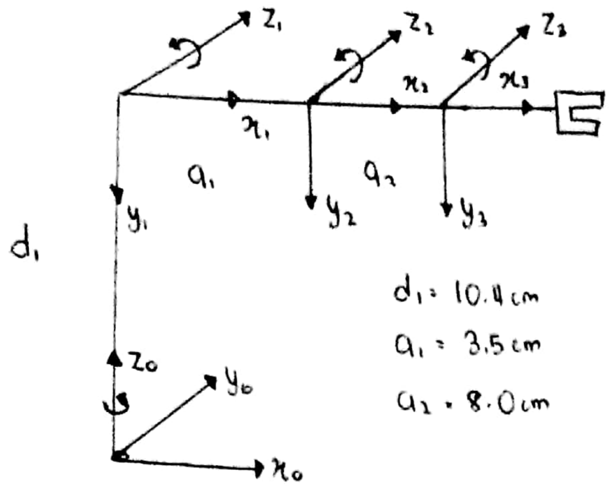

$d_1 = 10.4\,cm$

$a_1 = 3.5\,cm$

$a_2 = 8.0\,cm$

axis	θ	d	a	α
1	θ_1	d_1	0	-90
2	θ_2	0	a_1	0
3	θ_3	0	a_2	0

D-H TABLE

$${}^0_3T = {}^0_1T \; {}^1_2T \; {}^2_3T$$

$${}^0_1T = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^2_3T = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_2\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_2\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_1\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_1\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_1T\;{}^1_2T = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_1\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_1\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & -\sin\theta_1 & a_1\cos\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2 & -\sin\theta_1\sin\theta_2 & \cos\theta_1 & a_1\sin\theta_1\cos\theta_2 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & -a_1\sin\theta_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}^0_1T\;{}^1_2T\right]\left[{}^2_3T\right] = \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & -\sin\theta_1 & a_1\cos\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2 & -\sin\theta_1\sin\theta_2 & \cos\theta_1 & a_1\sin\theta_1\cos\theta_2 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & -a_1\sin\theta_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_2\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & a_2\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^0_1T\,{}^1_2T\,{}^2_3T = \begin{bmatrix} \cos\theta_1\cos\theta_2\cos\theta_3 & -\cos\theta_1\sin\theta_2\sin\theta_3 & -\sin\theta_1 & a_2\cos\theta_1\cos\theta_2\cos\theta_3 - a_2\cos\theta_1\sin\theta_2\sin\theta_3 + a_1\cos\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_1\sin\theta_2\sin\theta_3 & -\sin\theta_1\sin\theta_3\cos\theta_2 - \sin\theta_1\sin\theta_2\cos\theta_3 & \cos\theta_1 & a_1\sin\theta_1\cos\theta_2 + a_2\sin\theta_1\cos\theta_2\cos\theta_3 - a_2\sin\theta_1\sin\theta_2\sin\theta_3 \\ -\sin\theta_2\cos\theta_3 - \cos\theta_2\sin\theta_3 & \sin\theta_2\sin\theta_3 - \cos\theta_2\cos\theta_3 & 0 & -a_2\sin\theta_2\cos\theta_3 - a_1\sin\theta_2 \cancel{\cos} + d_1 - a_2\cos\theta_2\sin\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\begin{aligned}
P_x &= a_2\cos\theta_1\cos\theta_2\cos\theta_3 - a_2\cos\theta_1\sin\theta_2\sin\theta_3 + a_1\cos\theta_1\cos\theta_2 \\
&= a_2\cos\theta_1\cos(\theta_2+\theta_3) + a_1\cos\theta_1\cos\theta_2 \\
&= a_2\cos\theta_1\cos\theta_{23} + a_1\cos\theta_1\cos\theta_2
\end{aligned}
$$

$$
\begin{aligned}
P_y &= a_1\sin\theta_1\cos\theta_2 + a_2\sin\theta_1\cos\theta_2\cos\theta_3 - a_2\sin\theta_1\sin\theta_2\sin\theta_3 \\
&= a_1\sin\theta_1\cos\theta_2 + a_2\sin\theta_1\cos(\theta_2+\theta_3) \\
&= a_1\sin\theta_1\cos\theta_2 + a_2\sin\theta_1\cos\theta_{23}
\end{aligned}
$$

$$
\begin{aligned}
P_z &= -a_2\sin\theta_2\cos\theta_3 - a_1\sin\theta_2 + d_1 - a_2\cos\theta_2\sin\theta_3 \\
&= -a_2\sin\theta_2\cos\theta_3 - a_1\sin\theta_2 + d_1 - a_2\cos\theta_2\sin\theta_3 \\
&= -a_2\sin(\theta_2+\theta_3) - a_1\sin\theta_2 + d_1 \\
&= -a_2\sin\theta_{23} - a_1\sin\theta_2 + d_1
\end{aligned}
$$

$$
\begin{aligned}
r^2 &= P_x^2 + P_y^2 \\
r^2 &= (a_2\cos\theta_1\cos\theta_{23} + a_1\cos\theta_1\cos\theta_2)^2 + (a_1\sin\theta_1\cos\theta_2 + a_2\sin\theta_1\cos\theta_{23})^2 \\
r &= a_1\cos\theta_2 + a_2\cos\theta_{23}
\end{aligned}
$$

$$
\begin{aligned}
h &= P_z - d_1 \\
&= -a_2\sin\theta_{23} - a_1\sin\theta_2
\end{aligned}
$$

$$
\begin{aligned}
r^2+h^2 &= P_x^2 + P_y^2 + (P_z-h)^2 \\
&= a_1^2\cos^2\theta_2 + a_2^2\cos^2\theta_{23} + 2a_2a_1\cos\theta_2\cos\theta_{23} + a_1^2\sin^2\theta_2 \\
&\quad + 2a_1a_2\sin\theta_2\sin\theta_{23} + a_2^2\sin^2\theta_{23} \\
&= a_1^2[\cos^2\theta_2 + \sin^2\theta_2] + a_2^2[\cos^2\theta_{23} + \sin^2\theta_{23}] \cancel{+2a_1a_2} \\
&\quad + 2a_1a_2[\cos\theta_2\cos\theta_{23} + \sin\theta_2\sin\theta_{23}] \\
&= a_1^2 + a_2^2 + 2a_1a_2\cos(\cancel{\theta_2} + \theta_3 \cancel{-\theta_2}) \\
&= a_1^2 + a_2^2 + 2a_1a_2\cos\theta_3
\end{aligned}
$$

$$\therefore P_x^2 + P_y^2 + (P_z-d_1)^2 = r^2 + h^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\theta_3$$

$$\theta_3 = \frac{P_x^2 + P_y^2 + (P_z-d_1)^2 - a_1^2 + a_2^2}{2a_1a_2}$$

$$\cos^2\theta + \sin^2\theta = 1 \ ; \ \sin\theta = \pm(1-\cos^2\theta)^{\frac{1}{2}}$$

$$\therefore \theta_3 = ATAN2(\cancel{s\theta_3}\sin\theta_3, \cos\theta_3) \ \#$$

$\cancel{s\theta_1\cos\theta_3}$

$\cancel{\cos\theta_2\cos\theta_3}$

$r = a_1 \cos\theta_2 - a_2 \sin\theta_2 \sin\theta_3 + a_2 \cos\theta_2 \cos\theta_3$

$h = -a_1 \sin\theta_2 - a_2 \cos\theta_2 \sin\theta_3 - a_2 \sin\theta_2 \cos\theta_3$

let $x = a_2 \cos\theta_3 + a_1$

$y = a_2 \sin\theta_3$

$r = a_1 \cos\theta_2 - a_2 \sin\theta_2 \sin\theta_3 + a_2 \cos\theta_2 \cos\theta_3$

$= -y \sin\theta_2 + x \cos\theta_2$

$h = -a_1 \sin\theta_2 - a_2 \cos\theta_2 \sin\theta_3 - a_2 \sin\theta_2 \cos\theta_3$

$= -y \cos\theta_2 - x \sin\theta_2$

$H = \sqrt{x^2 + y^2}$ $\qquad \alpha = \tan^{-1}\left(\frac{y}{x}\right)$

$\therefore\ x = H \cos\alpha \ ; \ y = H \sin\alpha$

$r = H \cos\alpha \cos\theta_2 - H \sin\alpha \sin\theta_2$

$\frac{r}{H} \cancel{\frac{H}{H}} = \cos(\alpha + \theta_2)$

$h = -H \cos\alpha \sin\theta_2 - H \sin\alpha \cos\theta_2$

$\frac{h}{H} = -\sin(\alpha + \theta_2)$

$\therefore\ \frac{h}{r} = \frac{-\sin(\alpha+\theta_2)}{\cos(\alpha+\theta_2)} = -\tan(\alpha+\theta_2)$

$\tan(\alpha+\theta_2) = -h/r$

$\alpha + \theta_2 = \mathrm{ATAN2}(-h/r)$

$\therefore\ \theta_2 = \mathrm{ATAN2}(-h/r) - \mathrm{ATAN2}\left(\frac{y}{x}\right)$

$= \mathrm{ATAN2}(P_y, P_x)$ #

$\therefore\ \theta_1 = \mathrm{ATAN2}(P_y/P_x)$ #

Assume: $\theta_1 = 30^\circ$, $\theta_2 = 50^\circ$, $\theta_3 = 85^\circ$

$d_1 = 10.4\,cm$, $a_1 = 3.5\,cm$, $a_2 = 8.0\,cm$

$$
{}^0_1T\,{}^1_2T\,{}^2_3T = \begin{bmatrix}
\cos(30)\cos(50)\cos(85) & -\cos(30)\sin(50)\sin(85) & -\sin(30) & 8\cos(30)\cos(50)\cos(85) - 8\cos(30)\sin(50)\sin(85) + 3.5\cos(30)\cos(50) \\
\sin(30)\cos(50)\cos(85) & -\sin(30)\sin(85)\cos(50) - \sin(30)\sin(50)\cos(85) & \cos(30) & 3.5\sin(30) + 8\sin(30)\cos(50)\cos(85) - 8\sin(30)\sin(50)\sin(85) \\
-\sin(50)\cos(85) - \cos(50)\sin(85) & \sin(50)\sin(85) - \cos(50)\cos(85) & 0 & -8\sin(50)\cos(85) - 3.5\sin(50) + 10.4 - 8\cos(50)\sin(85) \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
{}^0_1T\,{}^1_2T\,{}^2_3T = \begin{bmatrix}
0.0485 & -0.6609 & -0.5 & -2.9506 \\
0.0280 & -0.3536 & 0.8660 & -1.0784 \\
-0.7071 & 0.7071 & 0 & 2.0620 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$\therefore$ the gripper will move to location

$x = -2.9506$

$y = -1.0784$

$z = 2.0620$