

Longest Common Prefix array

- $LCP[i] = \text{longest common prefixes}(\text{suffix}[i], \text{suffix}[i-1])$
 - Generate suffixes, compute suffix array
 - For every pair compute the LCP
 - $LCP(abcd, abckkk) = 3$
 - $LCP(abcd, \text{hello_abcd}) = 0$
- To compute LCP, let's introduce Rank array
 - It represents the reverse of the suffix positions?
 - Recall we have suffix array idx, and its original position
 - Which suffix at position 6 in suffix array? Suffix at 8 (bra)
 - Reverse it
 - Where is suffix generated from pos 8 (bra) ? at position 6

LCP and Rank arrays

Idx	Suffix String	LCP array	Suffix array	Suffix (rev) rank
0		0	11	3
1	a	0	10	7
2	abra	1	7	11
3	abracadabra	4	0	4
4	acadabra	1	3	8
5	adabra	1	5	5
6	bra	0	8	9
7	bracadabra	3	1	2
8	cadabra	0	4	6
9	dabra	0	6	10
10	ra	0	9	1
11	racadabra	2	2	0

$LCP(abra, abracadabra) = 4$

For $i = 6$, suffix at 8 exists (bra)
Let's **reverse** that:

Where is suffix at 8? Position 6

FOR i

$rank[suffix[i]] = i$

LCP

■ Observation

- Let $LCP[i] = k$ = longest common prefix between i th sorted suffix and it previous one
- $suffix[i] = abcdZZ, suffix[i-1] = abcdXX \Rightarrow k = 4$
- Notice that, if we removed 1 letter from both suffixes
- $suffix[j] = bcdZZ, suffix[j-1] = bcdXX \Rightarrow k = 3$
- So lcp between 2 suffixes, tell us lcp about their internal suffixes
- E.g. $lcp(bcdZZ, bcdXX) = 3, lcp(cdZZ, cdXX) = 2$
- Specifically, if $lcp = k$, it tell us values of **$k+1$ consecutive pairs**

LCP

■ Observation

- To make of this, we need to iterate over the order that remove letter by letter from a suffix
- This is the actual suffix generation order NOT suffix sorted array
- That is why we needed the rank array, the reverse of it
- So iterate over generation of suffixes order
- For current position, get suffix position using rank
- compute $LCP = \text{some } K$
- this K will be the answer for $K+1$ iterations

LCP

0	abracadabra
1	bracadabra
2	racadabra
3	acadabra
4	cadabra
5	adabra
6	dabra
7	abra
8	bra
9	ra
10	a
11	

Idx	Suffix String	LCP array
0		0
1	a	0
2	abra	1
3	abracadabra	4
4	acadabra	1
5	adabra	1
6	bra	0
7	bracadabra	3
8	cadabra	0
9	dabra	0
10	ra	0
11	racadabra	2

- Start at $i = 0$, abracadabra
- $\text{rank}[0] = 3$
- Its previous one, 2
- Compute $\text{LCP}(3, 2) = 4$
- $\text{lcp}[\text{rank}[0]] = \text{lcp}[3] = 4$
- Move to $i = 1$
- so 1 letter from suffix 0 removed
- Let current $\text{LCP} = 4 - 1 = 3$

LCP

0	abracadabra
1	bracadabra
2	racadabra
3	acadabra
4	cadabra
5	adabra
6	dabra
7	abra
8	bra
9	ra
10	a
11	

Idx	Suffix String	LCP array
0		0
1	a	0
2	abra	1
3	abracadabra	4
4	acadabra	1
5	adabra	1
6	bra	0
7	bracadabra	3
8	cadabra	0
9	dabra	0
10	ra	0
11	racadabra	2

- Once we move 1 step, the LCP = K, decrease by 1, as 1 letter is removed from the 2 new suffixes
- abracadabra -> bracadabra
- Move to i = 1
- LCP = LCP - 1
- LCP = 3
- rank[1] = 8
- lcp[8] = 3

LCP

0	abracadabra
1	bracadabra
2	racadabra
3	acadabra
4	cadabra
5	adabra
6	dabra
7	abra
8	bra
9	ra
10	a
11	

Idx	Suffix String	LCP array
0		0
1	a	0
2	abra	1
3	abracadabra	4
4	acadabra	1
5	adabra	1
6	bra	0
7	bracadabra	3
8	cadabra	0
9	dabra	0
10	ra	0
11	racadabra	2

- Move to $i = 2$
- $LCP = LCP - 1$
- $LCP = 2$
- $rank[2] = 11$
- $lcp[11] = 2$

LCP

0	abracadabra
1	bracadabra
2	racadabra
3	acadabra
4	cadabra
5	adabra
6	dabra
7	abra
8	bra
9	ra
10	a
11	

Idx	Suffix String	LCP array
0		0
1	a	0
2	abra	1
3	abracadabra	4
4	acadabra	1
5	adabra	1
6	bra	0
7	bracadabra	3
8	cadabra	0
9	dabra	0
10	ra	0
11	racadabra	2

- Move to $i = 3$
- $LCP = LCP - 1$
- $LCP = 1$
- $rank[3] = 4$
- $lcp[4] = 1$
-

LCP

0	abracadabra
1	bracadabra
2	racadabra
3	acadabra
4	cadabra
5	adabra
6	dabra
7	abra
8	bra
9	ra
10	a
11	

Idx	Suffix String	LCP array
0		0
1	a	0
2	abra	1
3	abracadabra	4
4	acadabra	1
5	adabra	1
6	bra	0
7	bracadabra	3
8	cadabra	0
9	dabra	0
10	ra	0
11	racadabra	2

- Move to $i = 4$
- $LCP = LCP - 1$
- $LCP = 0$
- $rank[4] = 8$
- $lcp[8] = 0$
-

LCP

0	abracadabra
1	bracadabra
2	racadabra
3	acadabra
4	cadabra
5	adabra
6	dabra
7	abra
8	bra
9	ra
10	a
11	

Idx	Suffix String	LCP array
0		0
1	a	0
2	abra	1
3	abracadabra	4
4	acadabra	1
5	adabra	1
6	bra	0
7	bracadabra	3
8	cadabra	0
9	dabra	0
10	ra	0
11	racadabra	2

- Move to $i = 5$
- $LCP = LCP - 1$
- $LCP = -1$
- $rank[5] = 5$
- Now $lcp = -1$, means we covered the $K+1$ suffixes
- So recompute LCP from current step
- $LCP(adabra, acadabra) = 1$
- $LCP = 1$ (tell us 2 suffixes)
- $lcp[5] = 1$

LCP

0	abracadabra
1	bracadabra
2	racadabra
3	acadabra
4	cadabra
5	adabra
6	dabra
7	abra
8	bra
9	ra
10	a
11	

Idx	Suffix String	LCP array
0		0
1	a	0
2	abra	1
3	abracadabra	4
4	acadabra	1
5	adabra	1
6	bra	0
7	bracadabra	3
8	cadabra	0
9	dabra	0
10	ra	0
11	racadabra	2

- Move to $i = 6$
- $LCP = LCP - 1$
- $LCP = 0$
- $rank[6] = 9$
- $lcp[9] = 0$

And so on

- If $LCP = K$
- We know answer of $K+1$ LCPs
- Build them
- Compute LCP of new missing pair
- and so on
- Overall $O(n)$ processing