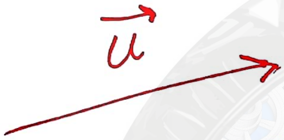
Revision

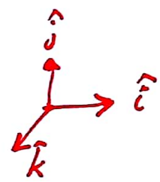
[1] Vector :- a quantity
- mag
- dir.



$\vec{u} =$
- $\vec{u} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$
- $\vec{u} = \langle u_x , u_y , u_z \rangle$

$(\quad x \quad)$

$[\quad x \quad]$



[2] mag. = length = norm $= \|\vec{u}\| = \sqrt{u_x^2 + u_y^2 + u_z^2}$

⇒ How to create a vector $= P_2 - P_1 = \langle 1, 2, 1 \rangle$

$P_1(0,0,0) \longrightarrow P_2(1,2,1)$

$\vec{u} = \langle 1, 2, 2 \rangle$

$\boxed{3}$ $\|k \cdot \vec{u}\| = \|k\| \cdot \|\vec{u}\|$

$\|\vec{u}\| = \sqrt{1 + 2^2 + 2^2}$

$= \sqrt{9} = \boxed{3}$

$2 \cdot \|\vec{u}\| = 2 * 3 = \boxed{6}$

$2 \cdot \|\vec{u}\| \stackrel{?}{=} \underbrace{\|2 \cdot \vec{u}\|}$

$2 \cdot \vec{u} = \langle 2, 4, 4 \rangle$

$\|2 \cdot \vec{u}\| = \sqrt{4 + 16 + 16}$

$= \sqrt{4 + 32}$

$= \sqrt{36} = 6$

$6 = 6$

$\boxed{2 \cdot \|\vec{u}\| = \|2 \cdot \vec{u}\|}$

4 dot Product $\cdot$

$\vec{u} = \langle u_x , u_y , u_z \rangle$

$\vec{V} = \langle V_x , V_y , V_z \rangle$

$\vec{u} \cdot \vec{V} \rightarrow u_x V_x + u_y \cdot V_y + u_z \cdot V_z = \text{scalar}$ (رقم)

$\vec{u} \cdot \vec{V} \rightarrow \|\vec{u}\| \cdot \|\vec{V}\| \cos\theta = \text{scalar}$ (رقم)

مهم

$\boxed{\Rightarrow}$ special case $\vec{u} \perp \vec{V}$ $\quad \theta = 90 \Rightarrow \vec{u} \cdot \vec{V} = |\vec{u}| \cdot |\vec{V}| \cos 90 = \underline{\underline{0}}$

$\Rightarrow \vec{u} \cdot \vec{V} = \vec{V} \cdot \vec{u}$

$\Rightarrow \vec{u} \cdot \vec{V} = \|\vec{u}\| \|\vec{V}\| \cos\theta \Rightarrow \cos\theta = \dfrac{\vec{u} \cdot \vec{V}}{\|\vec{u}\| \|\vec{V}\|} \Rightarrow \theta = \cos^{-1}\left(\dfrac{\vec{u} \cdot \vec{V}}{\|\vec{u}\| \|\vec{V}\|}\right)$

5 cross Product $\times$

$\vec{u} \times \vec{V} \rightarrow \begin{array}{c|ccc|} & \hat{i} & \hat{j} & \hat{k} \\ \vec{u} & u_x & u_y & u_z \\ \vec{V} & V_x & V_y & V_z \end{array} = (u_y \cdot V_z - u_z V_y)\hat{i} - (u_x V_z - u_z \cdot V_x)\hat{j} + (u_x \cdot V_y - u_y \cdot V_x)\hat{k}$

$\vec{u} \times \vec{V} \rightarrow \|\vec{u}\| \|\vec{V}\| \sin\theta$

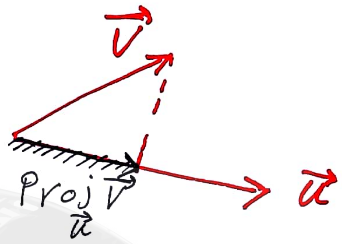
* if $\vec{u} \parallel \vec{V}$

$\hookrightarrow \vec{u} \times \vec{V} = 0$

* $\vec{u} \times \vec{V} = -\vec{V} \times \vec{u}$

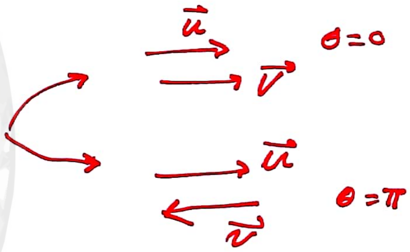
6 Projection (الاسقاط)

$$\Rightarrow \text{Proj}_{\vec u} \vec V = \frac{\vec u \cdot \vec V}{\|\vec u\|^2} \cdot \vec u$$



$$\hookrightarrow \|\text{Proj}_{\vec u} \vec V\| = \frac{\vec u \cdot \vec V}{\|\vec u\|}$$

7 Parallel Vectors $\vec u \parallel \vec V$



$\vec u \parallel \vec V$ → $\vec u \times \vec V = 0$

$\vec u \parallel \vec V$ → $\vec u = t \cdot \vec V$

$$\langle u_x, u_y, u_z \rangle = \langle t \cdot V_x, t \cdot V_y, t \cdot V_z \rangle$$

$$\Rightarrow u_x = t \cdot V_x$$
$$u_y = t \cdot V_y$$
$$u_z = t \cdot V_z$$

Ex) $\vec{u} = \langle -2, 1, 3 \rangle$

$\vec{V} = \langle 1, -2, 4 \rangle$

1) $\|\vec{u} + \vec{V}\| = \Rightarrow \vec{u} + \vec{V} = \langle -1, -1, 7 \rangle \Rightarrow \|\vec{u} + \vec{v}\| = \sqrt{1+1+49}$
$= \boxed{\sqrt{51}}$ #

2) $\vec{u} \cdot \vec{V} = (-2) + (-2) + (12) = \boxed{8}$ #

3) $$\vec{u} \times \vec{V} = \begin{array}{c|ccc|} & \hat{i} & \hat{j} & \hat{k} \\ \vec{u} & -2 & 1 & 3 \\ \vec{v} & 1 & -2 & 4 \end{array} = (4-(-6))\hat{i} - (-8-3)\hat{j} + (4-1)\hat{k}$$

$= \langle 10, 11, 3 \rangle$ #

4) $\|6 \cdot \vec{u}\| = 6 \cdot \|\vec{u}\| \Rightarrow \|\vec{u}\| = \sqrt{4+1+9} = \sqrt{14} \Rightarrow 6 \cdot \|\vec{u}\| = \boxed{6\sqrt{14}}$ #

5) $2\vec{u} + 3\vec{V} = \langle -4, 2, 6 \rangle + \langle 3, -6, 12 \rangle$

$= \langle -1, -4, 18 \rangle$ #

$\vec{u} = \langle -2, 1, 3 \rangle$

$\vec{v} = \langle 1, -2, 4 \rangle$

$\boxed{6}$ $\|\vec{u} \times \vec{v}\| = \Rightarrow \vec{u} \times \vec{v} = \langle 10, 11, 3 \rangle \Rightarrow |\vec{u} \times \vec{v}| = \sqrt{100 + 121 + 9}$ #

$\boxed{7}$ $\vec{a} = \langle 1, 1, -2 \rangle$

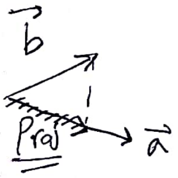
$\vec{b} = \langle 2, 2, -4 \rangle$

A) check if $\vec{a}$ and $\vec{b}$ are $\parallel$ or $\perp$

$\parallel \Rightarrow \vec{a} \overset{?}{=} t \cdot \vec{b} \Rightarrow \langle 1, 1, -2 \rangle \overset{?}{=} t \cdot \langle 2, 2, -4 \rangle$

$\perp \Rightarrow \vec{a} \cdot \vec{b} = 2 + 2 + 8 = 12$ $\quad \vec{a} = \frac{1}{2} \vec{b} \Rightarrow \boxed{\vec{b} = 2\vec{a}}$ # $\vec{a} \parallel \vec{b}$

$\vec{a} \cdot \vec{b} \cdot \perp$



B) $\text{Proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \cdot \vec{a} = \frac{(12) \cdot \langle 1, 1, -2 \rangle}{\left(\sqrt{1 + 1 + 4}\right)^2} = \frac{\cancel{12}^{\,2} \langle 1, 1, -2 \rangle}{\cancel{6}\ 1}$

$= \langle 2, 2, -4 \rangle$ #

Scanned with CamScanner