Autoregressive (AR) Signal Model

also known as

Linear Prediction

or All-pole Model

The difference equation is (P^{th} order model)

$$x[n] = a_1 x[n-1] + a_2 x[n-2] + \cdots + a_p x[n-p] + w[n]$$

modelling error, prediction error

$x[n]$ is prediction by a linear combination of its P past values, i.e., Linear Prediction.

Take the z-Transform

$$X(z) = a_1 z^{-1} X(z) + \cdots + a_p z^{-p} X(z) + W(z)$$

$$H(z) = \frac{X(z)}{W(z)} = \frac{1}{1 - a_1 z^{-1} - \cdots - a_p z^{-p}} = \frac{1}{A(z)}$$

All-pole model

$$w[n] \longrightarrow \boxed{H(z) = \frac{1}{A(z)}} \longrightarrow x[n]$$

We observe $x[n]$ over $n = 0, 1, \ldots, N-1$.

starting at $n = P$ we get

$$x[P] = a_1 x[P-1] + \cdots + a_p x[0] + w[P]$$
$$x[P+1] = a_1 x[P] + \quad + a_p x[1] + w[P+1]$$
$$\vdots \qquad\qquad \vdots$$
$$x[N-1] = a_1 x[N-2]$$

$$\begin{bmatrix} x[P] \\ x[P+1] \\ \vdots \\ x[N-1] \end{bmatrix} = \begin{bmatrix} x[P-1] & x[P-2] & \cdots & x[0] \\ x[P] & x[P-1] & & \vdots \\ \vdots & & \ddots & \vdots \\ x[N-2] & & & \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_p \end{bmatrix} + \begin{bmatrix} w[P] \\ \vdots \\ w[N-1] \end{bmatrix}$$

$$\underline{xvec} = X \underline{a} + \underline{w}$$

$$\underline{w} = \underline{xvec} - X \underline{a}$$

Sum of squared errors

$E = \underline{w}^T \underline{w}$ ← choose $\underline{a}$ to minimize E.

This is the same as the overdetermined LS solution.

In Matlab, make sure $\underline{x}$ is an $N \times 1$ column vector.

```
xvec = x( (P+1) : N);
Xfirstcolumn = x( P : (N-1) );
Xfirstrow = x( P : (-1) : 1 );
X = toeplitz (Xfirstcolumn, Xfirstrow);
ahatls = pinv( X) * xvec;
A = [1
     -ahatls ];
```

A is the vector of A(z) coefficients.