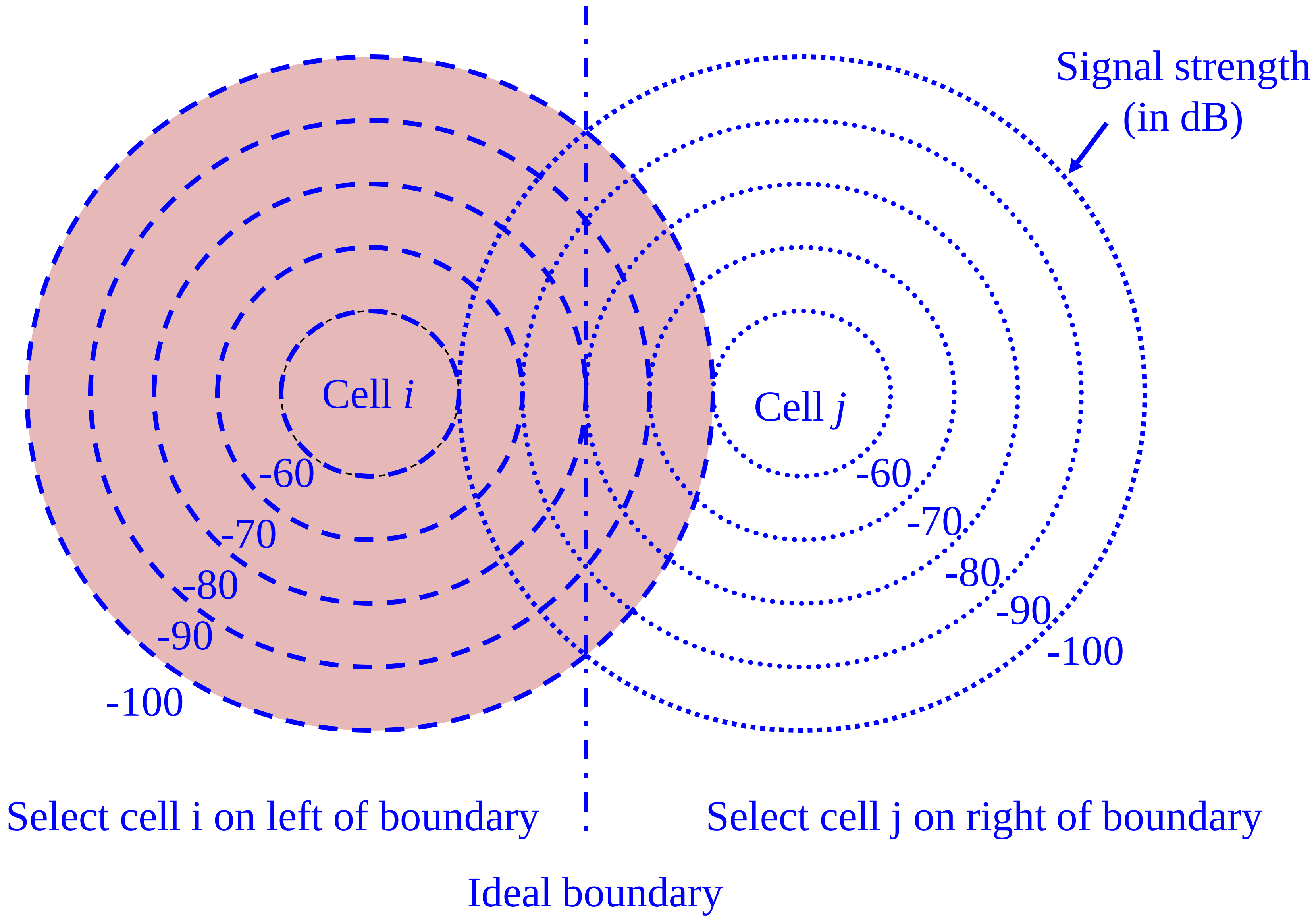
Chapter 5

The Cellular Concept

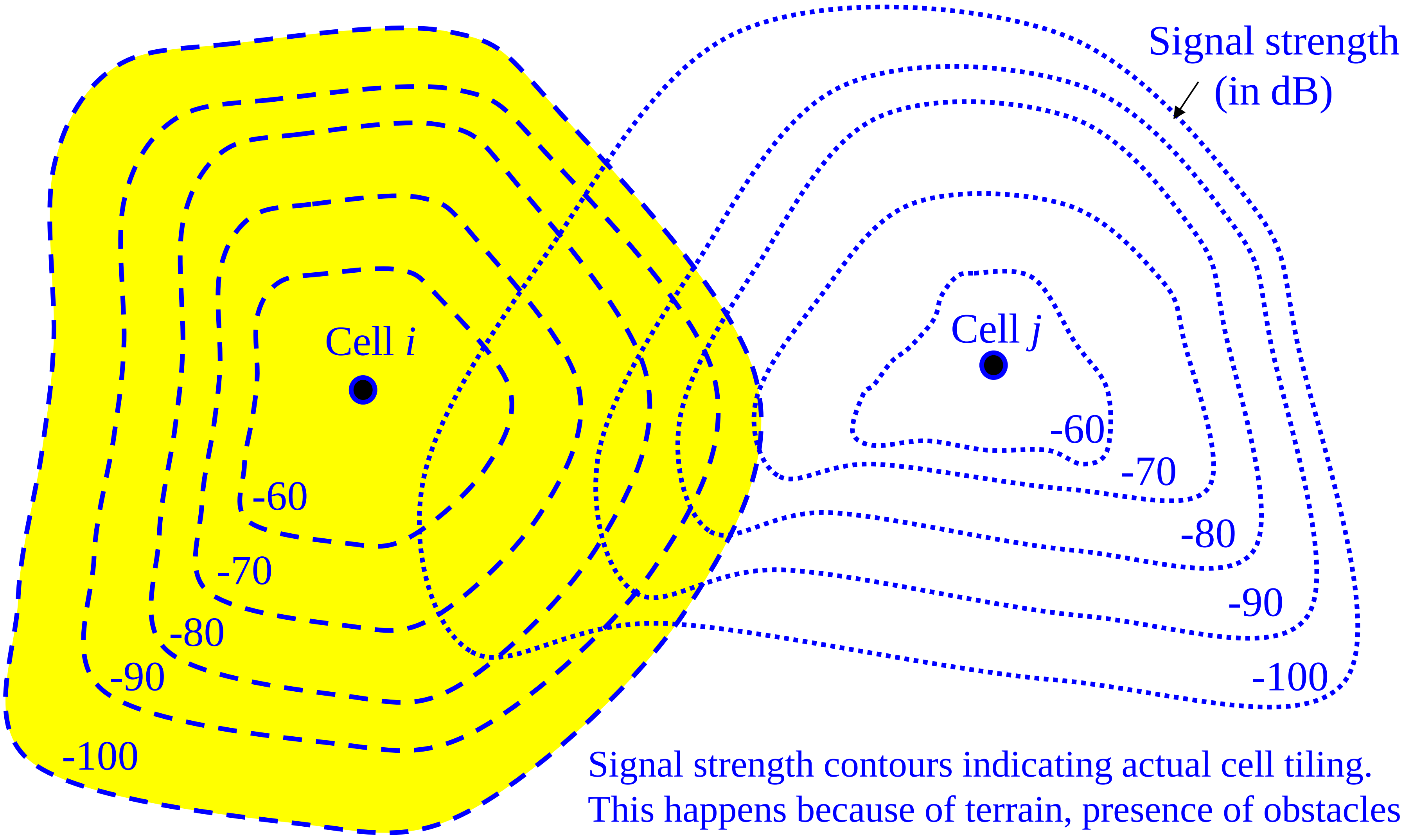
Outline

- Cell Area
  - Actual cell/Ideal cell
- Signal Strength
- Handoff Region
- Capacity of a Cell
  - Traffic theory
  - Erlang B and Erlang C
- Frequency Reuse
- How to form a Cluster
- Co-channel Interference
- Cell Splitting
- Cell Sectoring

Signal Strength

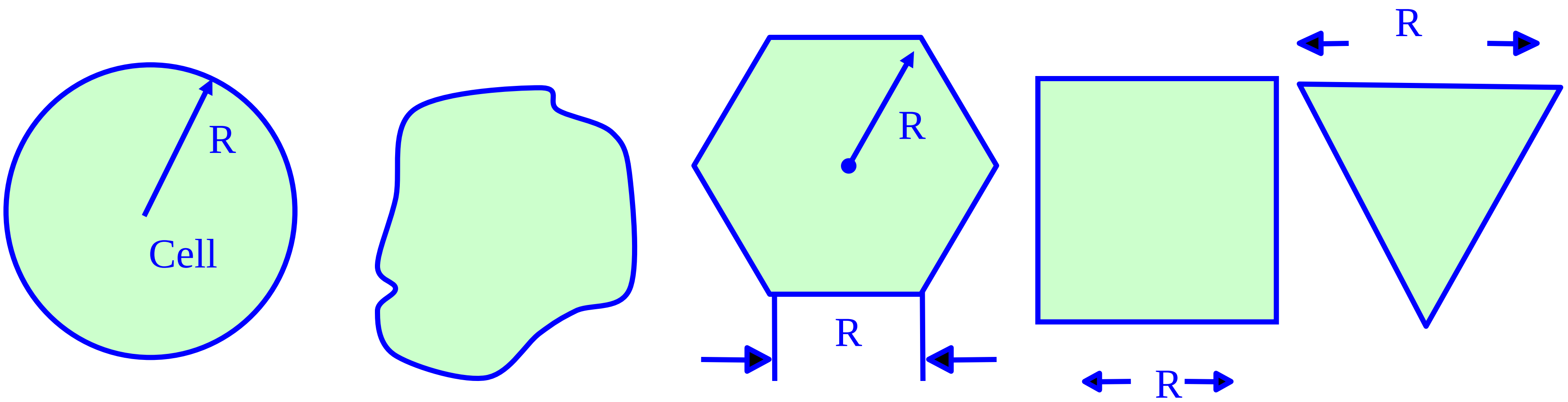


Actual Signal Strength



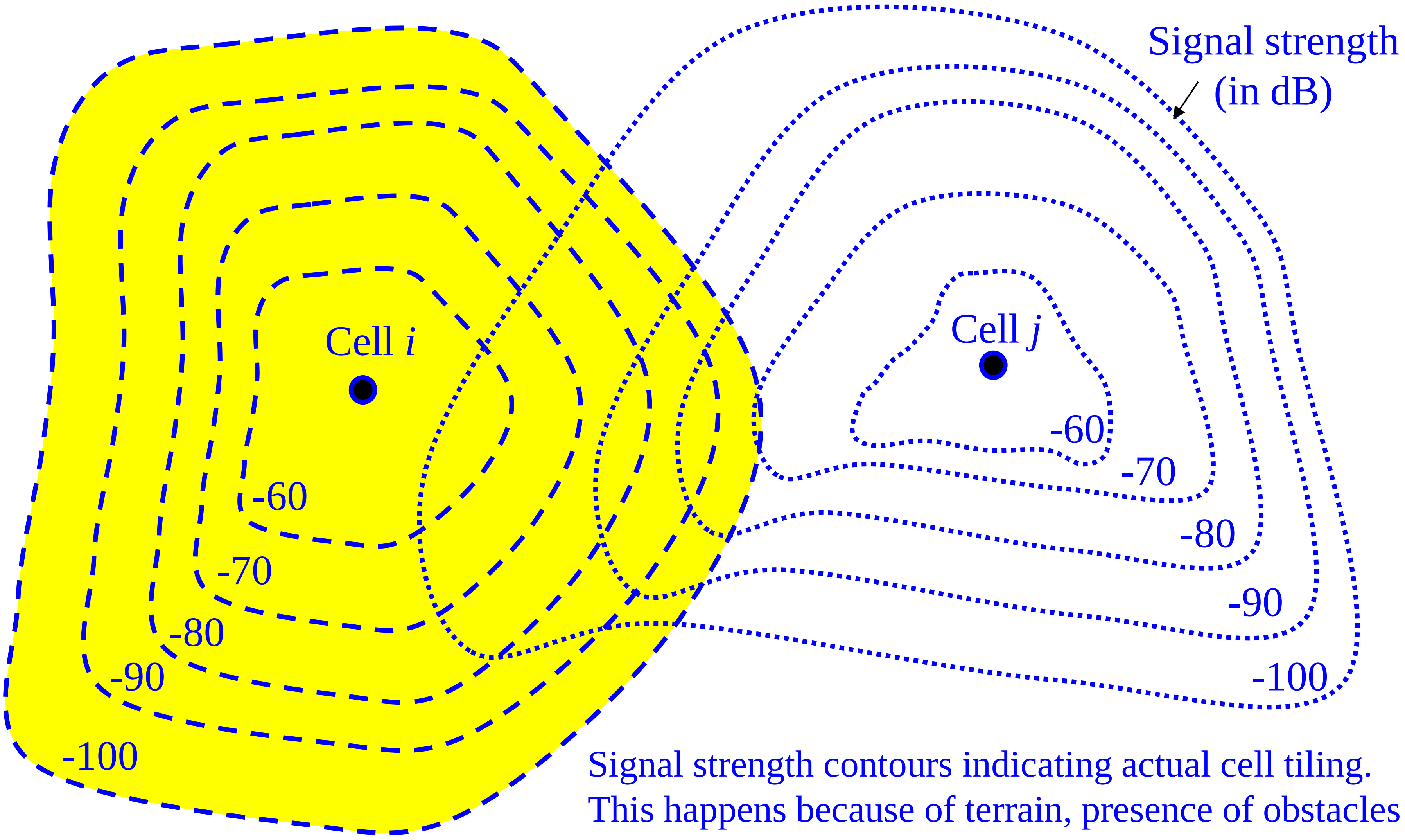
Signal strength contours indicating actual cell tiling. This happens because of terrain, presence of obstacles and signal attenuation in the atmosphere.

Cell Shape



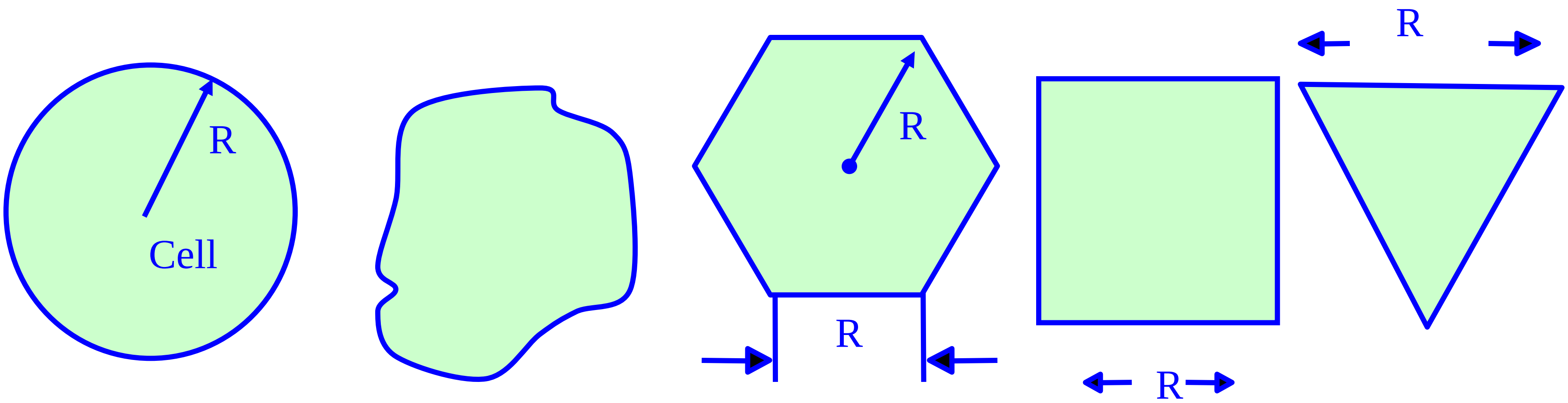
(a) Ideal cell (b) Actual cell (c) Different cell models

Actual Signal Strength



Signal strength contours indicating actual cell tiling. This happens because of terrain, presence of obstacles and signal attenuation in the atmosphere.

Cell Shape

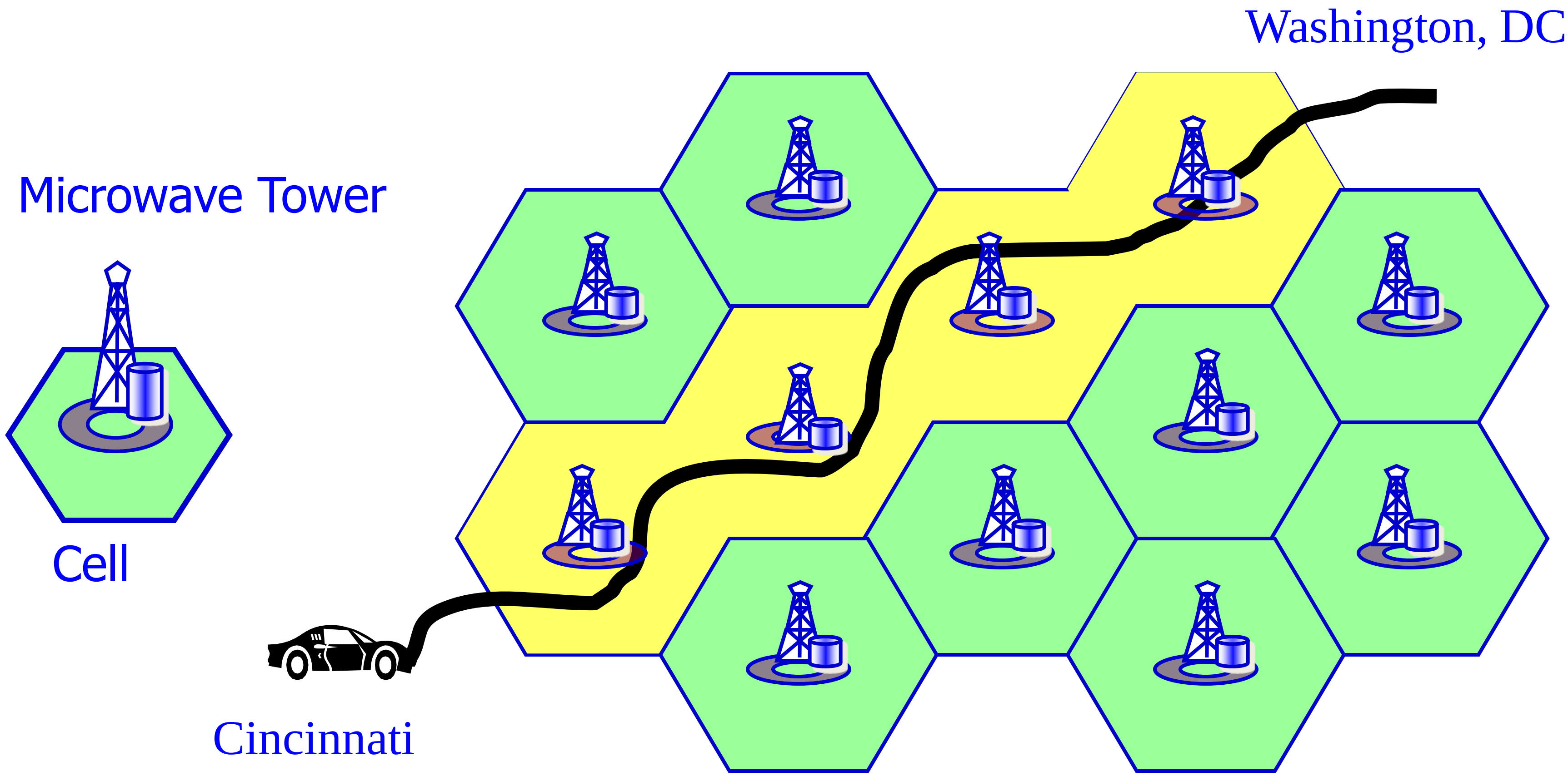


(a) Ideal cell (b) Actual cell (c) Different cell models

Impact of Cell Shape and Radius on Service Characteristics

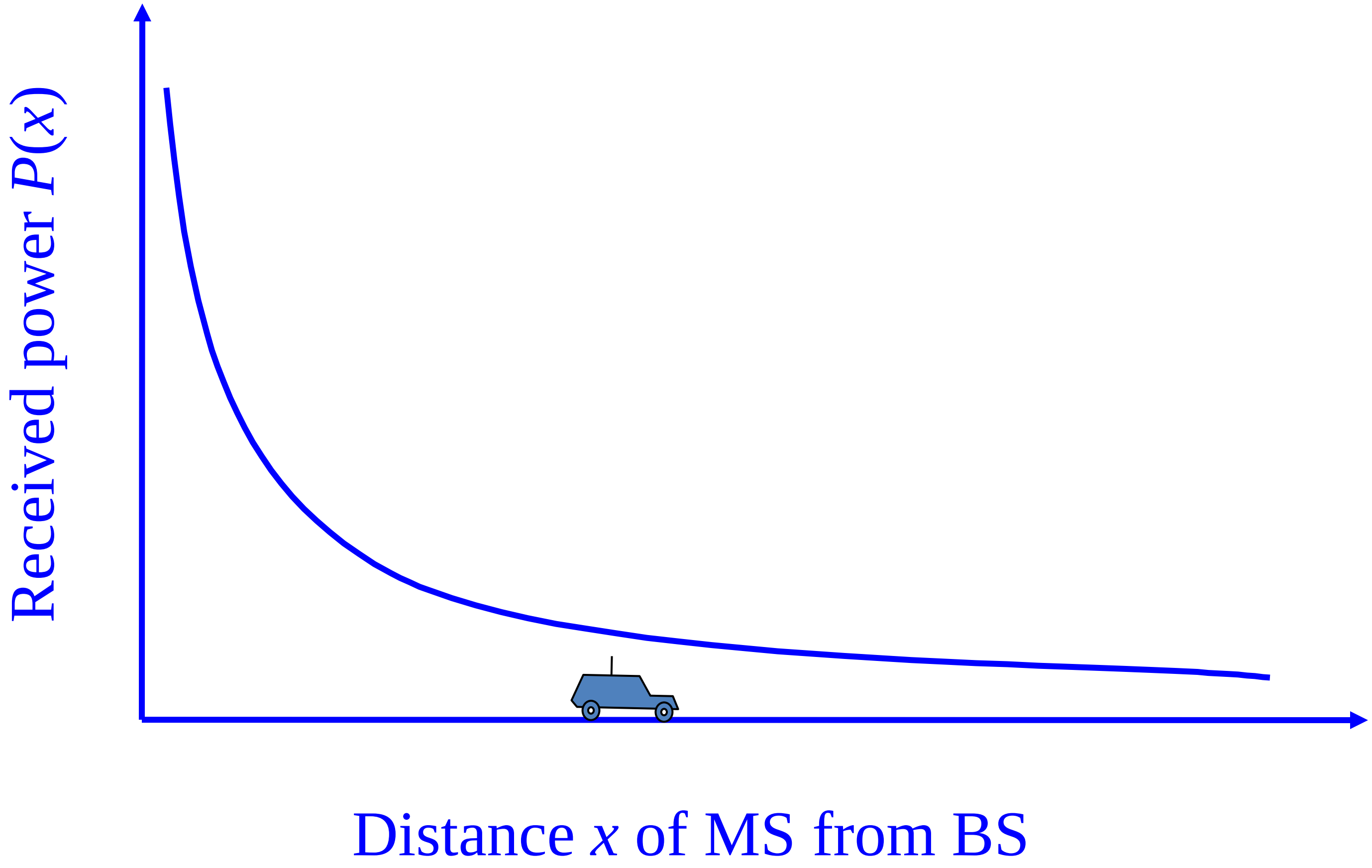
Shape of the Cell	Area	Boundary	Boundary Length/ Unit Area	Channels/ Unit Area with N Channels/ Cell	Channels/Unit Area when Number of Channels Increased by a Factor K	Channels/Unit Area when Size of Cell Reduced by a Factor M
Square cell (side =R)	R^2	4R	$\frac{4}{R}$	$\frac{N}{R^2}$	$\frac{KN}{R^2}$	$\frac{M^2N}{R^2}$
Hexagonal cell (side=R)	$\frac{3\sqrt{3}}{2}R^2$	6R	$\frac{4}{\sqrt{3}R}$	$\frac{N}{1.5\sqrt{3}R^2}$	$\frac{KN}{1.5\sqrt{3}R^2}$	$\frac{M^2N}{1.5\sqrt{3}R^2}$
Circular cell (radius=R)	πR^2	$2\pi R$	$\frac{2}{R}$	$\frac{N}{\pi R^2}$	$\frac{KN}{\pi R^2}$	$\frac{M^2N}{\pi R^2}$
Triangular cell (side=R)	$\frac{\sqrt{3}}{4}R^2$	3R	$\frac{4\sqrt{3}}{R}$	$\frac{4\sqrt{3}N}{3R^2}$	$\frac{4\sqrt{3}KN}{3R^2}$	$\frac{4\sqrt{3}M^2N}{3R^2}$

Universal Cell Phone Coverage

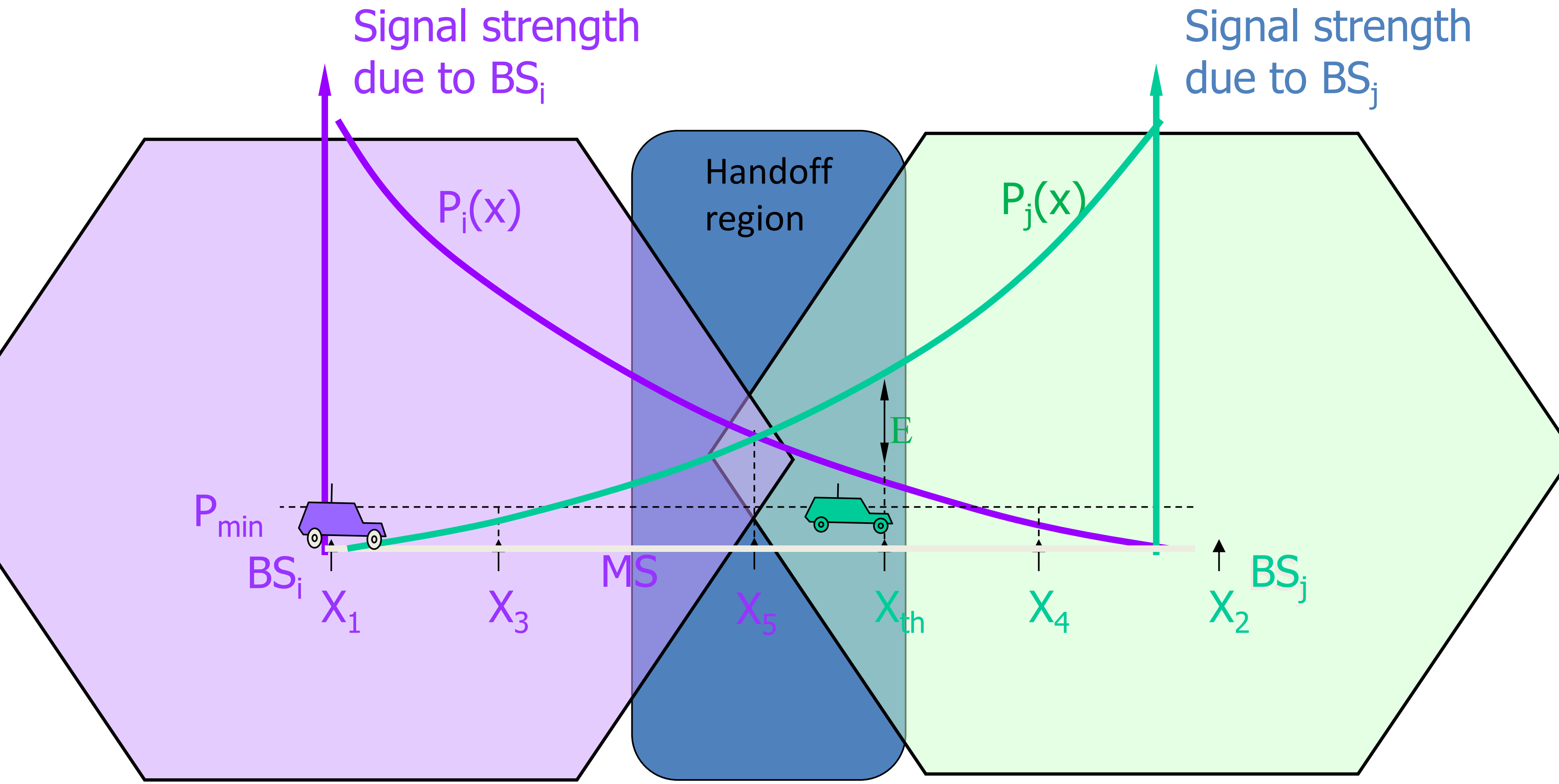


Maintaining the telephone number across geographical areas in a wireless and mobile system

Variation of Received Power

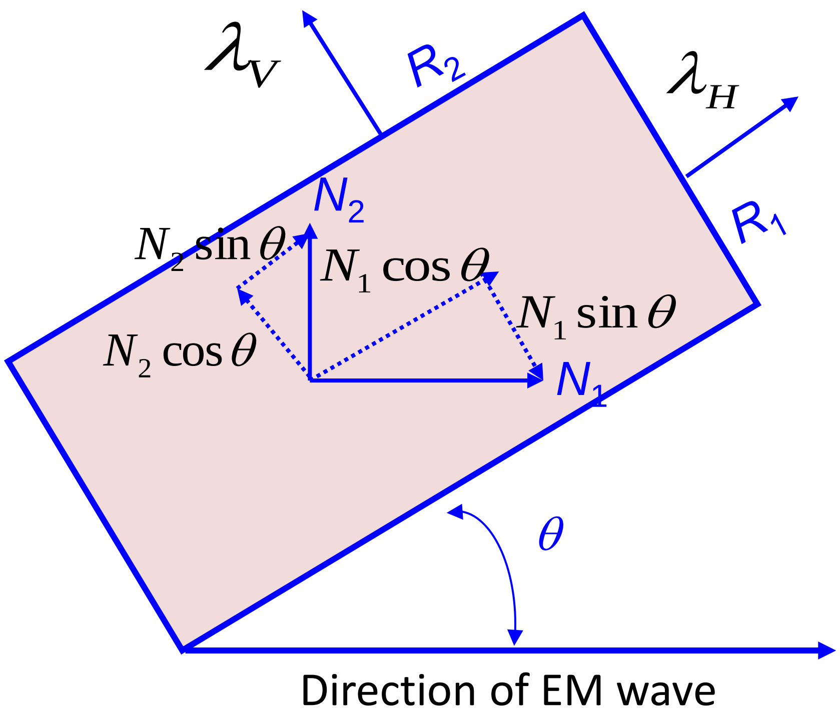


Handoff Region



By looking at the variation of signal strength from either base station it is possible to decide on the optimum area where handoff can take place

Handoff Rate in a Rectangular Area



N_1 is the number of MSs per unit length in horizontal direction

N_2 is the number of MSs per unit length in vertical direction

Since handoff can occur at sides R_1 and R_2 of a cell

$$\lambda_H = R_1\left(N_1\cos\theta + N_2\sin\theta\right) + R_2\left(N_1\sin\theta + N_2\cos\theta\right)$$

Assuming area $A=R_1 R_2$ is fixed, substitute $R_2= A/ R_1$, differentiating λ_H with respect to R_1 and equating to 0 gives

$$N_1 cosq + N_2 sinq - A/R_1^2\,(N_{1s}inq + N_2 cosq) = 0$$

Handoff Rate in a Rectangular Area

Thus, we have:

$$R_1^{\,2} = A\frac{N_1\sin\theta + N_2\cos\theta}{N_1\cos\theta + N_2\sin\theta} \qquad R_2^{\,2} = A\frac{N_1\cos\theta + N_2\sin\theta}{N_1\sin\theta + N_2\cos\theta}$$

Simplifying through few steps gives:

$$\lambda_H = 2\sqrt{A\left(N_1\cos\theta + N_2\sin\theta\right)\left(N_1\sin\theta + N_2\cos\theta\right)}$$

λ_H is minimized when $\theta = 0$, giving

$$\lambda_H = 2\sqrt{AN_1N_2} \qquad and \qquad \frac{R_1}{R_2} = \frac{N_1}{N_2}$$

Cell Capacity

- Average number of MSs requesting service (Average arrival rate): λ
- Average length of time MS requires service (Average holding time): T
- Offered load: $a = \lambda T$

e.g., in a cell with 100 MSs, on an average 30 requests are generated during an hour, with average holding time *T=360* seconds

Then, arrival rate λ*=30 requests/3600* seconds
=1/120 requests/sec

A channel kept busy for one hour is defined as one Erlang (*a*), i.e.,

$$a = \#calls * duration = \frac{30\ Calls}{3600\ Sec} \cdot \frac{360\ Sec}{call} = 3\ Erlangs$$

Example on Cell Capacity

- **Example 5.1:** A typical cluster has seven cells as shown in Figure 5.7 and each cell has a radius of 1 km, find the nearest frequency reuse distance and reuse factor.

Since N = 7 and R = 1 km, the reuse distance of frequency can be calculated by Equation (5.14) as

$$D = \sqrt{3N}R = \sqrt{3x7}x1 \approx 4.5826\,km$$

Based on Equation (5.15), the frequency reuse factor can be obtained by

$$q = \frac{D}{R} = \sqrt{3N} = \sqrt{3x7} \approx 4.5826$$

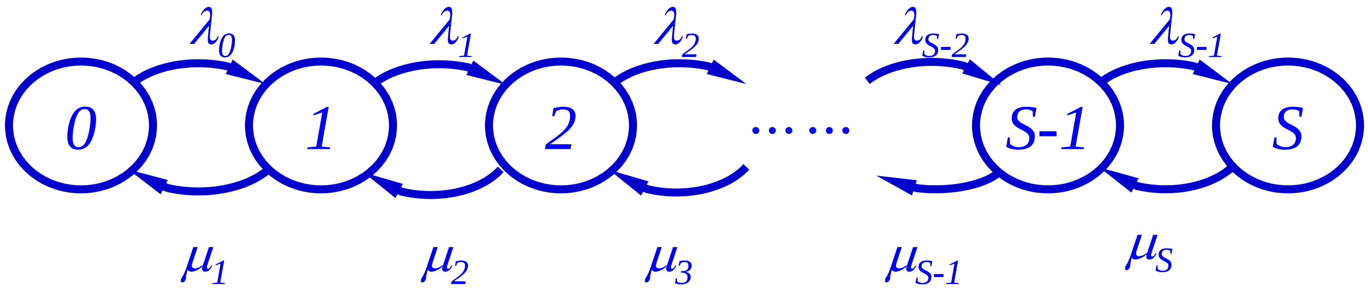
- Another popular cluster size of 4 (a rectangular)
- In general, the number of cells per cluster is given by

$$N = i^2 + ij + j^2$$

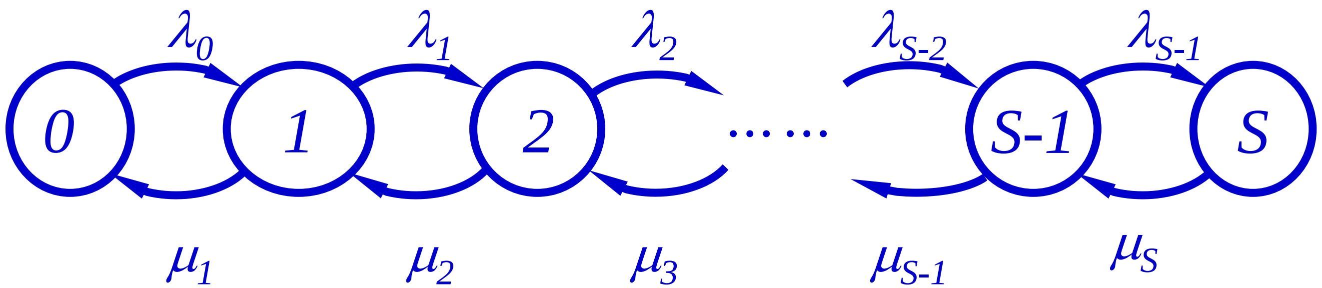
- Unless specified, a cluster of size 7 is assumed throughout this book

Cell Capacity

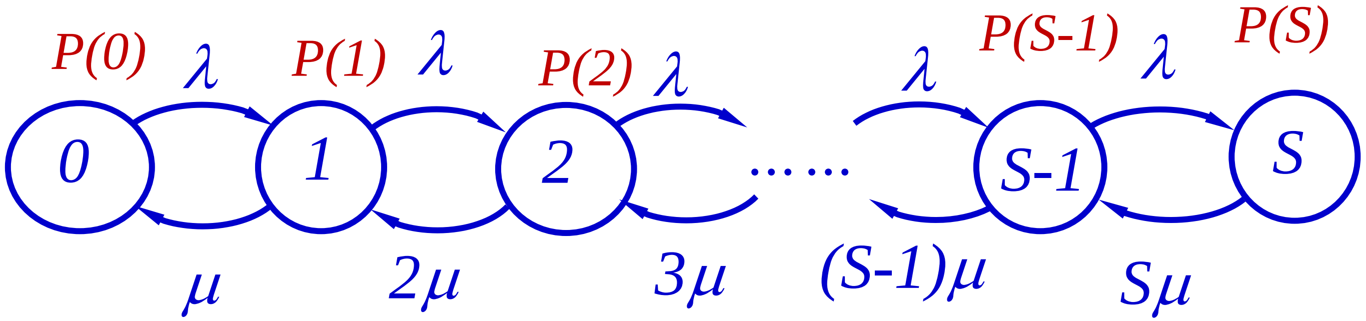
- Average arrival rate λ during a short interval t is given by $\lambda\, t$
- Average service (departure) rate is μ
- The system can be analyzed by a *M/M/S/S* queuing model, where *M* is interarrival time of users, *M* is distribution of service time, *S* is the number of channels, and *S* is the maximum number of users in the system
- The steady state probability $P(i)$ for this system in the form (for i =0, 1,……, *S*)



Cell Capacity



Assuming equal probability of an event

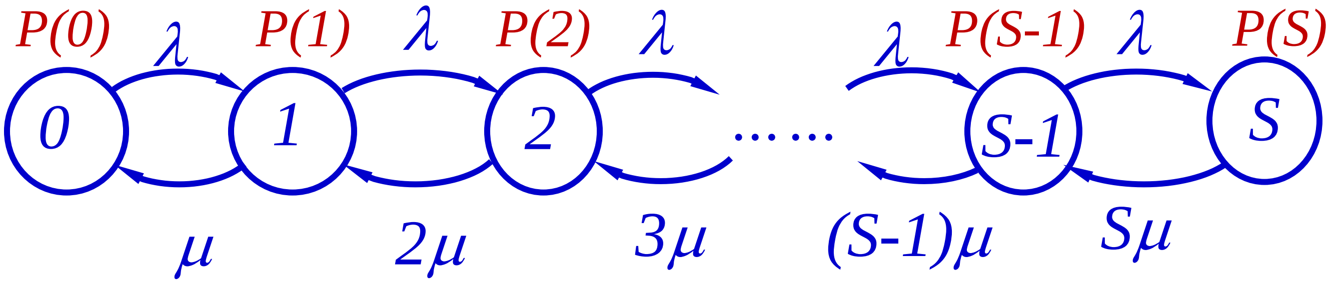


$$\lambda P(0) = \mu P(1)$$

$$(\lambda + \mu)P(1) = \lambda P(0) + 2\mu P(2)$$

$$(\lambda + i\mu)P(i) = \lambda P(i-1) + (i+1)\mu P(i+1),\ \ \mathrm{S} > i \geq 1$$

Cell Capacity



$$P(i) = \left(\tfrac{\lambda}{i\mu}\right)^i P(0), = \frac{a^i}{i!} P(0) \quad i \geq 1 \;\; where \; a = \tfrac{\mu}{\lambda}$$

This is steady state probability $P(i)$

As P(0)+P(1)+…P(S)=1; substituting in terms of $P(0)$ gives

$$P(0) + \frac{a}{1!} P(0) + \frac{a^2}{2!} P(0) + \frac{a^3}{3!} P(0) + \ldots + \frac{a^S}{S!} P(0) = 1$$

Therefore $$P(0) \left[\sum_{i=0}^{S} \frac{a^i}{i!} \right] = 1, or \; P(0) = \left[\sum_{i=0}^{S} \frac{a^i}{i!} \right]^{-1}$$

Capacity of a Cell

- The probability $P(S)$ of an arriving call being blocked is the probability that all S channels are busy

$$P(S) = \frac{a^S}{S!} P(0) = \frac{\frac{a^S}{S!}}{\sum_{i=0}^{S} \frac{a^i}{i!}}$$

- This is Erlang B formula B(S, a)
- In the previous example, if S=2 and a=3, the blocking probability B(2, 3) is

$$B(2,3) = \frac{\frac{3^2}{2!}}{\sum_{k=0}^{2} \frac{3^k}{k!}} = \frac{\frac{9}{2}}{1+3+9/2} = \frac{9}{19} = 0.529$$

- So, the number of calls blocked 30x0.529=15.87

Capacity of a Cell

$$\text{Efficiency} = \frac{\text{Traffic nonblocked}}{\text{Capacity}}$$

$$= \frac{\text{Erlangs x portions of used channel}}{\text{Number of channels}}$$

$$= \frac{3(1-0.529)}{2} = \frac{1.413}{2} = 0.7065$$

The probability of a call being delayed:

$$C(S,a) = \frac{\dfrac{a^S}{(S-1)!(S-a)}}{\dfrac{a^S}{(S-1)!(S-a)} + \sum_{i=0}^{S-1} \dfrac{a^i}{i!}}$$

This is Erlang C Formula

$$= \frac{S.B(S,a)}{S-a[1-B(S-a)]}$$

$$as\ B(S,a) = \frac{\dfrac{a^S}{S!}}{\sum_{i=0}^{S} \dfrac{a^i}{i!}}$$

For S=5, a=3, B(5,3)=0.11, gives C(5,3)=0.2360

Erlang B and Erlang C

- Probability of an arriving call being blocked is

$$B(S,a)=\frac{a^S}{S!}\cdot\frac{1}{\sum_{k=0}^{S}\frac{a^k}{k!}},$$ ← *Erlang B formula*

where S is the number of channels in a group

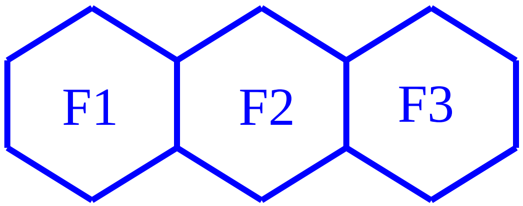
- Probability of an arriving call being delayed is

$$C(S,a)=\frac{\frac{a^S}{(S-1)!(S-a)}}{\frac{a^S}{(S-1)!(S-a)}+\sum_{i=0}^{S-1}\frac{a^i}{i!}},$$ ← *Erlang C formula*

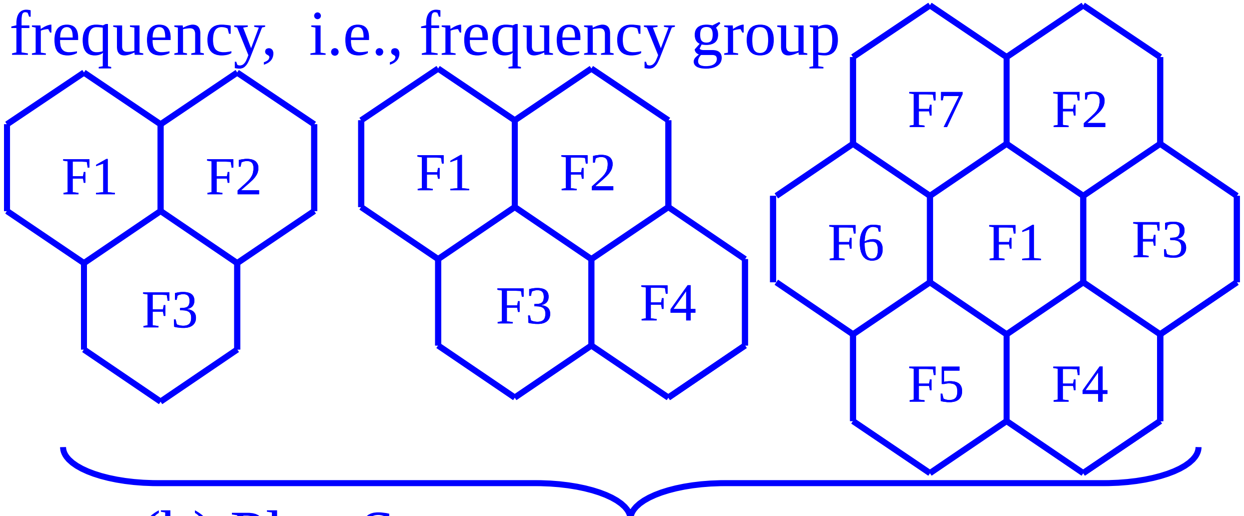
where $C(S, a)$ is the probability of an arriving call being delayed with a load and S channels

Cell Structure

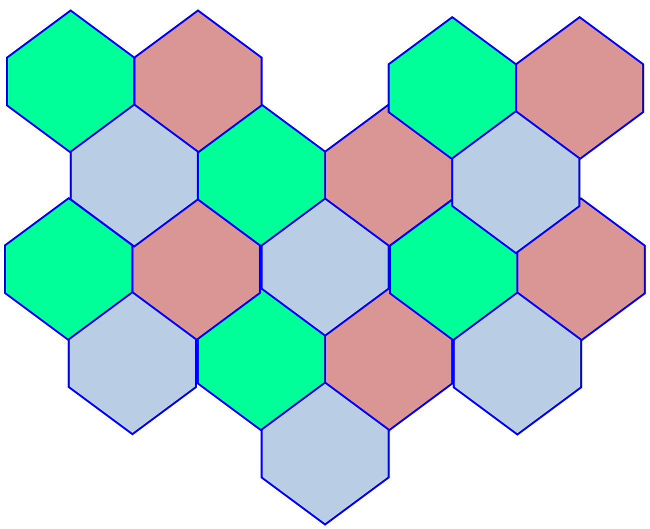
Note: Fx is set of frequency, i.e., frequency group

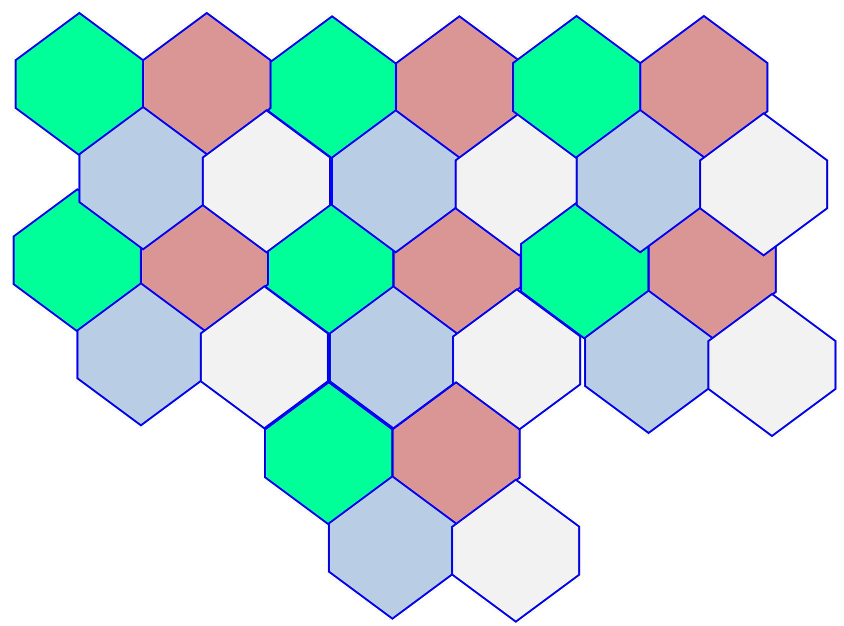


(a) Line Structure

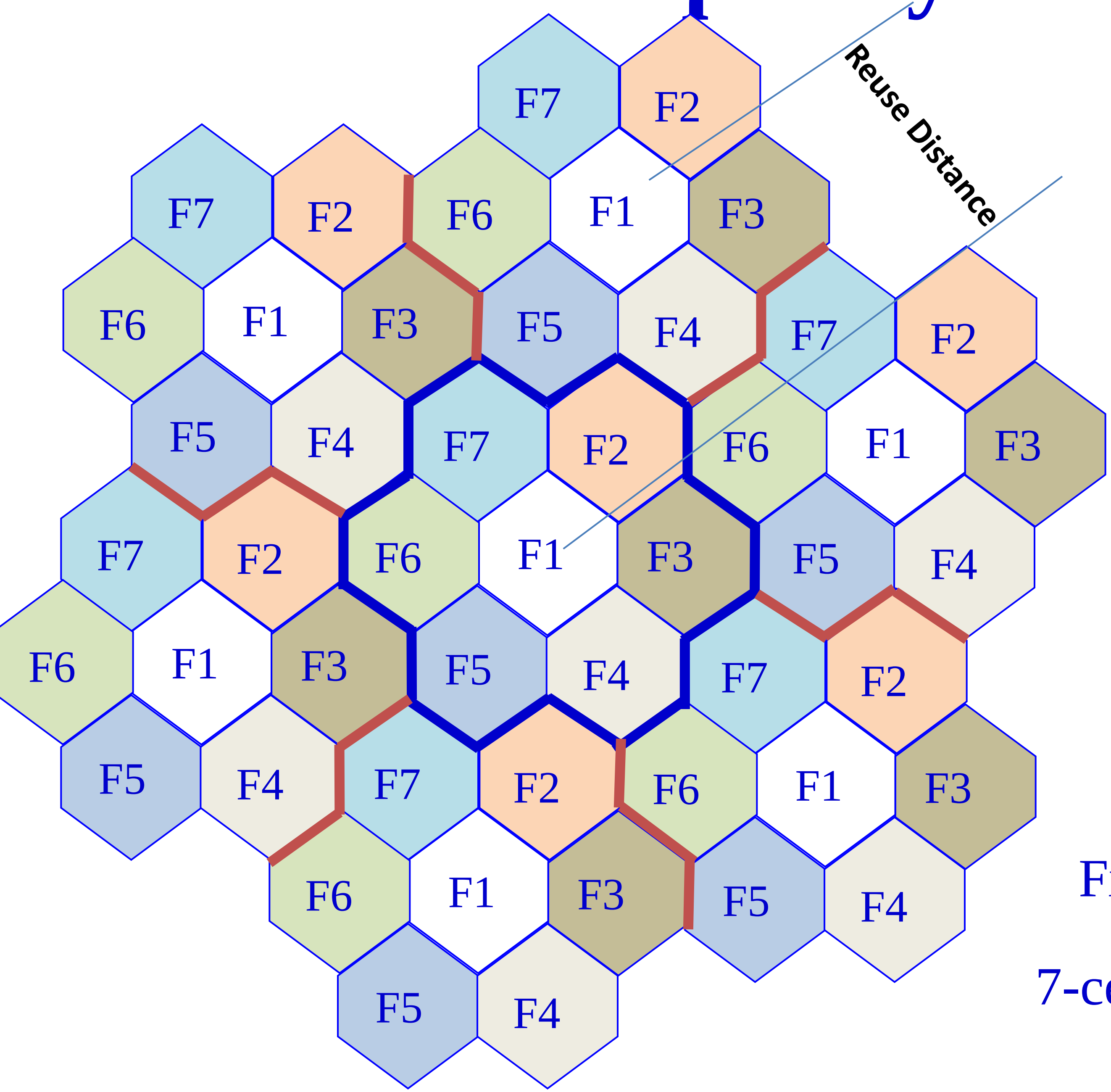


(b) Plan Structure





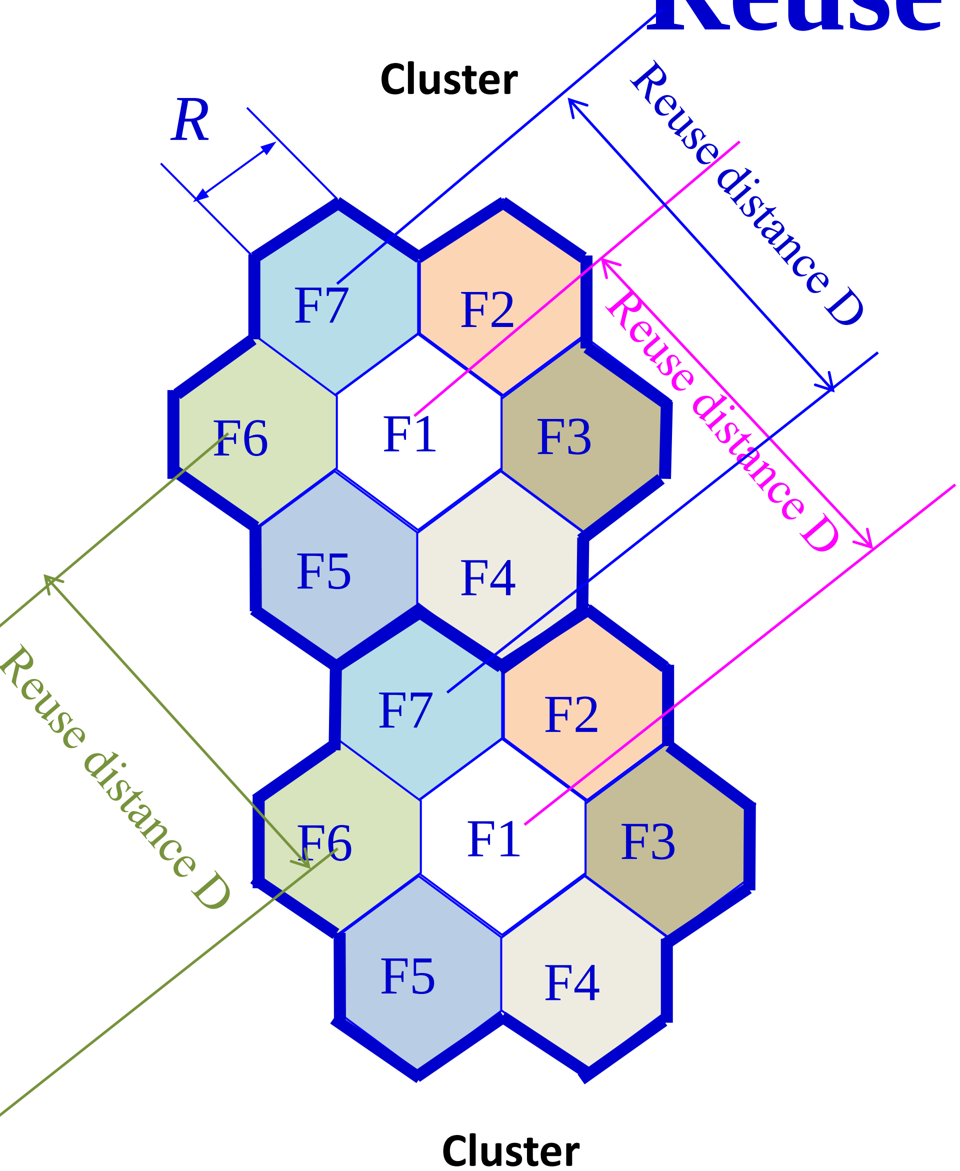
Frequency Reuse



Fx: Set of frequency

7-cell reuse cluster

Reuse Distance



- For hexagonal cells, the reuse distance is given by

$$D = \sqrt{3N}\,R$$

where R is cell radius and N is the reuse pattern (the cluster size or the number of cells per cluster).

- Reuse factor is

$$q \equiv \frac{D}{R} = \sqrt{3N}$$

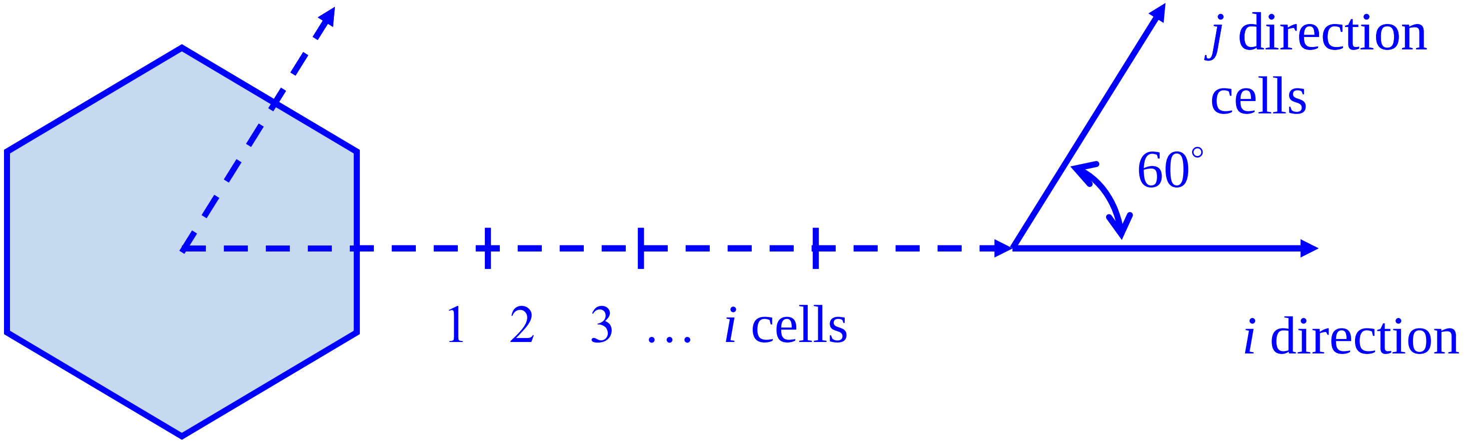
Reuse Distance (Cont'd)

- The cluster size or the number of cells per cluster is given by

$$N = i^2 + ij + j^2$$

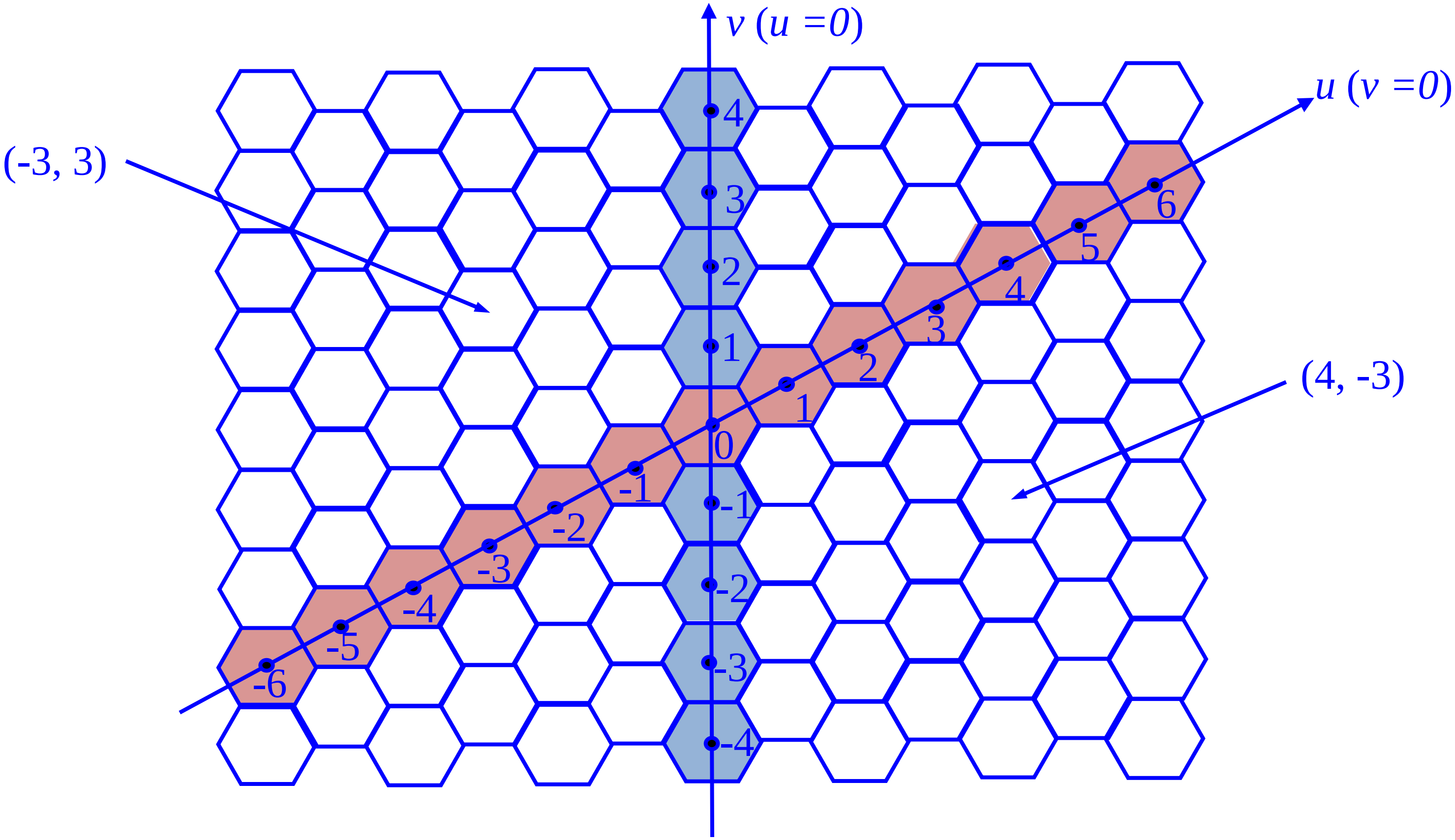
where *i* and *j* are positive integers, i.e. $0 \geq i, j < \infty$

- N = 1, 3, 4, 7, 9, 12, 13, 16, 19, 21, 28, ….
  The popular value of *N* being 4 and 7



Reuse Distance (Cont'd)

$$N = i^2 + ij + j^2$$ with *i* and *j* as integers



u and *v* coordinate representation of cells with (0,0) center

Reuse Distance and Channel set to use

- For **j=1**, the cluster size is given by $N = i^2 + i + 1$

Then defining $L = [(i+1)u + v] \bmod N$

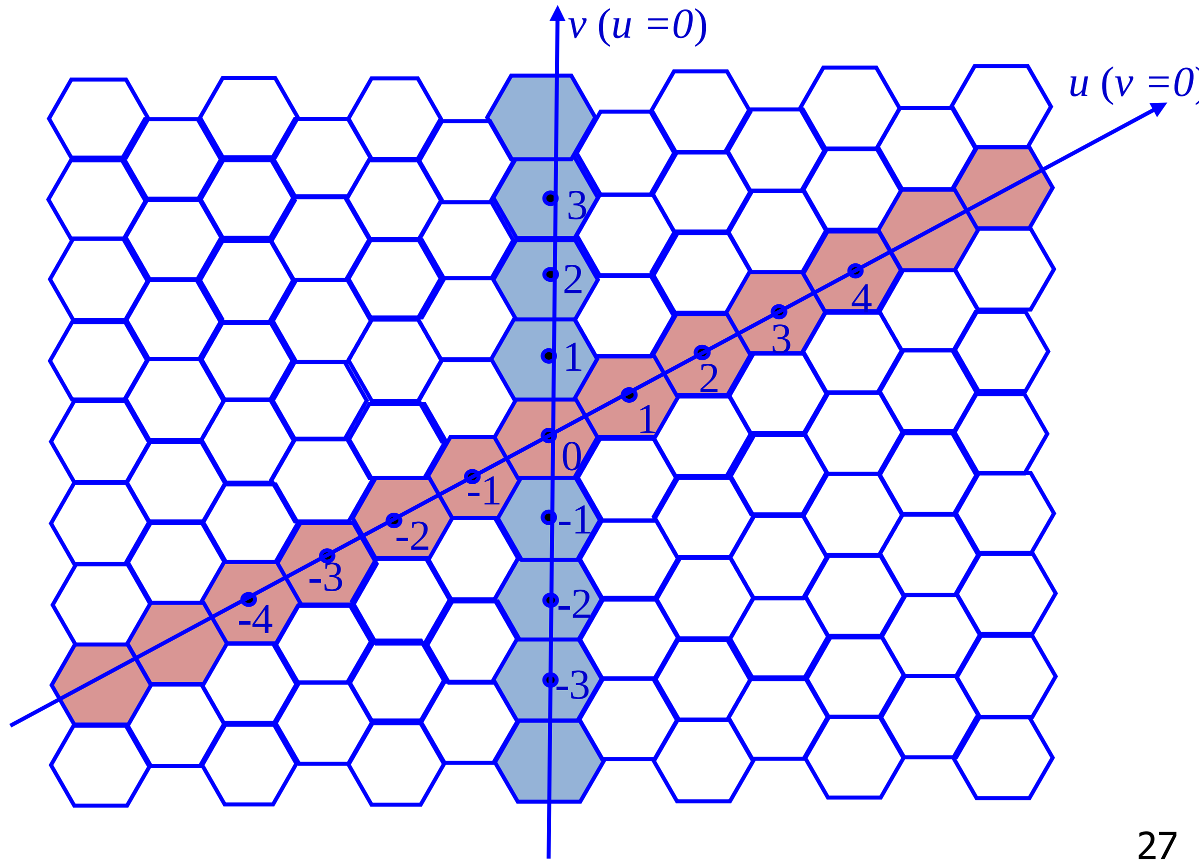
We can obtain label L for the cell whose center is at (u,v).

For N=7, with i= 2, j=1:

$$L = (3u + v) \bmod 7$$

u	**0**	**1**	**-1**	**0**	**0**	**1**	**-1**
v	**0**	**0**	**0**	**1**	**-1**	**-1**	**1**
L	**0**	**3**	**4**	**1**	**6**	**2**	**5**

Gives assignment of channels to use in different cells



Reuse Distance and Channel set to use

- For j=1, the cluster size is given by $N = i^2 + i + 1$

Then defining $L = [(i+1)u + v] \bmod N$

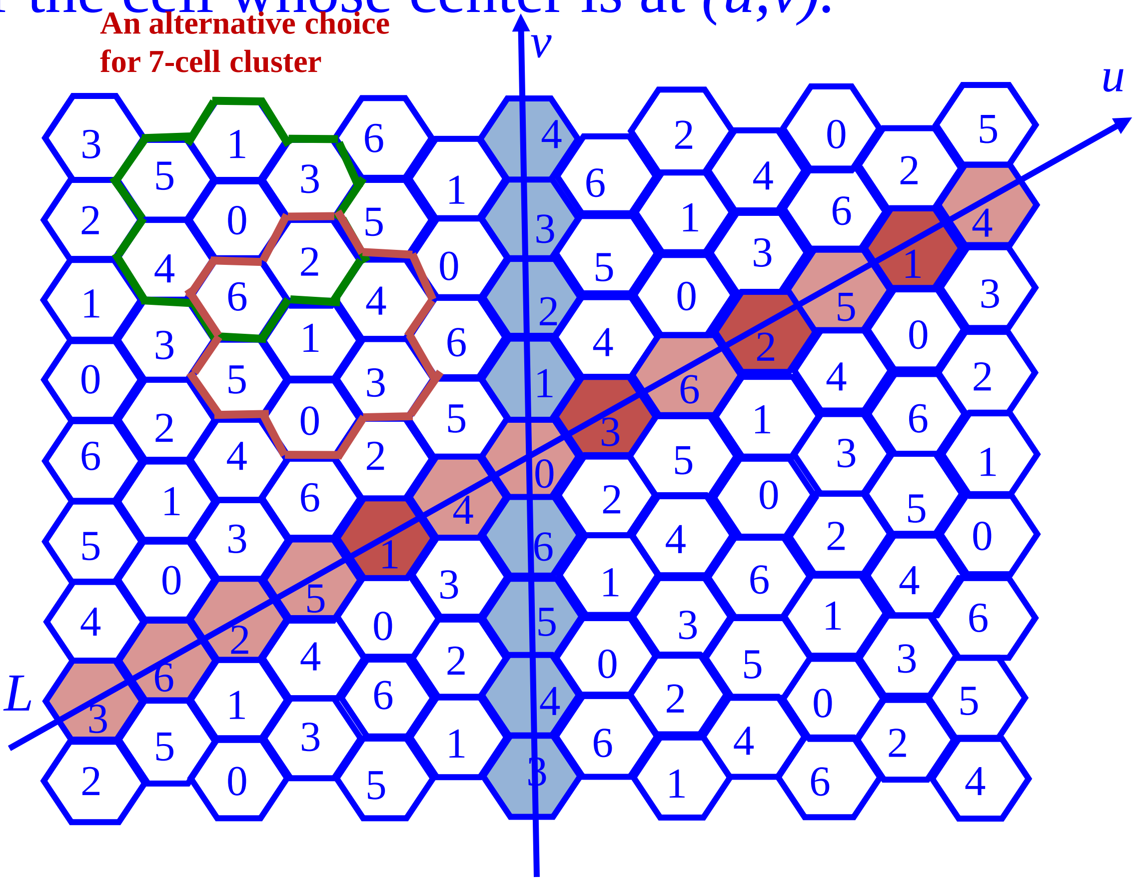
We can obtain label L for the cell whose center is at (u,v).

For N=7, with i= 2, j=1:

$$L = (3u + v) \bmod 7$$

u	0	1	-1	0	0	1	-1
v	0	0	0	1	-1	-1	1
L	0	3	4	1	6	2	5

Gives assignment of channels to use in different cells

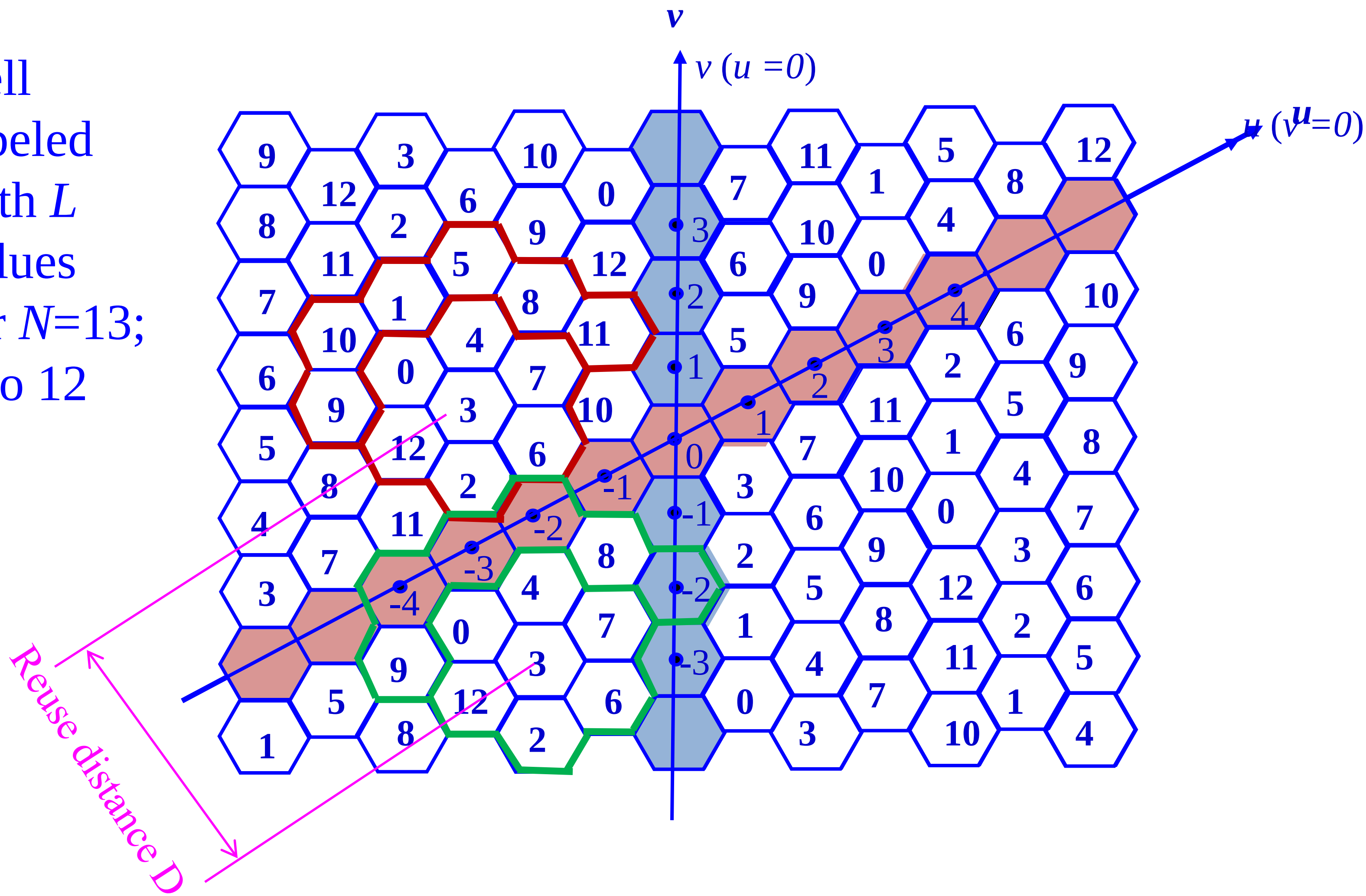


Labeling cells with L values for N=7

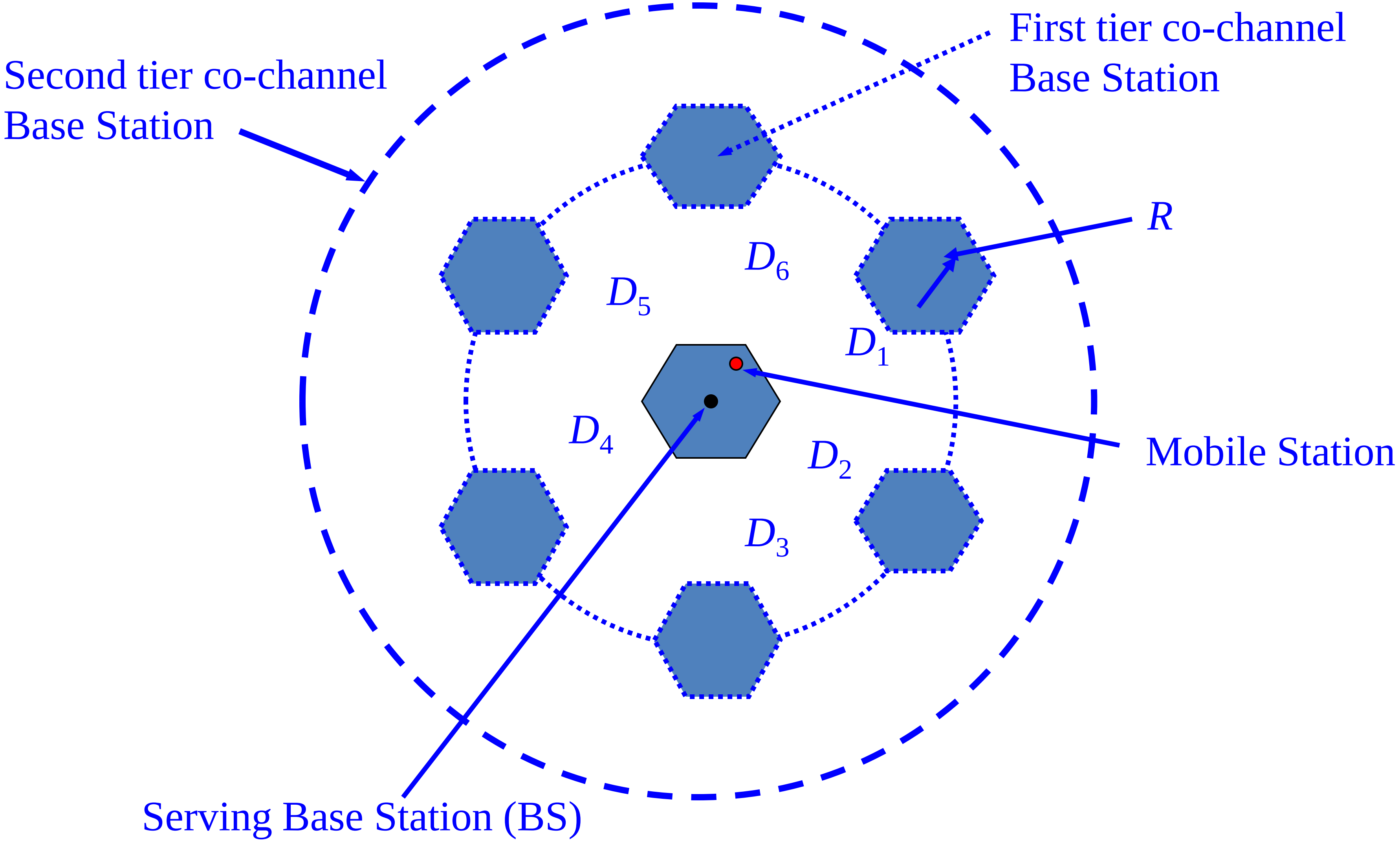
Reuse Distance and Channel set to use

For N=13, i=3, j=1; $$L = (4u + v) \bmod 13$$

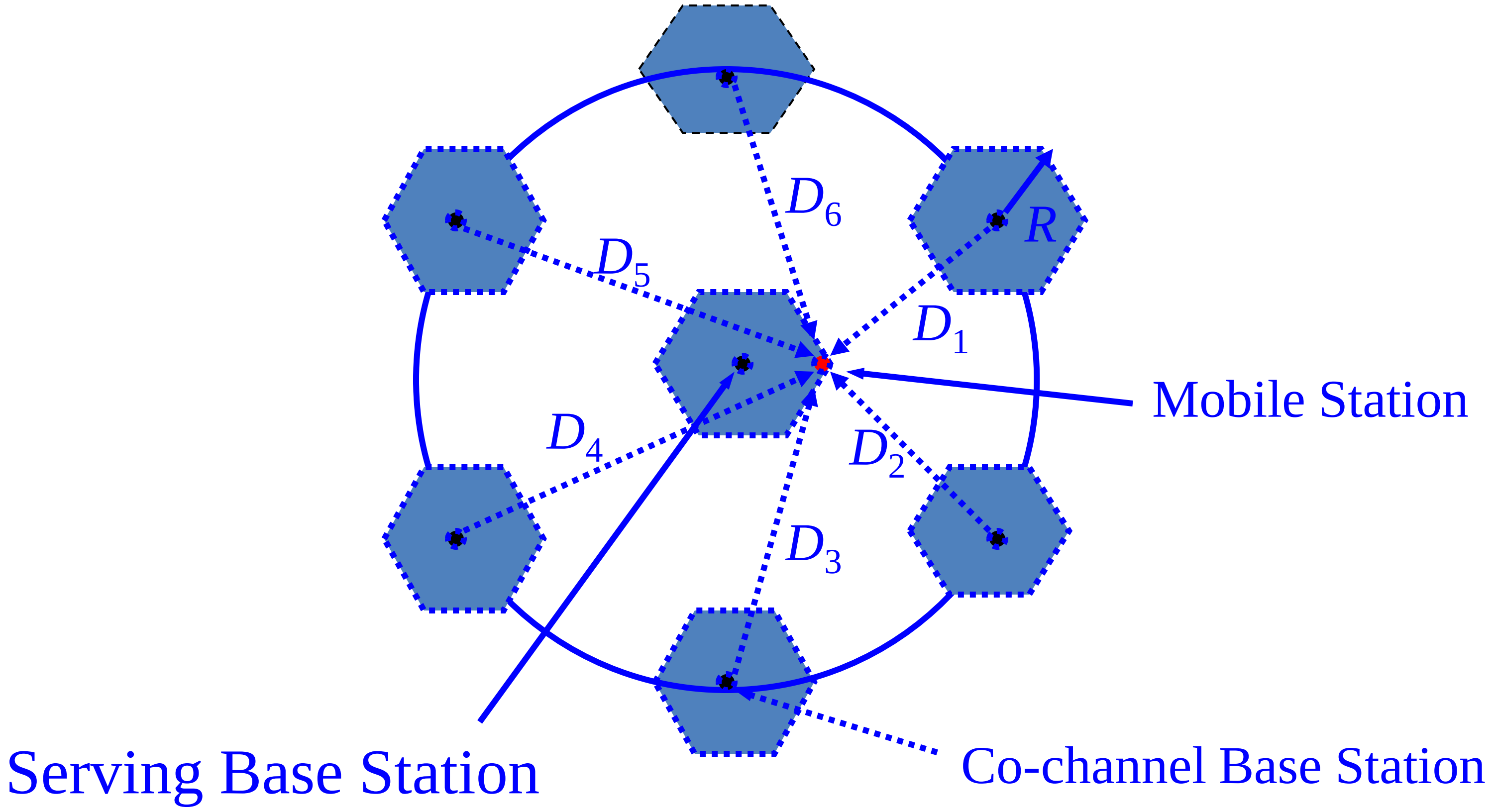
Cell labeled with L values for N=13; 0 to 12



Cochannel Interference



Worst Case of Cochannel Interference



Cochannel Interference

- Cochannel interference ratio is given by

$$\frac{C}{I} = \frac{Carrier}{Interference} = \frac{C}{\sum_{k=1}^{M} I_k}$$

where I is co-channel interference and M is the maximum number of co-channel interfering cells

For M = 6, C/I is given by:

$$\frac{C}{I} = \frac{C}{\sum_{k=1}^{M} \left(\frac{D_k}{R} \right)^{-\gamma}}$$

where γ is the propagation path loss slope and $\gamma = 2 \sim 5$

Example on Cochannel Interference

Example 5.2: Calculate the co-channel interference ratio in the worst case for the forward channel in Figure 5.14, given N = 7, R = 2 km, and γ = 1.5.

For this system, the frequency reuse factor q can be calculated as

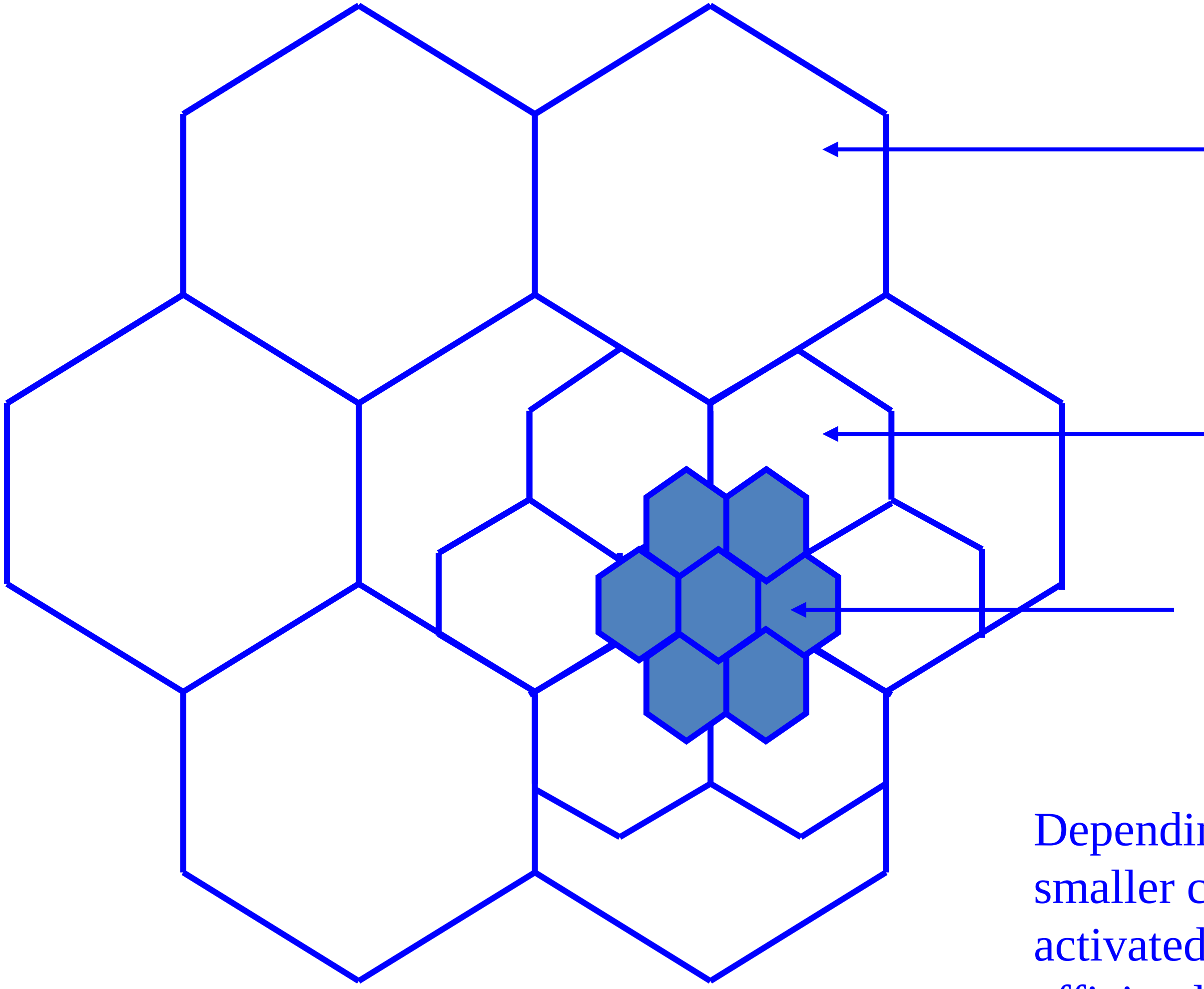
$$q = \sqrt{3N} = \sqrt{3x7} \approx 4.5826$$

Thus, the worst co-channel interference can be calculated by Equation (5.24) as

$$\frac{C}{I} = \frac{1}{2(q-1)^{-\gamma} + 2q^{-\gamma} + 2(q+1)^{-\gamma}}$$

$$= \frac{1}{2(4.5826-1)^{-1.5} + 2x4.5826^{-1.5} + 2(4.5826+1)^{-1.5}} \approx 1.5374$$

Cell Splitting



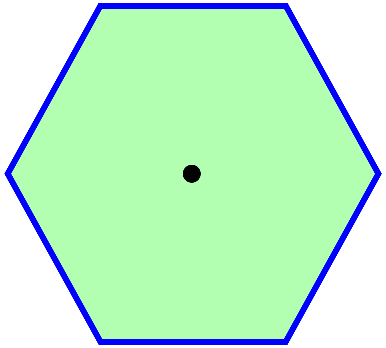
Large cell (low density)

Small cell (high density)

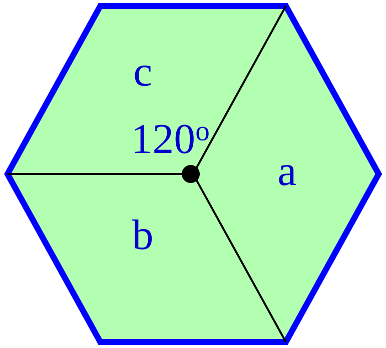
Smaller cell (higher density)

Depending on traffic patterns the smaller cells may be activated/deactivated in order to efficiently use cell resources.

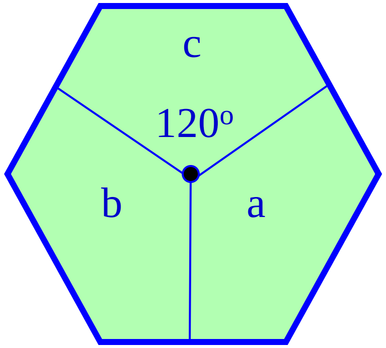
Cell Sectoring by Antenna Design



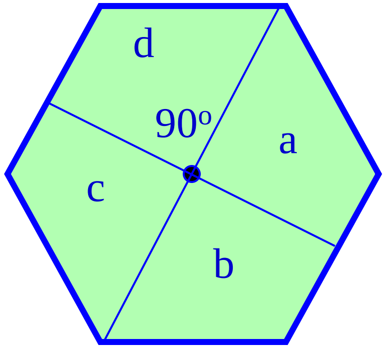
(a). Omni



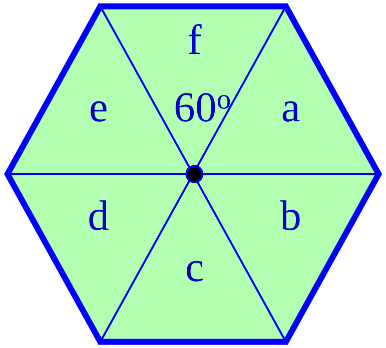
(b). 120^{o} sector



(c). 120^{o} sector (alternate)



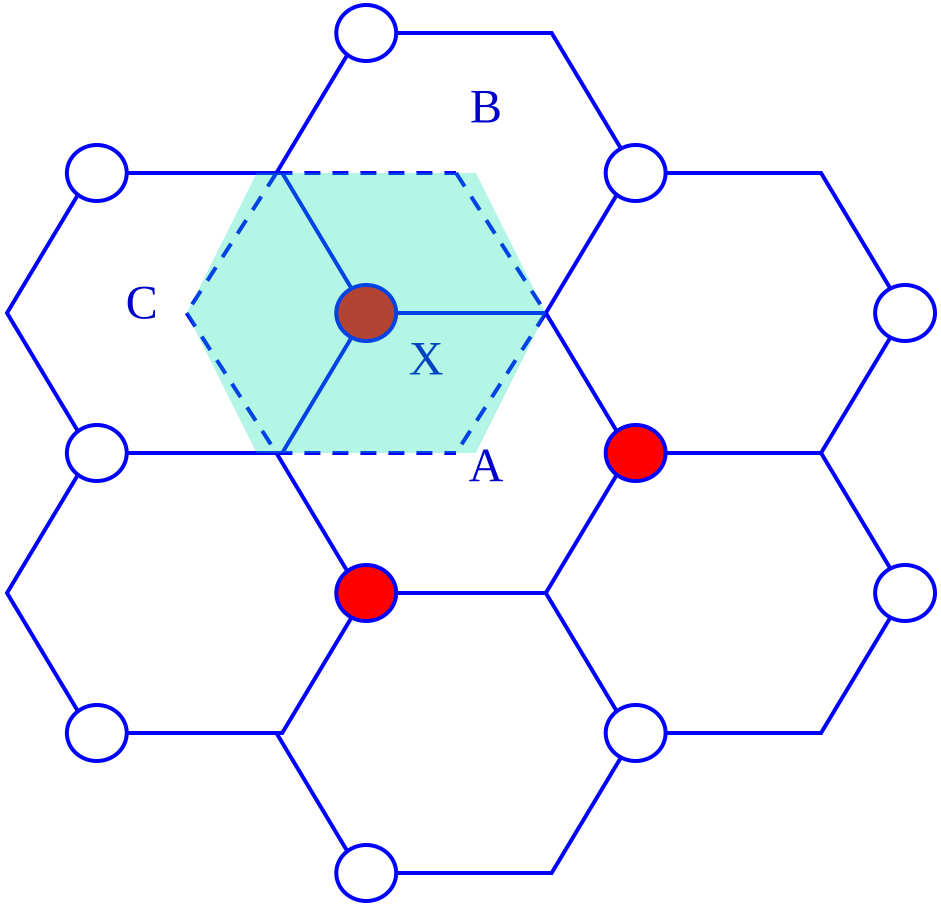
(d). 90^{o} sector



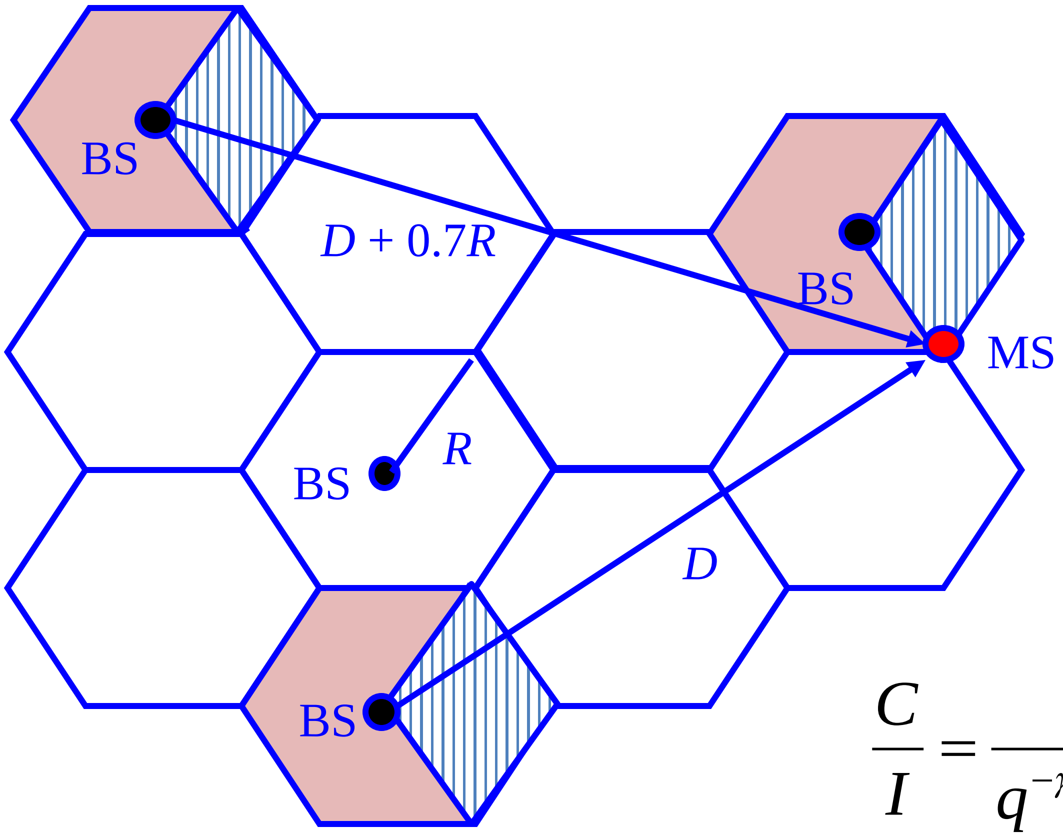
(e). 60^{o} sector

Cell Sectoring by Antenna Design

- Placing directional transmitters at corners where three adjacent cells meet



Worst Case for Forward Channel Interference in Three-sectors

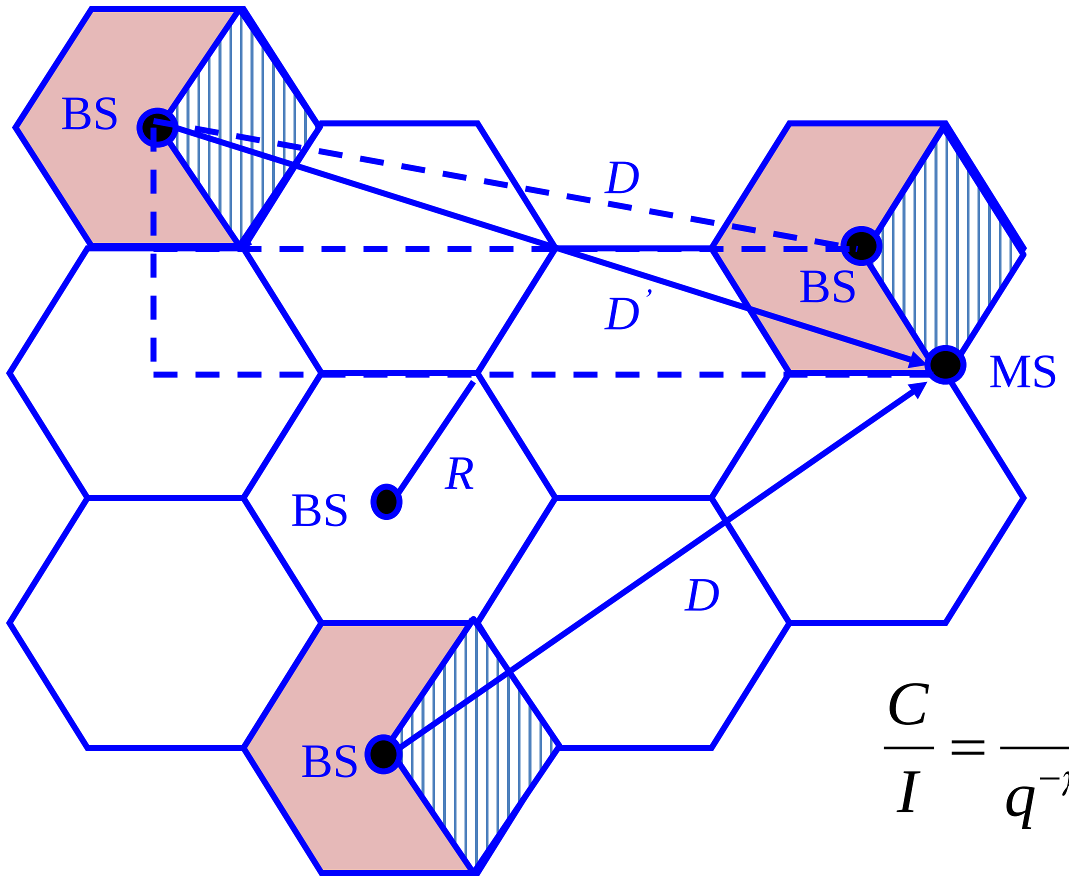


$$\frac{C}{I} = \frac{C}{q^{-\gamma} + (q+0.7)^{-\gamma}}$$

$$q = D/R$$

where γ is the propagation path loss slope and $\gamma = 2 \sim 5$

Worst Case for Forward Channel Interference in Three-sectors

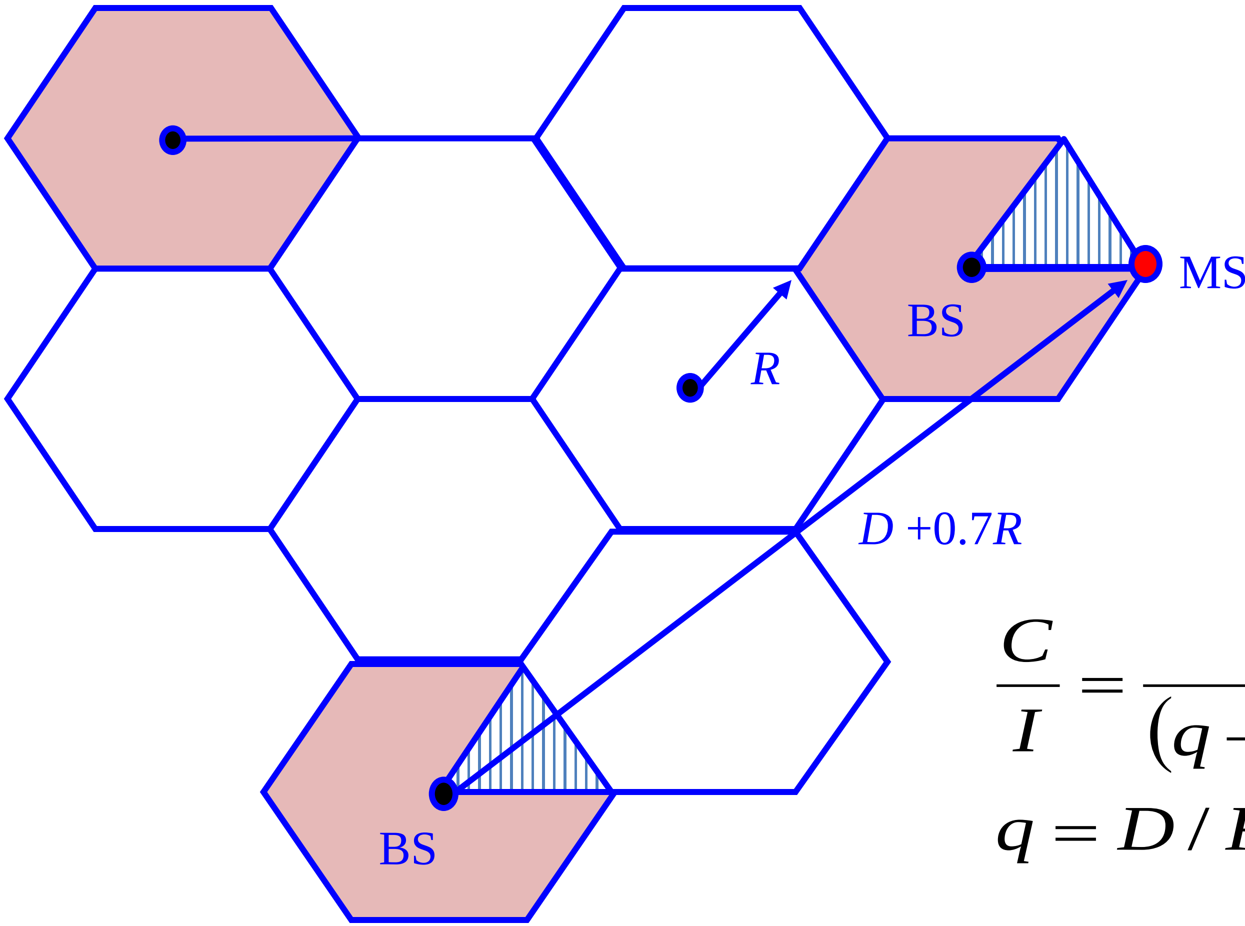


$$\frac{C}{I} = \frac{C}{q^{-\gamma} + (q+0.7)^{-\gamma}}$$

$$q = D/R$$

where γ is the propagation path loss slope and $\gamma = 2 \sim 5$

Worst Case for Forward Channel Interference in Six-sectors



$$\frac{C}{I} = \frac{C}{(q+0.7)^{-\gamma}}$$

$$q = D / R$$

where γ is the propagation path loss slope and $\gamma = 2 \sim 5$