Discrete Structures

* Lecture 4 *

- Division in the Integers:-

* Prime Numbers: Numbers which are only divisible by themselves and one. ex 13

* Notice that 2 is a prime number.

→ To test whether A number is prime or Not, we have many ways.

1) Divide the number N by every integer from 2 to n-1, if N is not divisible by any of these numbers, So N is prime.
Ex: we want to check whether 17 is prime or Not, So we divide 17 by the numbers from 2 to 16, if it is not divisible by any of them, So 17 is prime number. « Too Long way »

2) Divide the number N by numbers of range $1 < K \leq \sqrt{N}$, if N is not divisible by any of these numbers, So N is prime.
Ex: we want to check whether 17 is prime or Not, So we divide 17 by numbers from 2 to $\sqrt{17}$, if it is not divisible by any of them, So 17 is prime number. « Shorter way. »

3) Divide the number N by less than or equal its half.
Ex: 17 prime or Not? → we divide 17 by numbers from 2 to 8, if it is not divisible by any of them, So 17 is prime number.

4) If the number N is not divisible by 2, So it will not be divisible by any even number, So we will check only odd numbers.
Ex: The number 101 is not divisible by 2, So it will not be divisible by any even numbers, So we will only check if it is divisible by odd numbers from 2 to 10 « $\sqrt{101} = 10$ »

* GCD « Greatest Common Divisor » :-

- We want to know the GCD of 24, 36

→ 24 is divisible by: 1, 2, 3, 4, 6, 8, 12, 24

→ 36 is divisible by: 1, 2, 3, 4, 6, 9, 12, 18, 36

∵ 12 is the greatest number divisible by both of them

∴ 12 is the GCD of 24, 36.

- But this way is too long, So we will have another way to get GCD of two numbers called "Euclidean Algorithm"

* Euclidean Algorithm :-

→ We Express the 2 numbers in form of: $m = qn + r$
, where m, n have the same GCD and n, r have the same GCD

* Example: find GCD of 190, 34

→
$$190 = 5(34) + 20$$
$$34 = 1(20) + 14$$
$$20 = 1(14) + 6$$
$$14 = 2(6) + 2$$
$$6 = 3(2) + 0$$

∴ 2 is the GCD of 190, 34

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* We have another way to find GCD of 2 numbers called "Successive difference"

* Successive Difference :-

→ Find the GCD of 144, 166

a	b	a-b	b-a
144	166		22
144	22	122	
122	22	100	
100	22	78	
78	22	56	
56	22	34	
34	22	12	
12	22		10
12	10	2	
2	10		8
2	8		6
2	6		4
2	4		2
2	2		
	GCD		