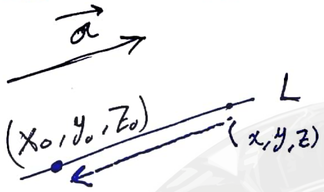
11.5 : Parametric equation of line



I need two things
1. Point lies on the line (x_0, y_0, z_0)
2. Parall vector $\vec{a}$

$\vec{a} = \langle a_x, a_y, a_z \rangle$

general form $\Rightarrow$
$$\left.\begin{aligned} x &= x_0 + a_x t \\ y &= y_0 + a_y t \\ z &= z_0 + a_z t \end{aligned}\right\}$$

الإثبات

$\Rightarrow \langle x - x_0, y - y_0, z - z_0 \rangle \parallel \vec{a}$

$= t \cdot \vec{a}$

$\langle x - x_0, y - y_0, z - z_0 \rangle = \langle a_x \cdot t, a_y \cdot t, a_z \cdot t \rangle$

$x - x_0 = a_x t$

$x = x_0 + a_x t$

$y - y_0 = a_y t$

$y = y_0 + a_y t$

$z - z_0 = a_z t$

$z = z_0 + a_z t$

- forms of Par. eq. of any line

① $$x = x_0 + a_x t$$
$$y = y_0 + a_y t$$
$$z = z_0 + a_z t$$

② $$L = \langle x_0 + a_x t \;,\; y_0 + a_y t \;,\; z_0 + a_z t \rangle$$

③ $$t = \frac{x - x_0}{a_x} = \frac{y - y_0}{a_y} = \frac{z - z_0}{a_z}$$

if we need write a Par. eq. of any line we need two things: Point, $\parallel$ Vector

9] examples :-

[x] find the Parametric equation of line that :-

[1] Passes $(2,-1,3)$ and Parallel to $\langle 3,2,4\rangle$

ans ① Point $(2,-1,3)$

② $\parallel$ Vector $=\langle 3,2,4\rangle$

$$x = x_0 + a_x t$$
$$y = y_0 + a_y t$$
$$z = z_0 + a_z t$$

$$\Rightarrow \boxed{\begin{aligned} x &= 2+3t \\ y &= -1+2t \\ z &= 3+4t \end{aligned}}$$

ans:-

$x = 2t + 2$

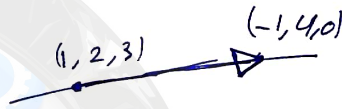
$y = 2t - 1$

$z = 4t + 3$

[2] Passes $(1,2,3)$ and $(-1,4,0)$

ans: ① Point $(1,2,3)$

② $\parallel$ Vector



$$\vec{a} = P_2 - P_1 = \langle -2, 2, -3\rangle$$

$$\boxed{\begin{aligned} x &= 1 + -2t \\ y &= 2 + 2t \\ z &= 3 - 3t \end{aligned}} \quad \#$$

ans :-

$x = 2t + 1$

$y = -2t + 2$

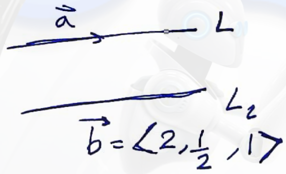
$z = 3t + 3$

$\boxed{3}$ Passes $(0,3,0)$ and Parallel to line

$x = 2t - 5 \quad , y = \frac{t+1}{2} \quad , z = 3 + t$

① Point $(0,3,0)$ ✓

② ∥ Vector



$\vec{a} = \vec{b} = \langle 2, \frac{1}{2}, 1 \rangle$

$x = 0 + 2t$

$y = 3 + \frac{1}{2}t$

$z = 0 + t$

#

ans:-

$x = 2t$

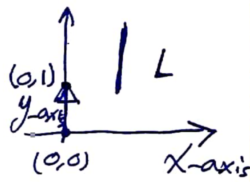
$y = \frac{1}{2}t + 3$

$z = t$

$\boxed{4}$ Passes through $(1,3,7)$ and Parallel to y-axis

① Point $(1,3,7)$

② ∥ Vector



$\vec{a} = P_2 - P_1$

$= \langle 0, 1, 0 \rangle$

$x = 1 + 0 = 1$

$y = 3 + t = 3 + t$

$z = 7 + 0 = 7$

#

ans:-

$x = 1$

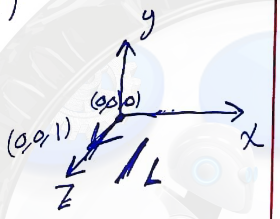
$y = t + 3$

$z = 7$

5 Passes (1, 3, 7) and orthogonal (⊥) to xy-Plane

① Point (1, 3, 7)

② // Vector



$\vec{a} = \langle 0, 0, 1 \rangle$

$$\Rightarrow \quad x = 1+0 = 1$$
$$y = 3+0 = 3$$
$$Z = 7+t = 7+t$$

ans :-
$x = 1$
$y = 3$
$Z = t + 7$

6 let $\ell \Rightarrow x = 2t \; ; \; y = 2 + 5t \; ; \; \cancel{z = 3+2t}$ $Z = 3 + 2t$

if (a, b) lies on L, find $(2a+b)^2$

ans :

$x = 2t = a$ — (1)

$y = 2 + 5t = 1$ — (2)

$Z = 3 + 2t = b$ — (3)

$\Rightarrow$ eq(2) $\Rightarrow$ $2 + 5t = 1$

$5t = -1$

$\boxed{t = \frac{-1}{5}}$

sub in eq(1) $\Rightarrow$ $x = 2\left(\frac{-1}{5}\right) = a$

sub in eq(3) $\Rightarrow$ $a = \frac{-2}{5}$

$3 + 2\left(\frac{-1}{5}\right) = b$

$* \; b = \frac{3}{1} - \frac{2}{5} = \frac{15 - 2}{5} = \boxed{\frac{13}{5}}$

$\left(2 \cdot \left(\frac{-2}{5}\right) + \frac{13}{5}\right)^2 = \left(-\frac{4}{5} + \frac{13}{5}\right)^2 = \left(\frac{9}{5}\right)^2 = \boxed{\frac{81}{25}}$ #

ans :- $t = -1/5 \Rightarrow (2a+b)^2 =$

7 find the Par. eq. of line that Passes (2, 1, 5) and Perpindicular to

$L_1 \Rightarrow x = 1-t,\ y = 2+t,\ z = 3-2t$

$L_2 \Rightarrow x = t,\ y = 3-t,\ z = 3+t$

① Point (2, 1, 5) ✓
② // vector, $\vec{a}$

$\vec{u} = \langle -1, +1, -2 \rangle$

$\vec{v} = \langle 1, -1, 1 \rangle$

$$\vec{a} = \vec{u} \times \vec{v} = \begin{vmatrix} -1 & 1 & -2 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= (1-2)\hat{i} - (-1-(-2))\hat{j} + (1-1)\hat{k}$$

$\vec{a} = \langle -1, -1, 0 \rangle$ ✓

$$\boxed{\begin{aligned} x &= 2-t \\ y &= 1-t \\ z &= 5+0 \end{aligned}}$$

#

8 The Values of (a) and (c) 12

2019a

So that the Point (a, −8, c) lies on the line that Passes through two Pts (a 2, 3) and (2, 7, 5) ?

ans| ⇒ find (a) and (c)

① find the eq.
② sub. the Point in eq.

Par. eq. ⇒ ① Point (0, 2, 3) ✓
② // Vector

$\vec{a} = P_2 - P_1 = \langle 2, 5, 2 \rangle$ ✓

⇒ $x = 2t,\ y = 2+5t,\ z = 3+2t$

* $-8 = 2+5t \Rightarrow 5t = -10 \Rightarrow \boxed{t = -2}$
* sub in eq(1) ⇒ $x = 2(-2) = \boxed{a = -4}$ ✓
* sub in eq(3) ⇒ $z = 3+2(-2) = \boxed{c = -1}$

#

ans:- $a = -4$
$c = -1$

$\boxed{9}$ let $L_1 \Rightarrow \begin{array}{l} x = 2t-1 \\ y = 3+t \\ z = t-\frac{1}{2} \end{array}$

find :-

a) Vector $\parallel$ to L_1

$\vec{a} = \langle 2, 1, 1 \rangle$ ✓

$\Rightarrow \vec{a} = \langle 4, 2, 2 \rangle$ ✓

b) Point lies on the line

$t = 0 \Rightarrow (-1, 3, -\frac{1}{2})$

$t = 1 \Rightarrow (1, 4, \frac{1}{2})$

$t = -1, \ldots$

c) when the line intersects xz-Plane

$y = 0$

$y = 3 + t = 0 \Rightarrow \boxed{t = -3}$

$x = -6 - 1 = \boxed{-7}$

$z = -3 - \frac{1}{2} = \boxed{-\frac{7}{2}}$

$(-7, 0, -\frac{7}{2})$

#

ans: $(-7, 0, -\frac{7}{2})$

d) when the line intersects zy-Plane

$x = 0 \Rightarrow x = 2t - 1 = 0$

$\boxed{t = \frac{1}{2}}$

$y = 3 + \frac{1}{2} = \boxed{\frac{7}{2}}$

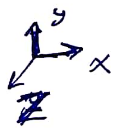
$z = \frac{1}{2} - \frac{1}{2} = \boxed{0}$

$(0, \frac{7}{2}, 0)$

#

ans:- $(0, \frac{7}{2}, 0)$

e) When the line intersects y-axis



$x = 0$

$z = 0$

$x = 2t - 1 = 0 \Rightarrow \boxed{t = \frac{1}{2}}$

$z = t - \frac{1}{2} = 0 \Rightarrow \boxed{t = \frac{1}{2}}$

$y = 3 + \frac{1}{2} = \frac{7}{2}$

$\boxed{(0, \frac{7}{2}, 0)}$

#

ans:- $(0, \frac{7}{2}, 0)$