



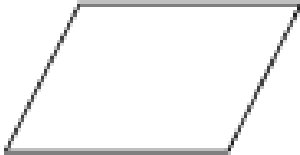
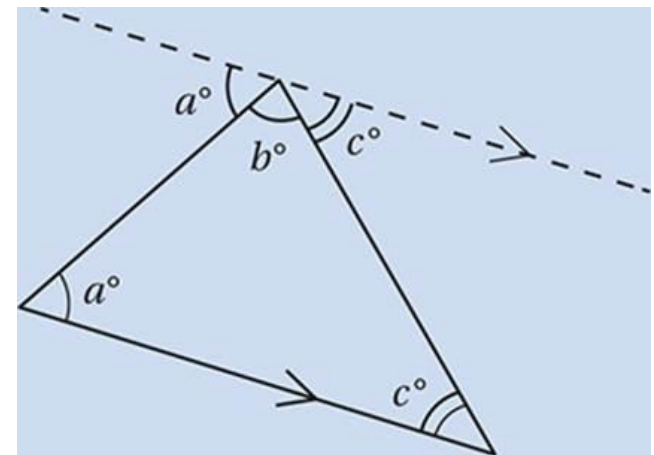
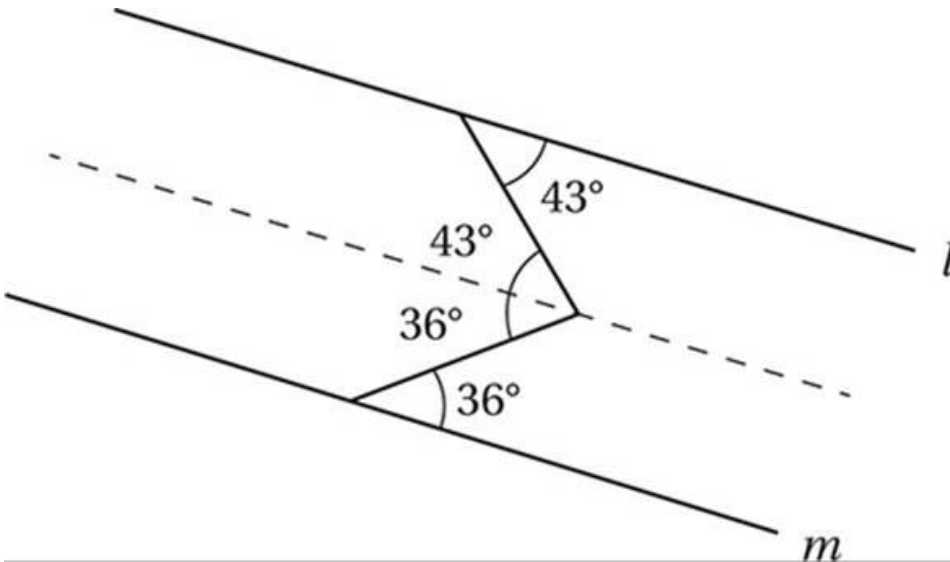
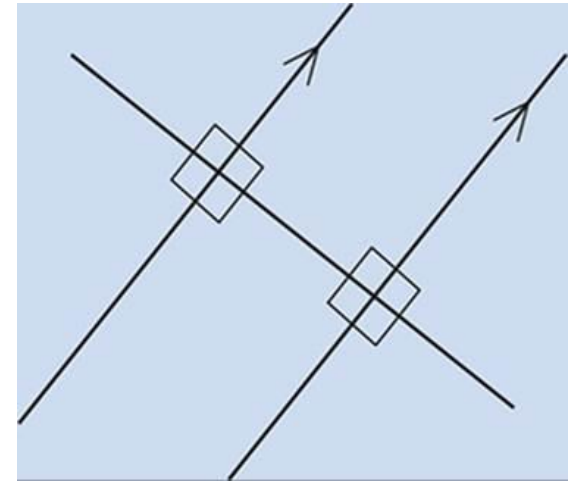
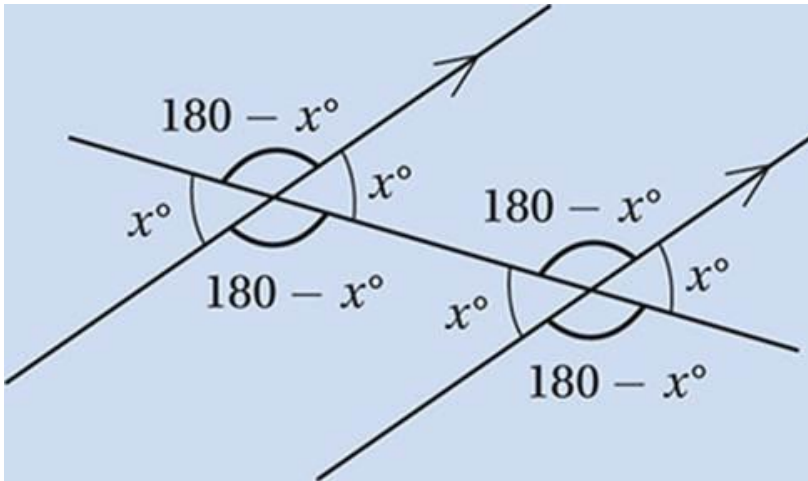


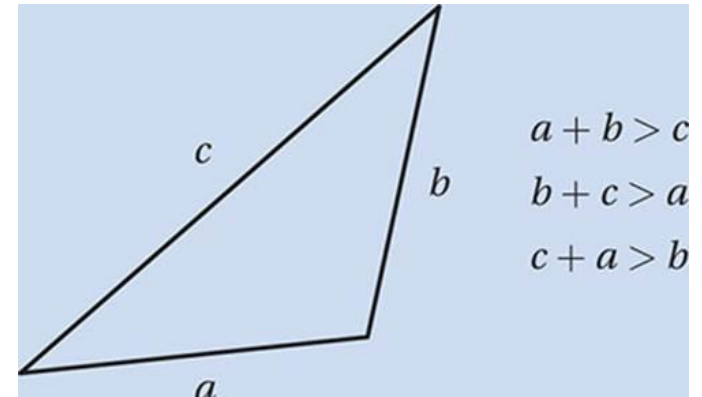
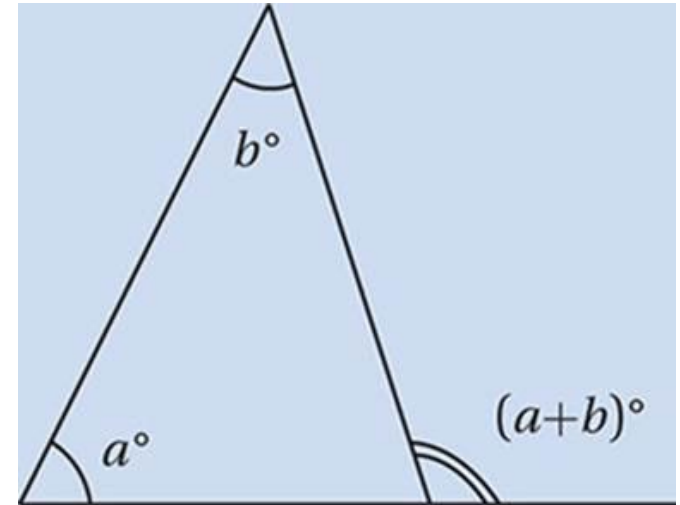
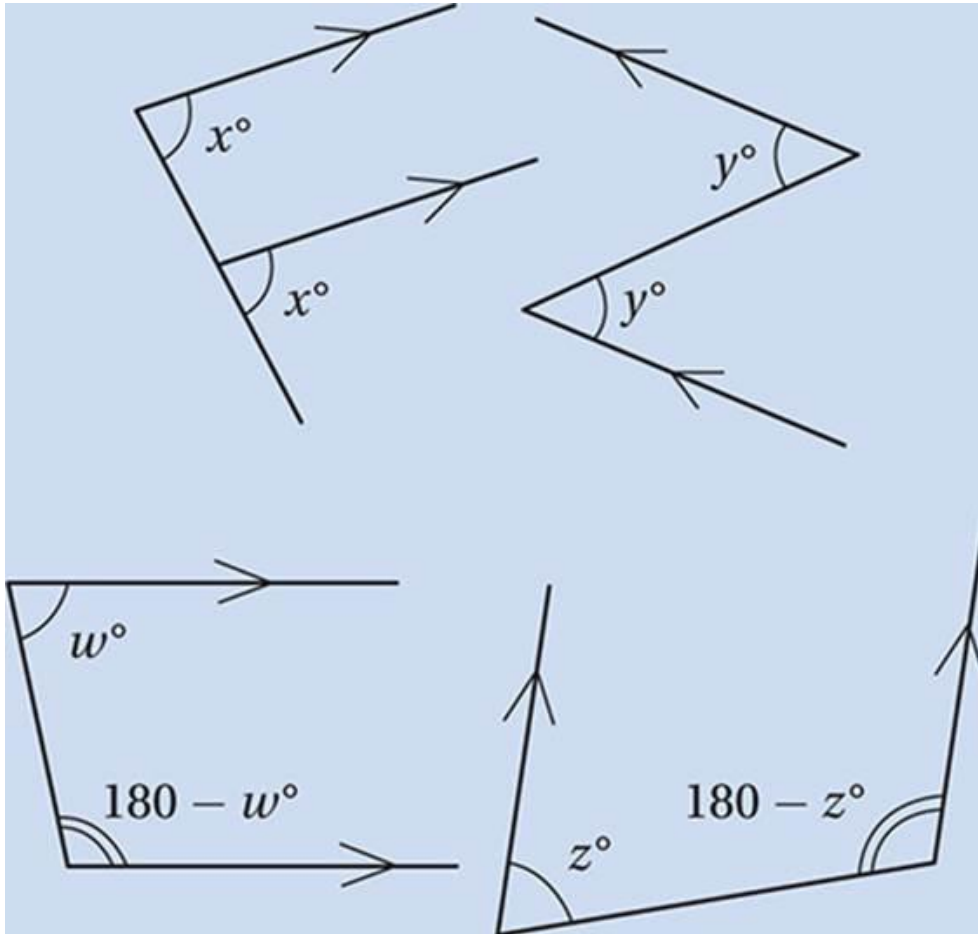
# Elements

Term	Dimensions	Graphic	Symbol
Point	Zero		$\cdot A$
Line Segment	One		$\overline{AB}$
Ray	One		$\overrightarrow{AB}$
Line	One		$\overleftrightarrow{AB}$
Plane	Two		Plane M

# Angles

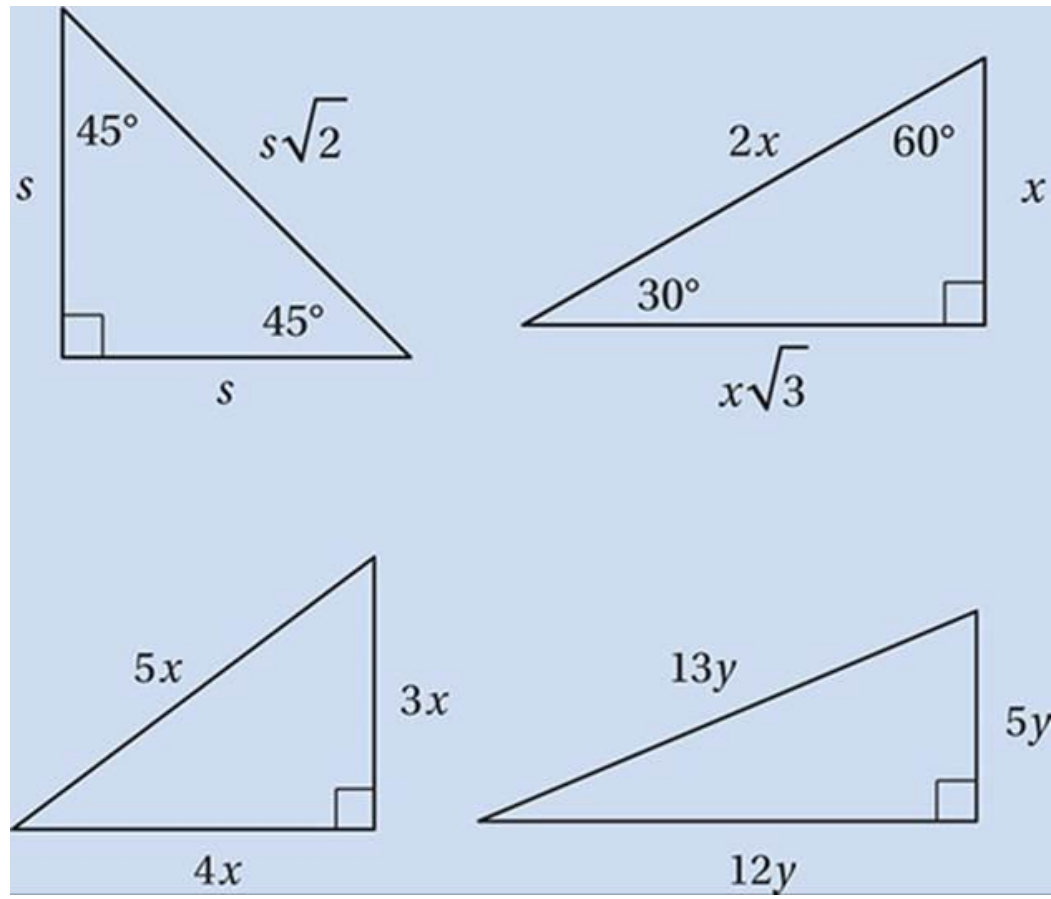


# Angles



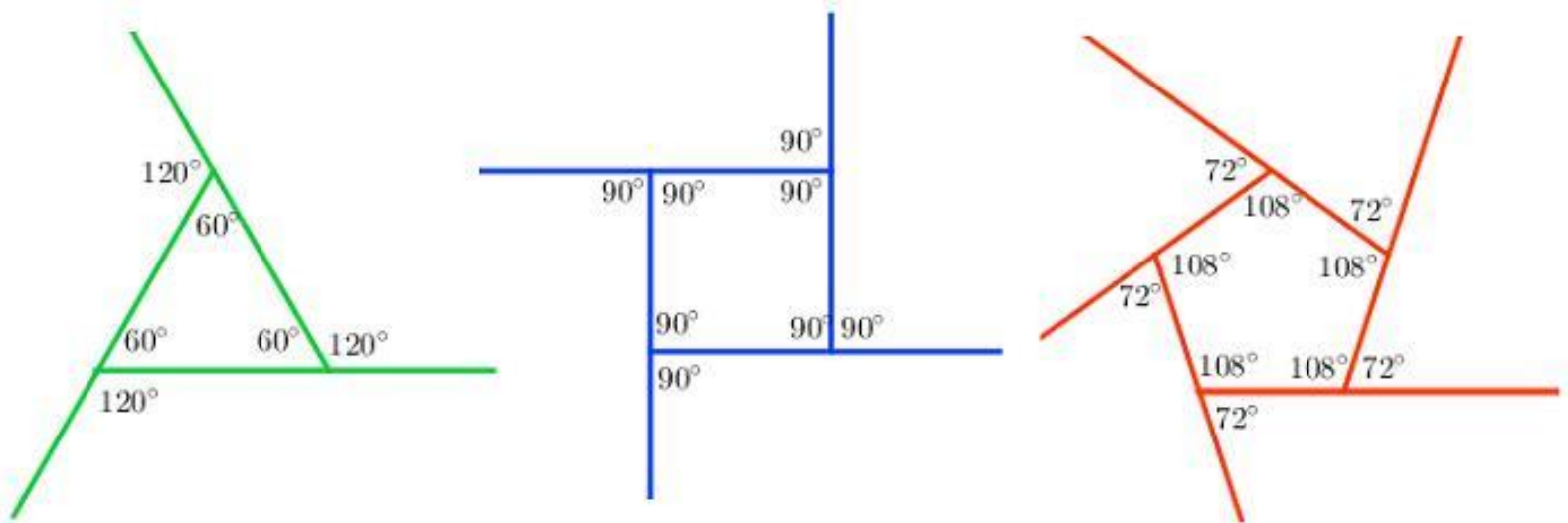
Src: [http://schoolbag.info/sat/sat\\_3/73.html](http://schoolbag.info/sat/sat_3/73.html)

# Angles



# Angles




$360 / \# \text{ sides if all equal}$






Src: <http://www.debate.org/photos/albums/1/6/5244/257096-5244-a4bzy-a.jpg>

# Triangles Types

## Triangles Based on Sides

<p>Scalene</p>  <p>Length of all sides are different</p>	<p>Isosceles</p>  <p>Length of two sides are equal</p>	<p>Equilateral</p>  <p>Length of all sides are equal</p>
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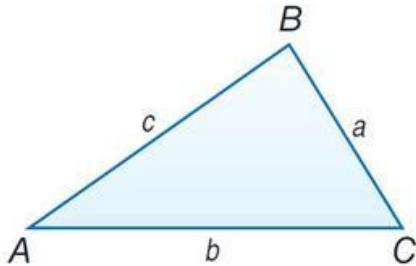
## Triangles Based on Angles

<p>Acute</p>  <p>Each angle is <math>&lt; 90^\circ</math></p>	<p>Right</p>  <p>One angle is <math>= 90^\circ</math></p>	<p>Obtuse</p>  <p>One angle is <math>&gt; 90^\circ</math></p>
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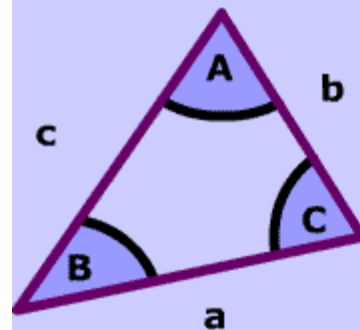
# Triangle Laws

## Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



## Law of Cosines



$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$$

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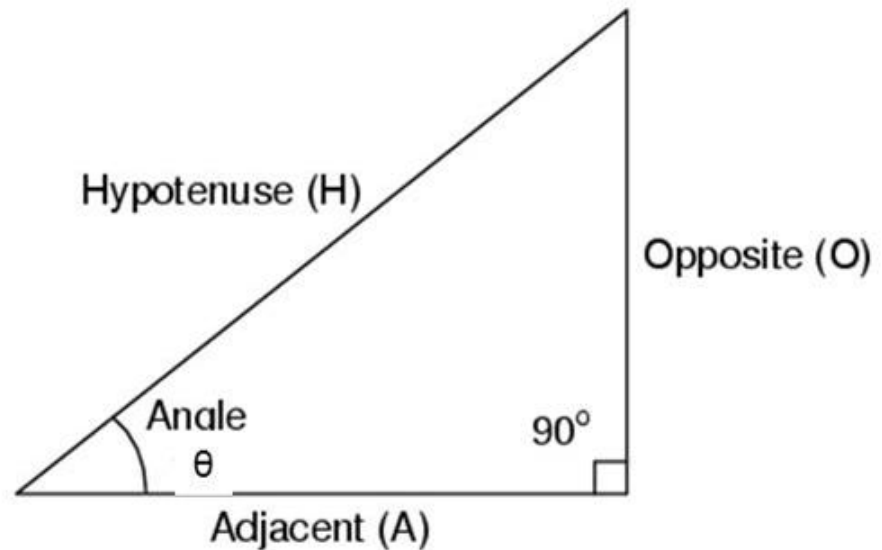
# Trigonometric functions

- $\sin \theta = \text{opposite/hypotenuse}$  Soh
- $\cos \theta = \text{adjacent/hypotenuse}$  Cah
- $\tan \theta = \text{opposite/adjacent}$  Toa

$$a^2 + b^2 = c^2$$

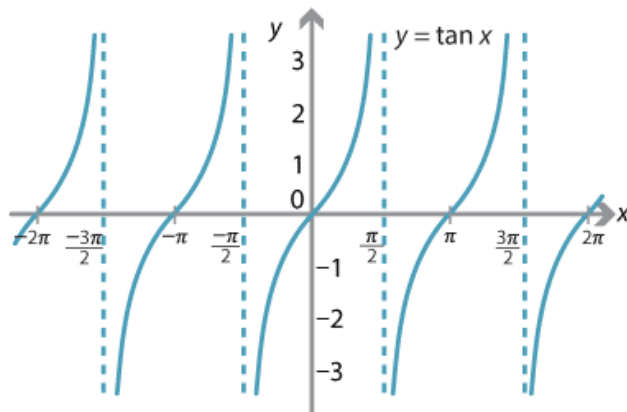
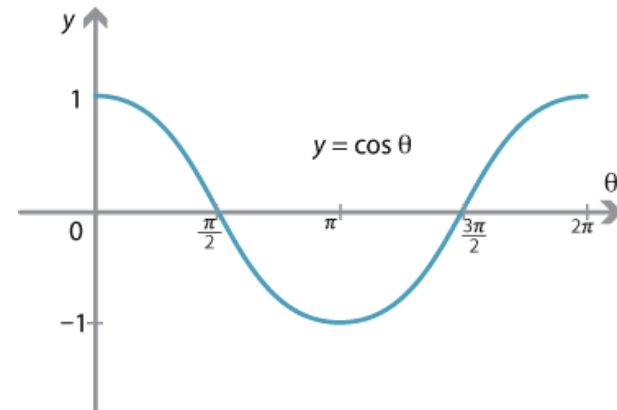
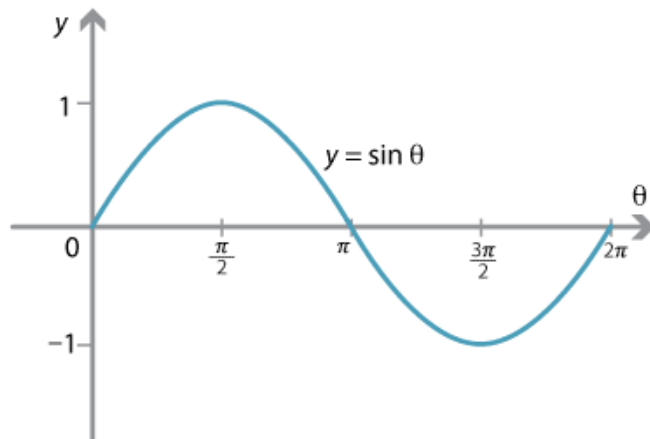
$\sin^{-1}$ ,  $\cos^{-1}$ , and  $\tan^{-1}$   
functions give  $\theta$

With any 2 values, you can  
find all sides and all angles





# Trigonometric functions



$$\sin\left(\frac{\pi}{2} - \theta\right) = +\cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = +\sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = +\cot \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = +\sec \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = +\csc \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = +\tan \theta$$

# Trigonometric formula

$$\sin (A + B) = \sin A \cos B + \sin B \cos A$$

$$\sin (A - B) = \sin A \cos B - \sin B \cos A$$

$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan (a - b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}$$

# Trigonometric functions in C++

- In `cmath` header .. all in **radians**
  - Revise input/output ranges...vary much

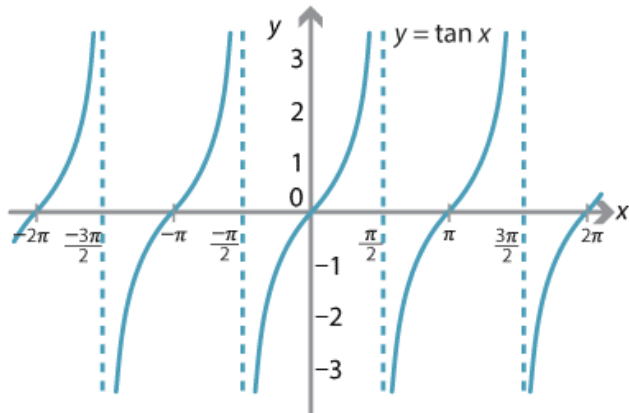
## Trigonometric functions

<code>cos</code>	Compute cosine (function )
<code>sin</code>	Compute sine (function )
<code>tan</code>	Compute tangent (function )
<code>acos</code>	Compute arc cosine (function )
<code>asin</code>	Compute arc sine (function )
<code>atan</code>	Compute arc tangent (function )
<code>atan2</code>	Compute arc tangent with two parameters (function )

## Hyperbolic functions

<code>cosh</code>	Compute hyperbolic cosine (function )
<code>sinh</code>	Compute hyperbolic sine (function )
<code>tanh</code>	Compute hyperbolic tangent (function )
<code>acosh</code> <small>C++11</small>	Compute area hyperbolic cosine (function )
<code>asinh</code> <small>C++11</small>	Compute area hyperbolic sine (function )
<code>atanh</code> <small>C++11</small>	Compute area hyperbolic tangent (function )

# Atan vs Atan 2



Quadrant	Angle	sin	cos	tan
I	$0 < \alpha < \pi/2$	$> 0$	$> 0$	$> 0$
II	$\pi/2 < \alpha < \pi$	$> 0$	$< 0$	$< 0$
III	$\pi < \alpha < 3\pi/2$	$< 0$	$< 0$	$> 0$
IV	$3\pi/2 < \alpha < 2\pi$	$< 0$	$> 0$	$< 0$

Atan range is  $[-\pi/2, \pi/2]$

Tan of either angles 45 or 135  $\Rightarrow$  positive values?!

How to know the quadrant! We need to use sin/cos too

$\text{atan2}(y, x)$  do that for us and return range  $[-\pi, \pi]$

# Atan vs Atan 2

$$\text{atan2}(y, x) = \begin{cases} \arctan(\frac{y}{x}) & x > 0 \\ \arctan(\frac{y}{x}) + \pi & y \geq 0, x < 0 \\ \arctan(\frac{y}{x}) - \pi & y < 0, x < 0 \\ \frac{\pi}{2} & y > 0, x = 0 \\ -\frac{\pi}{2} & y < 0, x = 0 \\ \text{undefined} & y = 0, x = 0 \end{cases}$$

```
(+1,+1) cartesian is (1.41421,0.785398) polar  
(+1,-1) cartesian is (1.41421,2.35619) polar  
(-1,-1) cartesian is (1.41421,-2.35619) polar  
(-1,1) cartesian is (1.41421,-0.785398) polar  
atan2(0, 0) = 0 atan2(0,-0) = 3.14159  
atan2(7, 0) = 1.5708 atan2(7,-0) = 1.5708
```

**Degree = Radian**

-----  
0 = 0

**90 = 1.5708**

**180 = 3.14159**

270 = 4.71239

360 = 6.28319

**45 = 0.785398**

**135 = 2.35619**

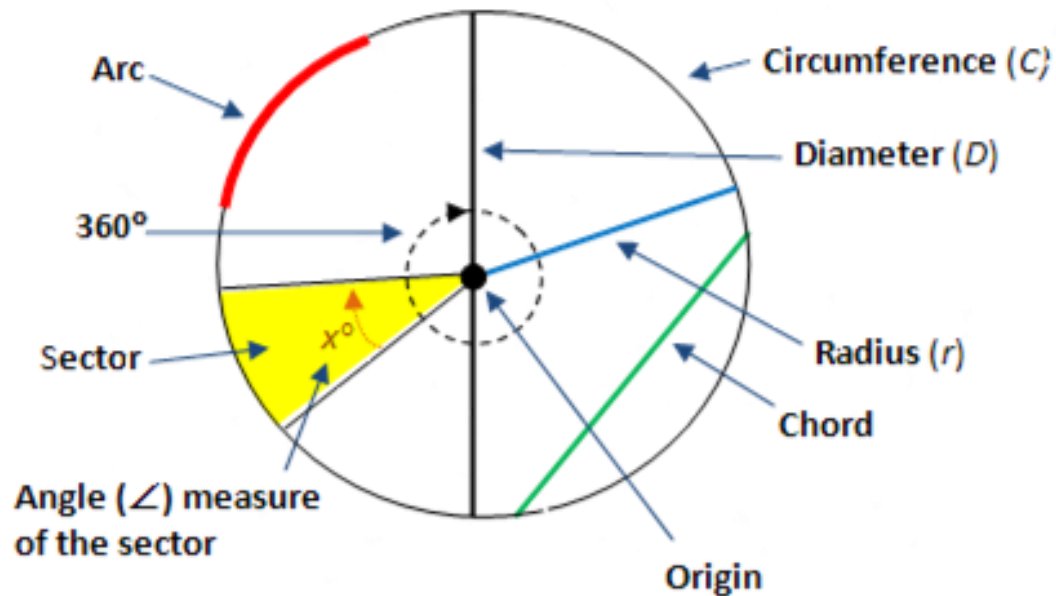
225 = 3.92699

315 = 5.49779

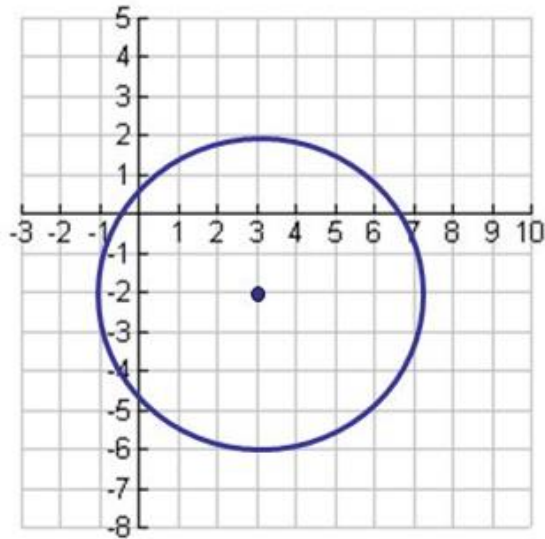
1.4 = sqrt(2)

# Circles

## Parts of a Circle



# Circles

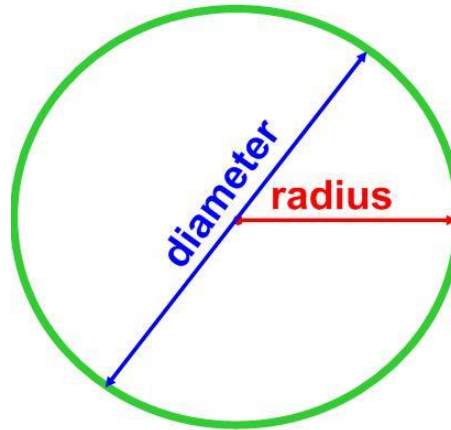
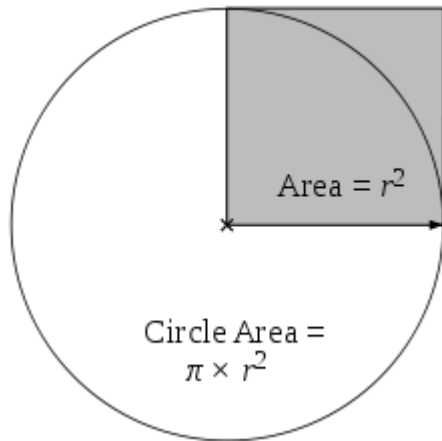


$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 3)^2 + (y - (-2))^2 = 4^2$$

$$(x - 3)^2 + (y + 2)^2 = 16$$

# Circles



Area of a circle  
=  $\pi \times \text{radius}^2$

Circumference of a  
circle =  $\pi \times \text{diameter}$

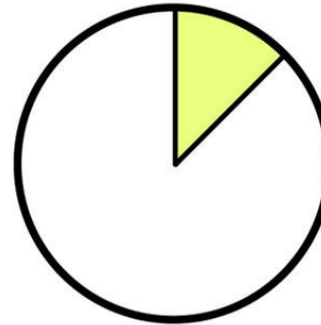
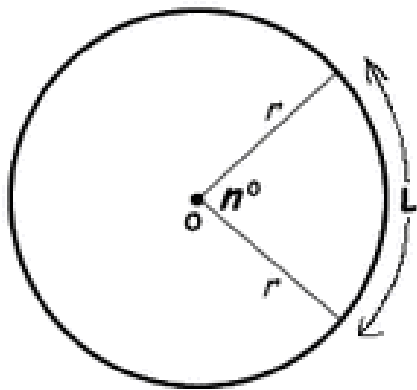
remember that the  
**diameter** = 2 x **radius**



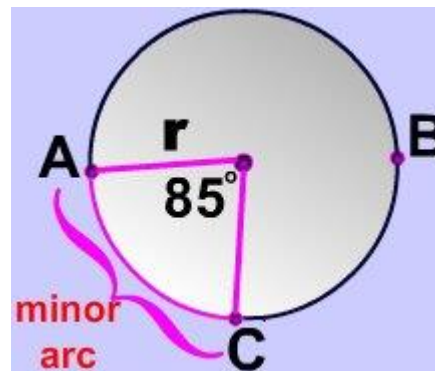
# Circles

## Length of an Arc Formula

$$\text{Length} = \frac{n^\circ}{360^\circ} \times 2\pi r$$

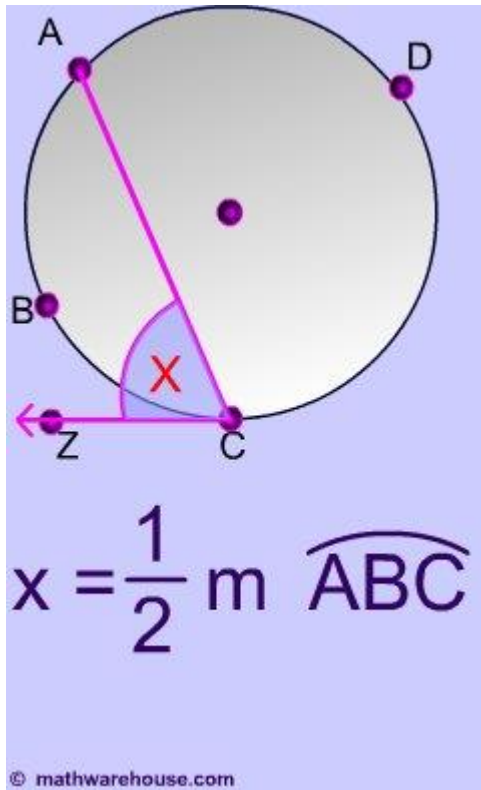


$$\text{sector area} = \frac{n}{360^\circ} \times (\pi \times r^2)$$

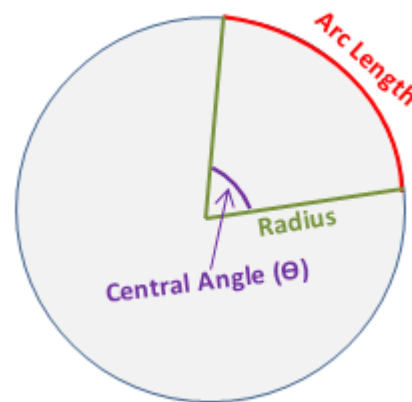
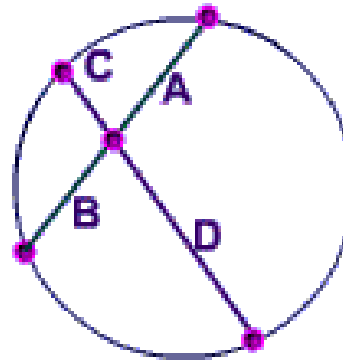


$\widehat{ABC}$  is the major arc  
 $\widehat{AC}$  is the minor arc

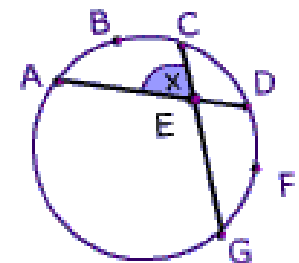
# Circles



$$A \cdot B = C \cdot D$$



$$\angle X = \frac{1}{2} (\widehat{ABC} + \widehat{DFG})$$



# Circles

