System Identification

This is similar to the AR signal, except we observe both the input $x[n]$ and output $y[n]$.

$$x[n] \longrightarrow \boxed{H(z) = \frac{B(z)}{A(z)}} \longrightarrow y[n]$$

$$B(z) = b_0 + b_1 z^{-1} + \cdots + b_Q z^{-Q}$$

$$A(z) = 1 - a_1 z^{-1} - \cdots - a_P z^{-P}$$

$$y[n] = a_1 y[n-1] + \cdots + a_P x[n-P] + b_0 x[n] + b_1 x[n-1] + \cdots + b_Q x[n-Q]$$

We define $L = \max(P, Q)$.

$$\begin{bmatrix} y[L] \\ \vdots \\ y[N-1] \end{bmatrix} = \begin{bmatrix} y[L-1] & y[L-2] \cdots y[L-P] \\ \vdots & \ddots \\ y[N-2] & \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_P \end{bmatrix} + \begin{bmatrix} x[L] & x[L-1] \cdots & x[L-Q] \\ \vdots & \ddots & \\ x[N-1] & & \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_Q \end{bmatrix} + \begin{bmatrix} w[L] \\ \vdots \\ w[N-1] \end{bmatrix}$$

Matlab

x, y are N-1 column vectors
and P and Q are known integers.

```
L = max(P, Q);
N = length(x);

yvec = y((L+1):N);
Yfirstcolumn = y(L:(N-1));
Yfirstrow = y(L:(-1):(L-P+1));
Y = toeplitz(Yfirstcolumn, Yfirstrow);
Xfirstcolumn = x((L+1):N);
Xfirstrow = x((L+1):(-1):(L-Q+1));
X = toeplitz(Xfirstcolumn, Xfirstrow);

D = [Y X];
theta = pinv(D) * yvec;
B = theta((P+1):length(theta));
ahat = theta(1:P);
A = [1
     -ahat];
```