* The relations b/w two lines L_1, L_2

- intersect
  - ∞ of Points (identical)
  - one Point
    - $\perp$ ($\theta = 90$)
      - L_2, L_1
      - $\vec{u} \cdot \vec{v} = 0$
    - $\theta \neq 90$
      - L_2, θ, L_1
      - $\theta = \cos^{-1}\left(\dfrac{\vec{u} \cdot \vec{v}}{\|u\| \|v\|}\right)$
- 3. does not intersect
  - Parallel $//$
    - $\vec{u} \times \vec{v} = 0$
    - $\vec{u} = k \cdot \vec{v}$
  - skew
    - L_2, L_1

Some exercises :-

1 determine if L_1, L_2 are $\perp$ or $\parallel$ or skew

$L_1 \Rightarrow x = 5+3t \; , \; y = 4t \; , \; z = +t$
$L_2 \Rightarrow x = 1+2t \; , \; y = t-5 \; , \; z = 3-10t$

ans |

$\vec{u} = \langle 3, 4, 1 \rangle$
$\vec{v} = \langle 2, 1, -10 \rangle$

$\vec{u} \overset{?}{=} k\vec{v} \quad \alpha \quad \vec{u} \nparallel \vec{v}$

$\vec{u} \cdot \vec{v} = 6 + 4 + -10 = 10 - 10 = 0$

$\vec{u} \cdot \vec{v} = 0 \Rightarrow \boxed{L_1, L_2 \text{ are } \perp}$

#

2 determine the angle b/w L_1 and L_2

$L_1 \Rightarrow x = 1-t_1 \; , \; y = 2+t_1 \; , \; z = 3-2t_1$
$L_2 \Rightarrow x = t_2 \; , \; y = 3-t_2 \; , \; z = 3+t_2$

ans |

$\vec{u} = \langle -1, +1, -2 \rangle$
$\vec{v} = \langle 1, -1, 1 \rangle$

$\theta = \cos^{-1}\left(\dfrac{-1 + -1 + -2}{\sqrt{6} \cdot \sqrt{3}}\right)$

$\theta = \cos^{-1}\left(\dfrac{-4}{\sqrt{18}}\right)$

#

$\boxed{3}$ determine if L_1 and L_2 are $\perp$, $\parallel$, skew?

$L_1 :- \langle 3t_1 , 4-t_1 , 2t_1 - 1 \rangle$

$L_2 :- \langle 2-6t_2 , 7+2t_2 , -4t_2 \rangle$

ans| $\vec{u} = \langle 3, -1, 2 \rangle$

$\vec{v} = \langle -6, 2, -4 \rangle$

$\vec{u} = k\vec{v}$

$-2 * \vec{u} = \langle -6, 2, -4 \rangle = \vec{v}$

~~if~~ ∴ L_1 and L_2 are $\parallel$

$\not\perp$

$\boxed{4}$ determine if L_1 and L_2 are $\perp$, $\parallel$, skew. $\boxed{15}$

$L_1 :- \langle 1+7t_1 , 3+t_1 , 5-3t_1 \rangle$

$L_2 :- \langle 4-t_2 , 6 , 7+3t_2 \rangle$

ans| $\vec{u} = \langle 7, 1, -3 \rangle$

$\vec{v} = \langle -1, 0, 3 \rangle$

$\rightarrow \vec{u} \neq k\vec{v} \Rightarrow \nparallel$

$\rightarrow \vec{u} \cdot \vec{v} \overset{?}{=} 0 = -7+0+-9 \neq 0$

$\not\perp$

① $x_1 = x_2$

$1+7t_1 = 4-t_2$

sub. $t_1=3$

$1+21 = 4-t_2$

$22 = 4-t_2$

$t_2 = 4-22 = \boxed{-18}$

$y_1 = y_2$

$3+t_1 = 6$

$\boxed{t_1 = 3}$

$z_1 \neq z_2$

$5-3(3) = 7+3(-18)$

$-4 \neq 7-54$

$z_1 \neq z_2$

∴ L_1, L_2 are skew.

5 find the intersection b/w L_1, L_2

$L_1 = \langle 1+2t_1, 2-t_1, 4-2t_1 \rangle$

$L_2 = \langle 9+t_2, 5+3t_2, -4-t_2 \rangle$

ans

$x_1 = x_2$, $y_1 = y_2$

$1+2t_1 = 9+t_2$, $2-t_1 = 5+3t_2$

$t_2 = 2t_1 - 8$ $\xrightarrow{sub}$ $2-t_1 = 5+3(2t_1-8)$

$2-t_1 = 5+6t_1-24$

$+19+2 = 7t_1$

$7t_1 = 21$

$\boxed{t_1 = 3}$

$t_2 = 2(3)-8$
$= 6-8$
$\boxed{t_2 = -2}$

$z_1 = 4-2t_1$
$= 4-2(3) = 4-6 = \boxed{-2}$

$z_2 = -4-t_2 = -4-(-2) = -4+2 = \boxed{-2}$

$x = 1+2(3) = 7$
$y = 2-3 = -1$
$z = -2$

ans:- $(7, -1, -2)$ $\boxed{(7,-1,-2)}$ #

2019a

6 The y-coordinate for the point of intersection b/w two lines $\ell_1 = \langle 4+t_1, 2+3t, 3+t \rangle$

$\ell_2 = \langle \frac{4+4s}{2}, 4+4s, 1+2s \rangle$

A) 8 (B) 9 C) 10 D) 17 E) 20

ans

$x_1 = x_2$, $z_1 = z_2$

$4+t = \frac{4+4s}{2}$, ~~[illegible]~~ $3+t = 1+2s$

$t = 2s-2$

$8+2t = 4+4s$

$8+2(2s-2) = 4+4s$

$8+4s-4 = 4+4s$

try another $z_1 = z_2$, $y_1 = y_2$

$t = 2s-2$ $2+3(2s-2) = 4+4s$

$2+6s-6 = 4+4s$

$2s = 8$

$\boxed{s = 4}$

sub $s = 4$ in (ℓ_2)

$x = 4 + (\cancel{2s})4 = 8$

$y = 4 + 4(4) = \boxed{20}$

$z = 1+2(4) = 9$

ans:- (E) $y = 20$ # $y = 20$

2018
[7] The Point of intersection of L_1 and L_2 is :-

A) (1,1,1)
B) (2,2,2)
C) (3,3,3)
D) (1,2,3)
E) (2,3,1)
F) (1,1,2)

L_1 :- $x = 3+t$
$y = 3-t$
$z = 3$

L_2 :- $x = 3$
$y = 3$
$z = 3$

ans:- $x_1 = x_2$

$3+t = 3$

$t=0$

sub $t=0$ in eq of L_1

$x = 3+0 = 3$
$y = 3-0 = 3$
$z = 3$

$(3,3,3)$

ans:- c (3,3,3)

[8] Show that L_1, L_2 are skew

2017 essay

L_1 :- $x = 2$, $y = 1+t_1$, $z = t_1$
L_2 :- $x = t_2$, $y = t_2$, $z = t_2$

$\vec{u} = \langle 0, 1, 1 \rangle$
$\vec{v} = \langle 1, 1, 1 \rangle$

$\Rightarrow \vec{u} \neq \alpha \vec{v}$ ∴ they are not Parallel

$\Rightarrow \vec{u} \cdot \vec{v} = 0+1+1 \neq 0$ ∴ they aren't $\perp$

$\Rightarrow x_1 = x_2$ $\boxed{t_2 = 2}$ $\rightarrow$ $z_1 = z_2$ $\boxed{t_1 = t_2 = 2}$

$y_1 \overset{?}{=} y_2 \Rightarrow 1+2 \overset{?}{=} 2$
$\neq$

so, they are skew

[9] find the Parametric eq. of the line that Passes through the intersection b/w L_1 and L_2 and Perpindicular to them. $L_1 :- \langle 1-t, 2+t, 3-2t \rangle$

$L_2 :- \langle s, 3-s, 3+s \rangle$

ans| ① Point

$x_1 = x_2 \Rightarrow 1-t = s$ → sub (2)

$y_1 = y_2 \Rightarrow 2+t = 3-s \Rightarrow 2+t = 3-(1-t)$

$2+\cancel{t} = 2+\cancel{t}$

$z_1 = z_2 \Rightarrow 3-2t = 3+s \Rightarrow \boxed{s = -2t}$

→ sub in q $(x_1 = x_2)$

$1-t = -2t$

$1 = -t \Rightarrow \boxed{t=-1}$

$x = 1-(-1) = 2$

$y = 2+(-1) = 1$

$z = 3-2(-1) = 5$

$\boxed{(2, 1, 5)}$

② // Vector

$$\vec{a} = \vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & -1 & 1 \end{vmatrix}$$

$= (1-2)\hat{i} - (-1-(-2))\hat{j} + (1-1)\hat{k}$

$= \langle -1, -1, 0 \rangle$

eq ⇒ $\{x = 2-t, \ y = 1+-t, \ z = 5+0\}$ #