

Q1: Suppose a genetic algorithm uses chromosomes of the form $x = abcdefgh$ with a fixed length of eight genes. Each gene can be any digit between 0 and 9. Let the fitness of individual x be calculated as:

$$f(x) = (a + b) - (c + d) + (e + f) - (g + h),$$

and let the initial population consist of four individuals with the following chromosomes:

$$x_1 = 6\ 5\ 4\ 1\ 3\ 5\ 3\ 2$$

$$x_2 = 8\ 7\ 1\ 2\ 6\ 6\ 0\ 1$$

$$x_3 = 2\ 3\ 9\ 2\ 1\ 2\ 8\ 5$$

$$x_4 = 4\ 1\ 8\ 5\ 2\ 0\ 9\ 4$$

a) Evaluate the fitness of each individual, showing all your workings, and arrange them in order with the fittest first and the least fit last.

Answer:

$$f(x_1) = (6 + 5) - (4 + 1) + (3 + 5) - (3 + 2) = 9$$

$$f(x_2) = (8 + 7) - (1 + 2) + (6 + 6) - (0 + 1) = 23$$

$$f(x_3) = (2 + 3) - (9 + 2) + (1 + 2) - (8 + 5) = -16$$

$$f(x_4) = (4 + 1) - (8 + 5) + (2 + 0) - (9 + 4) = -19$$

The order is x_2 , x_1 , x_3 and x_4 .

b) Perform the following crossover operations:

i) Cross the fittest two individuals using one-point crossover at the middle point.

Answer: One-point crossover on x_2 and x_1 :

$$x_2 = 8\ 7\ 1\ 2\ 6\ 6\ 0\ 1$$

$$x_1 = 6\ 5\ 4\ 1\ 3\ 5\ 3\ 2$$

$$O_1 = 8\ 7\ 1\ 2\ 3\ 5\ 3\ 2$$

$$O_2 = 6\ 5\ 4\ 1\ 6\ 6\ 0\ 1$$

ii) Cross the second and third fittest individuals using a two-point crossover (points b and f).

Answer: Two-point crossover on x_1 and x_3

$$x_1 = 6\ 5\ 4\ 1\ 3\ 5\ 3\ 2$$

$$x_3 = 2\ 3\ 9\ 2\ 1\ 2\ 8\ 5$$

$$O_3 = 6\ 5\ 9\ 2\ 1\ 2\ 3\ 2$$

$$O_4 = 2\ 3\ 4\ 1\ 3\ 5\ 8\ 5$$

iii) Cross the first and third fittest individuals (ranked 1st and 3rd) using a uniform crossover.

Answer: In the simplest case uniform crossover means just a random exchange of genes between two parents. For example, we may swap genes at positions a, d and f of parents x_2 and x_3 :

$$x_2 = 8\ 7\ 1\ 2\ 6\ 6\ 0\ 1$$

$$x_3 = 2\ 3\ 9\ 2\ 1\ 2\ 8\ 5$$

$$O_5 = 2\ 7\ 1\ 2\ 6\ 2\ 0\ 1$$

$$O_6 = 8\ 3\ 9\ 2\ 1\ 6\ 8\ 5$$

c) Suppose the new population consists of the six offspring individuals received by the crossover operations in the above question. Evaluate the fitness of the new population, showing all your workings. Has the overall fitness improved?

Answer: The new population is:

$$O_1 = 8\ 7\ 1\ 2\ 3\ 5\ 3\ 2$$

O₂ = 6 5 4 1 6 6 0 1

O₃ = 6 5 9 2 1 2 3 2

O₄ = 2 3 4 1 3 5 8 5

O₅ = 2 7 1 2 6 2 0 1

O₆ = 8 3 9 2 1 6 8 5

Now apply the fitness function $f(x) = (a+b)-(c+d)+(e+f)-(g+h)$:

$$f(O_1) = (8 + 7) - (1 + 2) + (3 + 5) - (3 + 2) = 15$$

$$f(O_2) = (6 + 5) - (4 + 1) + (6 + 6) - (0 + 1) = 17$$

$$f(O_3) = (6 + 5) - (9 + 2) + (1 + 2) - (3 + 2) = -2$$

$$f(O_4) = (2 + 3) - (4 + 1) + (3 + 5) - (8 + 5) = -5$$

$$f(O_5) = (2 + 7) - (1 + 2) + (6 + 2) - (0 + 1) = 13$$

$$f(O_6) = (8 + 3) - (9 + 2) + (1 + 6) - (8 + 5) = -6$$

The overall fitness has improved.

d) By looking at the fitness function and considering that genes can only be digits between 0 and 9 find the chromosome representing the optimal solution (i.e. with the maximum fitness). Find the value of the maximum fitness.

Answer: The optimal solution should have a chromosome that gives the maximum of the fitness function

$$\max f(x) = \max [(a + b) - (c + d) + (e + f) - (g + h)] .$$

Because genes can only be digits from 0 to 9, the optimal solution should be:

$$x_{\text{optimal}} = 9 \ 9 \ 0 \ 0 \ 9 \ 9 \ 0 \ 0 ,$$

and the maximum fitness is

$$f(x_{\text{optimal}}) = (9 + 9) - (0 + 0) + (9 + 9) - (0 + 0) = 36$$

e) By looking at the initial population of the algorithm can you say whether it will be able to reach the optimal solution without the mutation operator?

Answer: No, the algorithm will never reach the optimal solution without mutation. The optimal solution is $x_{\text{optimal}} = 9 \ 9 \ 0 \ 0 \ 9 \ 9 \ 0 \ 0$. If mutation does not occur, then the only way to change genes is by applying the crossover operator. Regardless of the way crossover is performed, its only outcome is an exchange of genes of parents at certain positions in the chromosome. This means that the first gene in the chromosomes of children can only be either 6, 8, 2 or 4 (i.e. first genes of x_1 , x_2 , x_3 and x_4), and because none of the individuals in the initial population begins with gene 9, the crossover operator alone will never be able to produce an offspring with gene 9 in the beginning. One can easily check that a similar problem is present at several other positions. Thus, without mutation, this GA will not be able to reach the optimal solution.

Question 2

What two requirements should a problem satisfy in order to be suitable for solving it by a GA?

Answer: GA can only be applied to problems that satisfy the following requirements:

- The fitness function can be well-defined.
- Solutions should be decomposable into steps (building blocks) which could be then encoded as chromosomes.

Question 6

A budget airline company operates 3 planes and employs 5 cabin crews. Only one crew can operate on any plane on a single day, and each crew cannot work for more than two days in a row. The company uses all planes every day. A Genetic Algorithm is used to work out the best combination of crews on any particular day.

a) Suggest what chromosome could represent an individual in this algorithm?

Answer: On each day, a solution is a combination of 3 cabin crews assigned to 3 airplanes. Thus, a chromosome of 3 genes could be used in this algorithm with each gene representing a crew on a certain plane.

b) Suggest what could be the alphabet of this algorithm? What is its size?

Answer: The alphabet of genes representing the crews can be used. Thus, its size is 5.

c) Suggest a fitness function for this problem.

Answer: You may come up with different versions, but it is important for the fitness to take into account the condition that cabin crews cannot work more than 2 days in a row. For example, the fitness function can take into account how many days each crew has left before a day off (e.g. 1 or 0). The fitness could be calculated as the sum of these numbers for all drivers in the chromosome.

d) How many solutions are in this problem? Is it necessary to use Genetic Algorithms for solving it? What if the company operated more planes and employed more crews?

Answer: The number of solutions is the number of times 3 crews can be selected out of 5 without replacement and without taking into account their order. The first crew can be selected in 5 different ways, the second in 4 ways and the third in 3 different ways. These numbers multiplied together will give us total number times how 3 crews can be selected randomly out of 5: $5 \times 4 \times 3 = 60$ times. However, there are 6 possible combinations in which 3 crews can be ordered, and because the order does not matter the answer is $60/6 = 10$. Thus, there are 10 possible solutions for this problem.

It is not really necessary to use GA for a problem with such a small population, because solutions can be checked explicitly. However, as the number of crews and airplanes increases, so does the number of solutions, and the use of GA can be the only option. In fact, if n is the number of cabin crews and k is the number of airplanes, then the number of solutions is

$$\frac{n!}{k!(n-k)!} \cdot$$

For example, if the company operated 10 airplanes and employed 20 cabin crews, then the number of solutions would be

$$\frac{20!}{10!(20-10)!} = 184,756$$