

20CS6037 Machine Learning

MLE, MAP, Bayesian Reasoning - Chapter 3 & 5 (Lecture 5: 9/9/14)

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Section 1: Conditional Independence

Section 2: Transformation of Random Variables

Section 3: General Transformations

Section 4: Monte Carlo Approximation

Section 5: Entropy

Section 6: Mutual Information

Section 1:

Conditional Independence

X, Y r.v. $X \perp Y$: X and Y are independent

Def

$$X \perp Y \Leftrightarrow P(X, Y) = P(X)P(Y)$$

This really means $\{w \in S \mid X(w) = a\}$, $\{w \in S \mid Y(w) = b\}$ are independent event for all $a \in \text{Range}(x)$; $b \in \text{Range}(Y)$

Notation

$X \perp Y \mid Z$: X and Y are **Conditionally Independent (CI)** given Z

$$P(X \perp Y \mid Z) \Leftrightarrow P(X, Y \mid Z) = P(X \mid Z)P(Y \mid Z)$$

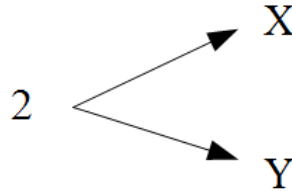


Figure 1: X and Y are CI given Z

$\underline{P}(X, Y)$: Joint Distribution of X and Y

$$\{w \in S \mid X(w) = a\}, \{w \in S \mid Y(w) = b\}$$

Can extend to $\underline{P}(X_1, \dots, X_D)$: CDF, PDF/PMF

$$\begin{aligned} \text{COV}(X, Y) &\triangleq E[(X - E(X))(Y - E(Y))] \\ &= E[XY - XE(Y) - YE(X) + E(X)E(Y)] \\ &= E(XY) - E(X)E(Y) \end{aligned}$$

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