Fall 2011  
20EECS6037: Machine Learning   
Instructor: Anca Ralescu

# Homework assignment 1

# 15 points

Assigned: September 2, 2014

Due on BlackBoard by 11:59PM September 8, 2014

This is a *“pencil-and-paper”* (i.e. non-programming) assignment on Chapter 2. Please use the Latex template provided on BB, to write up your solution.

* **Please name your files as follows:**
  + **Name1Name2Assignment1**
  + **Name1Name2Name3Assignment1**
* **In the actual file please include the names and student ID numbers of team members clearly marked on the first page of the assignment.**

**Problem 1 (7 points)** Pairwise independence does not imply mutual independence. Two random variables, Xi, i=1,2 are independent if

P(Xi | Xj) = P(Xi), for i,j=1,2, i <> j

and therefore

P(Xi, Xj) = P(Xj) P(Xi | Xj) = P(Xi)P(Xj)

Now, given *n* random variables, we say that there are mutually independent if

P(Xi| XS) = P(Xi) for all subsets S of {1, 2, …, n} which do not contain i,

and therefore P(X1, …, Xn) = P(X1) x…xP(Xn)

1. Show that pairwise independence between all pairs of variables (Xi, Xj), does NOT imply mutual independence. Note: it is enough to give an example.

SOLUTION 1

SUPPOSE A BOX CONTAINS 4 TICKETS LABELLED BY

331 323 233 333

LET US CHOOSE ONE TICKET AT RANDOM, AND CONSIDER THE RANDOM EVENTS

A1={1 OCCURS AT THE FIRST PLACE}

A2={1 OCCURS AT THE SECOND PLACE}

A3={1 OCCURS AT THE THIRD PLACE}

P(A1)=1/2 P(A2)=1/2 P(A3)=1/2

A1A2={112} A1A3={121} A2A3={211}

P(A1A2)=P(A1A3)=P(A2A3)=1/4.

So we conclude that the three events A1, A2, A3 are pairwise independent.

However

A1A2A3=f

P(A1A2A3)=0¹P(A1)P(A2)P(A3)=(1/2)3

**CONCLUSION:** Pairwise independence of a given set of random events does not imply that these events are mutually independent.

1. Show mutual independence implies pairwise independence.

Solution:

For example, for four events A, B, C, D to be mutually independent, we must have

P(A∩B∩C∩D) = P(A)P(B)P(C)P(D),

P(A∩B∩C) = P(A)P(B)P(C),

P(A∩B∩D) = P(A)P(B)P(D),

P(A∩C∩D) = P(A)P(C)P(D),

P(B∩C∩D) = P(B)P(C)P(D),

P(A∩B) = P(A)P(B), P(A∩C) = P(A)P(C), P(A∩D) = P(A)P(D),

P(B∩C) = P(B)P(C), P(B∩D) = P(B)P(D), P(C∩D) = P(C)P(D).

**Problem 2(8 points)** Let X and Y be two discrete random variables which are identically distributed but not necessarily independent. Define

R = 1 – H(Y|X) / H(X)

1. Show that R = I(X,Y) / H(X)
2. Show that 0 <= R <= 1
3. When is R = 0?
4. When is R = 1?