega}_0$ and another at $\hat{\omega} = -\hat{\omega}_0$.