"Discrete Structures"

lecture 5

* GCD VS LCM

→ Find GCD and LCM between (504, 540)

2	504
2	252
2	126
3	63
3	21
7	7
	1

- We divide both numbers by possible prime numbers.

2	540
2	270
3	135
3	45
3	15
5	5
	1

→ $504 = 2^3 * 3^2 * 5^0 * 7^1$

→ $540 = 2^2 * 3^3 * 5^1 * 7^0$

$\therefore GCD(504, 540) = 2^2 * 3^2 * 5^0 * 7^0$ → Least power

, $LCM(504, 540) = 2^3 * 3^3 * 5^1 * 7^1$ → highest power

* Notes:-

- To get GCD of two numbers, we take the least power, But we take the highest power to get LCM.

* we Can get LCM of a, b by this Law:

$$LCM(a,b) = \frac{a * b}{GCD(a,b)}$$

* Matrix *

$$A = \begin{bmatrix} 1 & 23 & 13 \\ 0 & 4 & 10 \\ 2 & 5 & 6 \\ 7 & 3 & 8 \\ 22 & 9 & 11 \end{bmatrix}$$

→ $A_{22} = 4$, 22 means row 2, Columb 2

1) Square Matrix: Number of rows = Number of Columbs.

2) Diagonal Matrix: All Element are 0 Except the main diagonal. « squared »

- Example:

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

3) Equal Matrices: $A = \begin{bmatrix} a & 9 \\ 5 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & b \\ 5 & 4 \end{bmatrix}$

→ If $A = B$, So $a = 3$, $b = 9$

4) zero Matrix: All elements are zeros

5) Identity Matrix: All elements are zeros Except main diagonal equals ones. « Squared. »

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* if $A = \begin{bmatrix} 9 & 8 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 10 & 4 \end{bmatrix}$

$\therefore A + B = \begin{bmatrix} 12 & 10 \\ 12 & 5 \end{bmatrix}$, $A - B = \begin{bmatrix} 6 & 6 \\ -8 & -3 \end{bmatrix}$

* Product of Matrices *

$$\underset{m \times \underline{n}}{A} \qquad \underset{\underline{n} \times p}{B}$$

- Number of Columbs of first Matrix must be equal to Number of rows of the Second.

- Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \end{bmatrix} , \quad B = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 0 & 3 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

$$2 \times \underline{3} \qquad \underline{3} \times 4$$

Result

$$\therefore A * B = \begin{bmatrix} 1\times1 + 2\times2 + 3\times4 & 1\times2 + 2\times0 + 3\times3 & 1\times1 + 2\times3 + 3\times2 & 1\times0 + 2\times0 + 3\times1 \\ 4\times1 + 0\times2 + 2\times4 & 4\times2 + 0\times0 + 2\times3 & 4\times1 + 0\times3 + 2\times2 & 4\times0 + 0\times0 + 2\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & 11 & 13 & 3 \\ 12 & 14 & 8 & 2 \end{bmatrix} \rightarrow 2 \times 4$$

---

* Notes:-

→ $\underset{3\times\underline{4}}{A} * \underset{\underline{4}\times 2}{B}$ Can be Calcaulated.

→ $\underset{4\times 2}{B} * \underset{3\times 4}{A}$ Can't be Calcaulated

$$\therefore AB \neq BA$$

→ Find A Case where $AB = BA$.

* Important Notes: -

1) $\underset{3\times3}{A}\,\underset{3\times3}{I} = \underset{3\times3}{I}\,\underset{3\times3}{A} = \underset{3\times3}{A}$

2) $\underset{4\times3}{A}\,\underset{3\times3}{I} = \underset{4\times4}{I}\,\underset{4\times3}{A} = \underset{4\times3}{A}$

3) $A(B+C) = AB + AC$

4) $AB(C) = A(BC)$

5) $(A+B)C = AC + BC$

6) → if $\underset{n\times n}{A}$, So $A^{p} = (AAA\ldots)$ power times

7) $(A^{p})^{q} = A^{p\times q}$

8) $A^{p} \times A^{q} = A^{p+q}$

9) $(A^{T})^{T} = A$

* Transposition *

→ Every raw becomes Columb.

- Example: if $\underset{3\times2}{A} = \begin{bmatrix} 5 & 3 \\ 2 & 7 \\ 10 & 4 \end{bmatrix}$, So $\underset{2\times3}{A^{T}} = \begin{bmatrix} 5 & 2 & 10 \\ 3 & 7 & 4 \end{bmatrix}$

* Prove by Example that: $(A+B)^{T} = A^{T} + B^{T}$, $(AB)^{T} = B^{T} \cdot A^{T}$

6) Symmetric Matrix: $A = A^{T}$ «Squared.»

* Example:

$$A = \begin{bmatrix} 1 & 5 & 10 \\ 5 & 3 & 2 \\ 10 & 2 & 7 \end{bmatrix}$$