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20CS6037 — Fall 2014 **Homework 1** Due: 11:59PM 9/10/2014

Homework Notes: Add information here on your study group, number of hours you spent on the homework, and other relevant information.

Problem 1

You can write aligned equations as follows:

$$\begin{aligned} a &\sim p(a) \\ b &\sim p(b). \end{aligned}$$

(7 points) Pairwise independence does not imply mutual independence. Two random variables, X_i, i=1,2 are independent if
P(X_i | X_j) = P(X_i), for i,j=1,2, i $\neq$ j
and therefore

P(X_i, X_j) = P(X_j)
P(X_i | X_j) = P(X_i)P(X_j)

Now, given n random variables, we say that there are mutually independent if
P(X_i— X_S) = P(X_i) for all subsets S of {1, 2, $\cdots$, n}which do not contain i, and therefore
P(X_1, $\cdots$, X_n) = xP(X_1) $\cdots$ xP(X_n)

You can write inline equations: $a \sim p(b)$, or one line equations:

$$a \sim p(a).$$

(1) **Show that pairwise independence between all pairs of variables (X_i, X_j), does NOT imply mutual independence. Note: it is enough to give an example.**

Here is an example obtained from:
http://mnstats.morris.umn.edu/introstat/stat2611/independence.html

SUPPOSE A BOX CONTAINS 4 TICKETS LABELLED BY
331 323 233 333

LET US CHOOSE ONE TICKET AT RANDOM, AND CONSIDER THE RANDOM EVENTS
A1=1 OCCURS AT THE FIRST PLACE
A2=1 OCCURS AT THE SECOND PLACE
A3=1 OCCURS AT THE THIRD PLACE

P(A1)=1/2 P(A2)=1/2 P(A3)=1/2

A1A2=112 A1A3=121 A2A3=211
P(A1A2)=P(A1A3)=P(A2A3)=1/4.
So we conclude that the three events A1, A2, A3 are pairwise independent.

However

A1A2A3=f
P(A1A2A3)=0
P(A1)P(A2)P(A3)=$(1/2)^3$

$\therefore$ P(A1A2A3) $\neq$ P(A1)P(A2)P(A3)

(2) **Show mutual independence implies pairwise independence.**

For four events A, B, C, D to be mutually independent, by definition:

P(ABCD) = P(A)P(B)P(C)P(D),
P(ABC) = P(A)P(B)P(C),
P(ABD) = P(A)P(B)P(D),
P(ACD) = P(A)P(C)P(D),
P(BCD) = P(B)P(C)P(D),
P(AB) = P(A)P(B), P(AC) = P(A)P(C), P(AD) = P(A)P(D),
P(BC) = P(B)P(C), P(BD) = P(B)P(D), P(CD) = P(C)P(D).

Last two conditions must satisfy in order for these events to be pairwise independent.

Problem 2

(8 points) Let X and Y be two discrete random variables which are identically distributed but not necessarily independent.
Define
R = 1 H(Y|X) / H(X)

(a) **Show that R = I(X,Y) / H(X)**

R = 1 - H(Y|X)/H(X) = (H(X)-H(Y|X))/ H(X)
I(X,Y) / H(X) = 1 - H(X|Y) / H(X)

Because X and Y are identically distributed,
H(Y|X) = H(X|Y) R = 1 - H(Y|X)/H(X)
= (H(X)-H(Y|X)) / H(X)
= (H(X)-H(X|Y)) / H(X)
= I(X,Y) / H(X)

(b) **Show that 0 <= R <= 1**
R = 1 - H(X|Y)/H(X)
0 <= H(X|Y) <= H(X)
$\therefore$ 0 <= H(X|Y)/H(X) <= 1 0 <= 1 - H(X|Y)/H(X) <= 1
0 <= R <= 1

(c) **When is R = 0?**
R = 0 iff I(X,Y) = 0
X and Y are independent

(d) **When is R = 1?**
R = 1 iff H(X|Y) = 0
X is a function of Y