FirstName LastName Mnumber

20CS6037 — Fall 2014 **Homework 1** Due: Enter Due date

Homework Notes: Add information here on your study group, number of hours you spent on the homework, and other relevant information.

Problem 1

You can write aligned equations as follows:

$$\begin{aligned} a &\sim p(a) \\ b &\sim p(b). \end{aligned}$$

(7 points) Pairwise independence does not imply mutual independence. Two random variables, X_i, i=1,2 are independent if
$P(X_i \mid X_j) = P(X_i)$, for i,j=1,2, i $\neq$ j
and therefore

$P(X_i, X_j) = P(X_j)$
$P(X_i \mid X_j) = P(X_i)P(X_j)$

Now, given n random variables, we say that there are mutually independent if
$P(X_i — X_S) = P(X_i)$ for all subsets S of $\{1, 2, \cdots, n\}$which do not contain i, and therefore
$P(X_1, \cdots, X_n) = xP(X_1) \cdots xP(X_n)$

You can write inline equations: $a \sim p(b)$, or one line equations:

$$a \sim p(a).$$

(1) **Show that pairwise independence between all pairs of variables (X_i, X_j), does NOT imply mutual independence. Note: it is enough to give an example.**

SUPPOSE A BOX CONTAINS 4 TICKETS LABELLED BY 331 323 233 333 LET US CHOOSE ONE TICKET AT RANDOM, AND CONSIDER THE RANDOM EVENTS A1=1 OCCURS AT THE FIRST PLACE A2=1 OCCURS AT THE SECOND PLACE A3=1 OCCURS AT THE THIRD PLACE P(A1)=1/2 P(A2)=1/2 P(A3)=1/2

A1A2=112 A1A3=121 A2A3=211 P(A1A2)=P(A1A3)=P(A2A3)=1/4. So we conclude that the three events A1, A2, A3 are pairwise independent. However A1A2A3=f P(A1A2A3)=0P(A1)P(A2)P(A3)=(1/2)3

(2) **Show mutual independence implies pairwise independence.**

For example, for four events A, B, C, D to be mutually independent, we must have P(ABCD) = P(A)P(B)P(C)P(D), P(ABC) = P(A)P(B)P(C), P(ABD) =

P(A)P(B)P(D), P(ACD) = P(A)P(C)P(D), P(BCD) = P(B)P(C)P(D), P(AB) = P(A)P(B), P(AC) = P(A)P(C), P(AD) = P(A)P(D), P(BC) = P(B)P(C), P(BD) = P(B)P(D), P(CD) = P(C)P(D).

Problem 2

(8 points) Let X and Y be two discrete random variables which are identically distributed but not necessarily independent.
Define
R = 1 H(Y|X) / H(X)

(a) **Show that R = I(X,Y) / H(X)**

(b) **Show that 0 <= R <= 1**

(c) **When is R = 0?**

(d) **When is R = 1?**

Problem 3

You can also add bullet points:

(a) This is the first bullet point. It can be the solution to the first part of the question

(b) This is the solution to the second part of the question.

You can also add figures (Figure 1).

Figure 1: example caption

And tables (Table 1)

```
SELECT  S.Course_number,
        COUNT(*) as Number_students
FROM    SECTION S, GRADE_REPORT G
WHERE   G.Section_identifier = S.Section_identifier AND
        S.Instructor='King'
GROUP   BY S.Course_number;
```

Problem 4

Problem 5

Table 1: Table captions are better up top

Animal	Description	Price ($)
Gnat	per gram	13.65
	each	0.01
Gnu	stuffed	92.50
Emu	stuffed	33.33
Armadillo	frozen	8.99