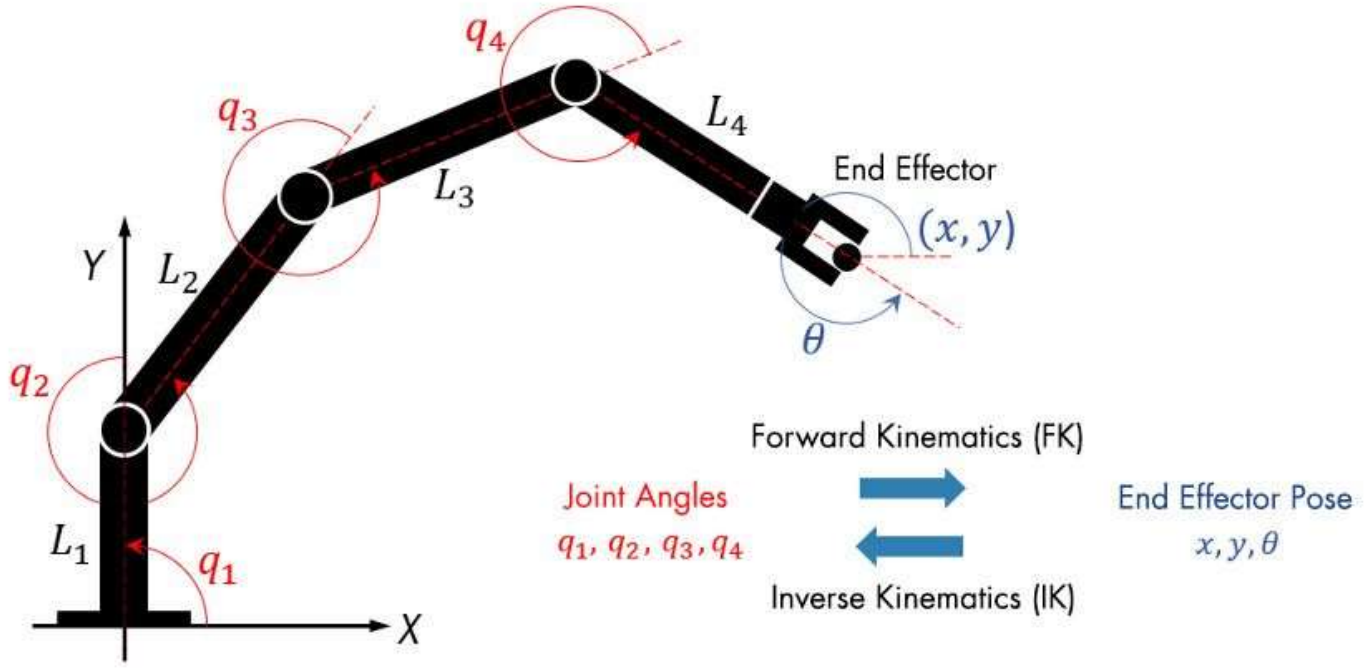
Alexandria University
Faculty of Engineering
Electrical Engineering Department

Forward Kinematics Assignment



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2. Consider the three-link planar manipulator shown in Figure 3.12. Derive

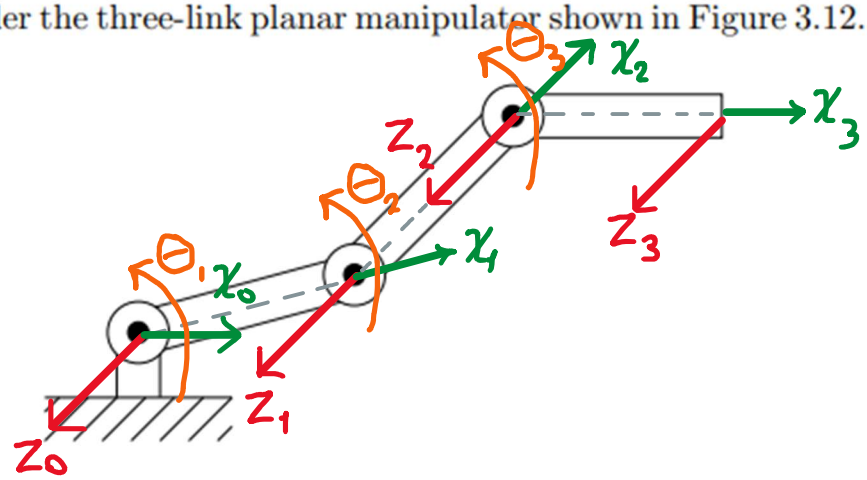

Figure 3.12: Three-link planar arm of Problem 3-2.

the forward kinematic equations using the DH-convention.

Link	a_i	α_i	d_i	θ_i
1	L_1	0	0	θ_1
2	L_2	0	0	θ_2
3	L_3	0	0	θ_3

$$A_i = R_z(\theta_i) \cdot T_z(d_i) \cdot T_x(a_i) \cdot R_x(\alpha_i)$$

From MATLAB script I build:

$$A_i = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
>> FK_Assignment
A =
[cos(th), -cos(alph)*sin(th),  sin(alph)*sin(th), a*cos(th)]
[sin(th),  cos(alph)*cos(th), -sin(alph)*cos(th), a*sin(th)]
[      0,          sin(alph),          cos(alph),         d]
[      0,                  0,                  0,         1]
```

To get H = A1 * A2 * A3

```
A1 =
[cos(th1), -sin(th1), 0, L1*cos(th1)]
[sin(th1),  cos(th1), 0, L1*sin(th1)]
[       0,         0, 1,          0]
[       0,         0, 0,          1]

=======================================
A2 =
[cos(th2), -sin(th2), 0, L2*cos(th2)]
[sin(th2),  cos(th2), 0, L2*sin(th2)]
[       0,         0, 1,          0]
[       0,         0, 0,          1]

=======================================
A3 =
[cos(th3), -sin(th3), 0, L3*cos(th3)]
[sin(th3),  cos(th3), 0, L3*sin(th3)]
[       0,         0, 1,          0]
[       0,         0, 0,          1]
```

```
H =
[cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)),
[cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)),
[                                                                                                  0,
[                                                                                                  0,

- cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) - sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)), 0,
  cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)), 0,
                                                                                                0, 1,
                                                                                                0, 0,
```

```
L1*cos(th1) + L3*cos(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) - L3*sin(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + L2*cos(th1)*cos(th2) - L2*sin(th1)*sin(th2)]
L1*sin(th1) + L3*cos(th3)*(cos(th1)*sin(th2) + cos(th2)*sin(th1)) + L3*sin(th3)*(cos(th1)*cos(th2) - sin(th1)*sin(th2)) + L2*cos(th1)*sin(th2) + L2*cos(th2)*sin(th1)]
                                                                                                                                                                  0]
                                                                                                                                                                  1]
```

---

3. Consider the two-link cartesian manipulator of Figure 3.13. Derive

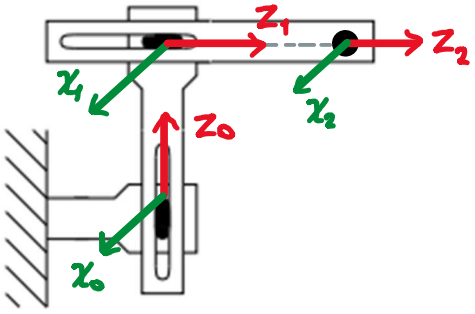

Figure 3.13: Two-link cartesian robot of Problem 3-3.

the forward kinematic equations using the DH-convention.

Link	a_i	α_i	d_i	θ_i
1	0	-90	d_1	0
2	0	0	d_2	0

H = A1 * A2

```
A =
[cos(th), -cos(alph)*sin(th),  sin(alph)*sin(th), a*cos(th)]
[sin(th),  cos(alph)*cos(th), -sin(alph)*cos(th), a*sin(th)]
[      0,           sin(alph),          cos(alph),        d]
[      0,                   0,                  0,        1]
```

```
A1 =
[1,  0, 0,  0]
[0,  0, 1,  0]
[0, -1, 0, d1]
[0,  0, 0,  1]
```

```
A2 =
[1, 0, 0,  0]
[0, 1, 0,  0]
[0, 0, 1, d2]
[0, 0, 0,  1]
```

```
H =
[1,  0, 0,  0]
[0,  0, 1, d2]
[0, -1, 0, d1]
[0,  0, 0,  1]
```

4. Consider the two-link manipulator of Figure 3.14 which has joint 1 revolute and joint 2 prismatic. Derive the forward kinematic equations using the DH-convention.

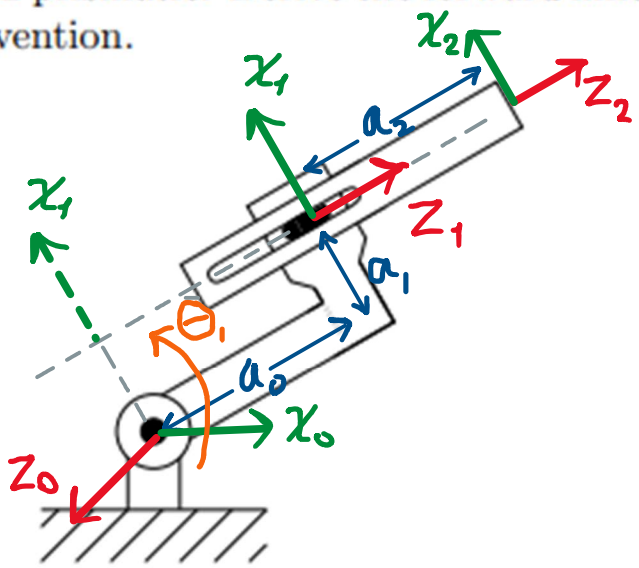

Figure 3.14: Two-link planar arm of Problem 3-4.

Link	a_i	α_i	d_i	θ_i
1	a_0	90	0	θ_1
2	0	0	d_2	0

Command Window

```
A =
[cos(th), -cos(alph)*sin(th),  sin(alph)*sin(th), a*cos(th)]
[sin(th),  cos(alph)*cos(th), -sin(alph)*cos(th), a*sin(th)]
[      0,           sin(alph),           cos(alph),        d]
[      0,                   0,                   0,        1]

============================================================
A1 =
[cos(th1), 0,  sin(th1), a0*cos(th1)]
[sin(th1), 0, -cos(th1), a0*sin(th1)]
[       0, 1,         0,          0]
[       0, 0,         0,          1]

============================================================
A2 =
[1, 0, 0,  0]
[0, 1, 0,  0]
[0, 0, 1, d2]
[0, 0, 0,  1]

============================================================
H =
[cos(th1), 0,  sin(th1), a0*cos(th1) + d2*sin(th1)]
[sin(th1), 0, -cos(th1), a0*sin(th1) - d2*cos(th1)]
[       0, 1,         0,                        0]
[       0, 0,         0,                        1]

fx >>
```

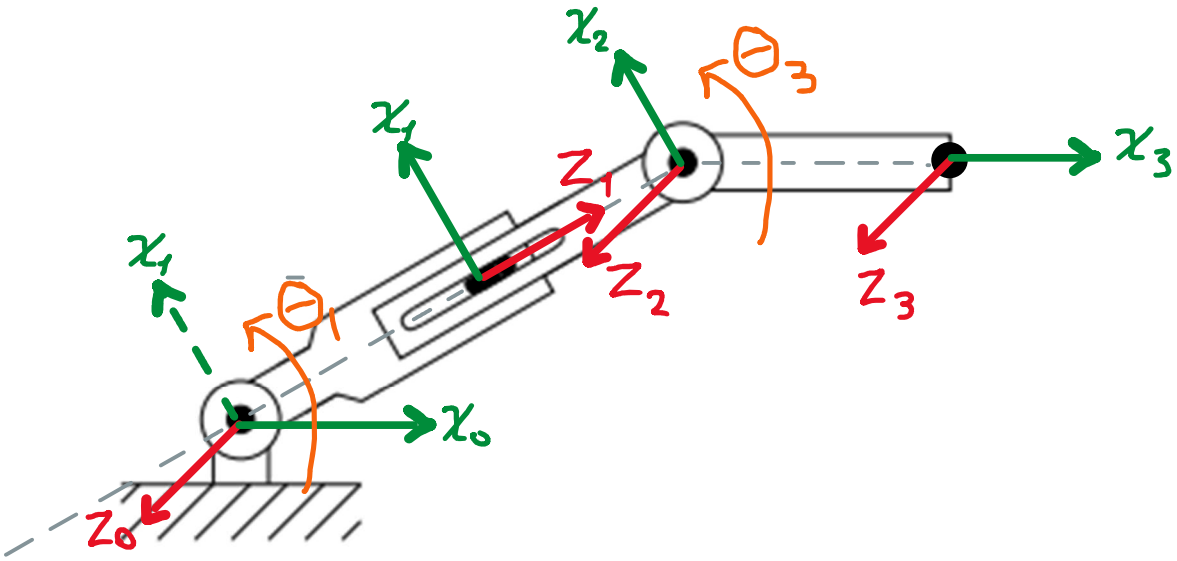

Figure 3.15: Three-link planar arm with prismatic joint of Problem 3-5.

Link	a_i	α_i	d_i	θ_i
1	0	90	0	θ_1
2	0	-90	d_2	0
3	L_3	0	0	θ_3

H = A1 * A2 * A3

```
A1 =
[cos(th1),  0,  sin(th1), 0]
[sin(th1),  0, -cos(th1), 0]
[       0,  1,         0, 0]
[       0,  0,         0, 1]

A2 =
[1,  0,  0,  0]
[0,  0,  1,  0]
[0, -1,  0, d2]
[0,  0,  0,  1]

A3 =
[cos(th3), -sin(th3), 0, L3*cos(th3)]
[sin(th3),  cos(th3), 0, L3*sin(th3)]
[       0,         0, 1,          0]
[       0,         0, 0,          1]
```

```
H =
[cos(th1)*cos(th3) - sin(th1)*sin(th3), - cos(th1)*sin(th3) - cos(th3)*sin(th1), 0, d2*sin(th1) + L3*cos(th1)*cos(th3) - L3*sin(th1)*sin(th3)]
[cos(th1)*sin(th3) + cos(th3)*sin(th1),   cos(th1)*cos(th3) - sin(th1)*sin(th3), 0, L3*cos(th1)*sin(th3) - d2*cos(th1) + L3*cos(th3)*sin(th1)]
[                                    0,                                       0, 1,                                                          0]
[                                    0,                                       0, 0,                                                          1]
```

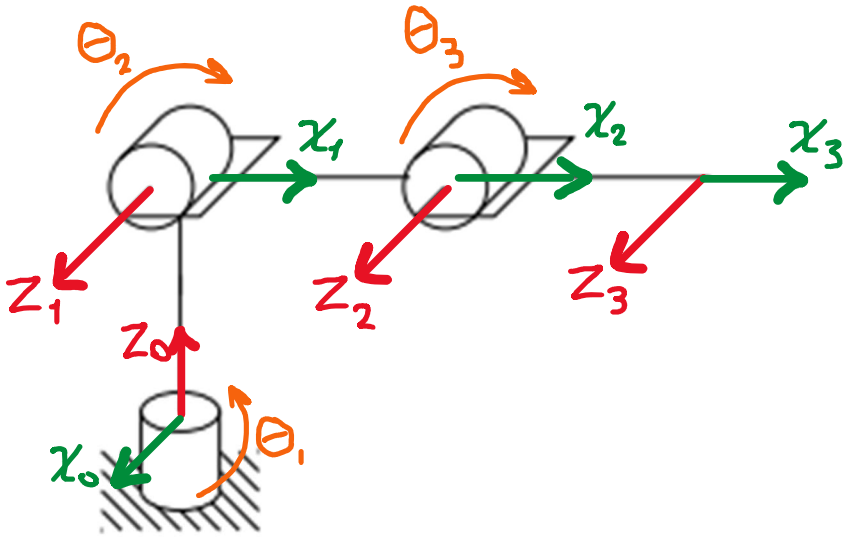

Figure 3.16: Three-link articulated robot.

Link	a_i	α_i	d_i	θ_i
1	0	90	L_1	θ_1
2	L_2	0	0	θ_2
3	L_3	0	0	θ_3

H = A1 * A2 * A3

```
A1 =
[cos(th1),  0,  sin(th1),  0]
[sin(th1),  0, -cos(th1),  0]
[       0,  1,         0, L1]
[       0,  0,         0,  1]
```

```
A2 =
[cos(th2), -sin(th2), 0, L2*cos(th2)]
[sin(th2),  cos(th2), 0, L2*sin(th2)]
[       0,         0, 1,          0]
[       0,         0, 0,          1]
```

```
A3 =
[cos(th3), -sin(th3), 0, L3*cos(th3)]
[sin(th3),  cos(th3), 0, L3*sin(th3)]
[       0,         0, 1,          0]
[       0,         0, 0,          1]
```

```
H =
[cos(th1)*cos(th2)*cos(th3) - cos(th1)*sin(th2)*sin(th3), - cos(th1)*cos(th2)*sin(th3) - cos(th1)*cos(th3)*sin(th2),
[cos(th2)*cos(th3)*sin(th1) - sin(th1)*sin(th2)*sin(th3), - cos(th2)*sin(th1)*sin(th3) - cos(th3)*sin(th1)*sin(th2),
[                 cos(th2)*sin(th3) + cos(th3)*sin(th2),                   cos(th2)*cos(th3) - sin(th2)*sin(th3),
[                                                     0,                                                       0,

 sin(th1), L2*cos(th1)*cos(th2) + L3*cos(th1)*cos(th2)*cos(th3) - L3*cos(th1)*sin(th2)*sin(th3)]
-cos(th1), L2*cos(th2)*sin(th1) + L3*cos(th2)*cos(th3)*sin(th1) - L3*sin(th1)*sin(th2)*sin(th3)]
        0,                        L1 + L2*sin(th2) + L3*cos(th2)*sin(th3) + L3*cos(th3)*sin(th2)]
        0,                                                                                        1]
```

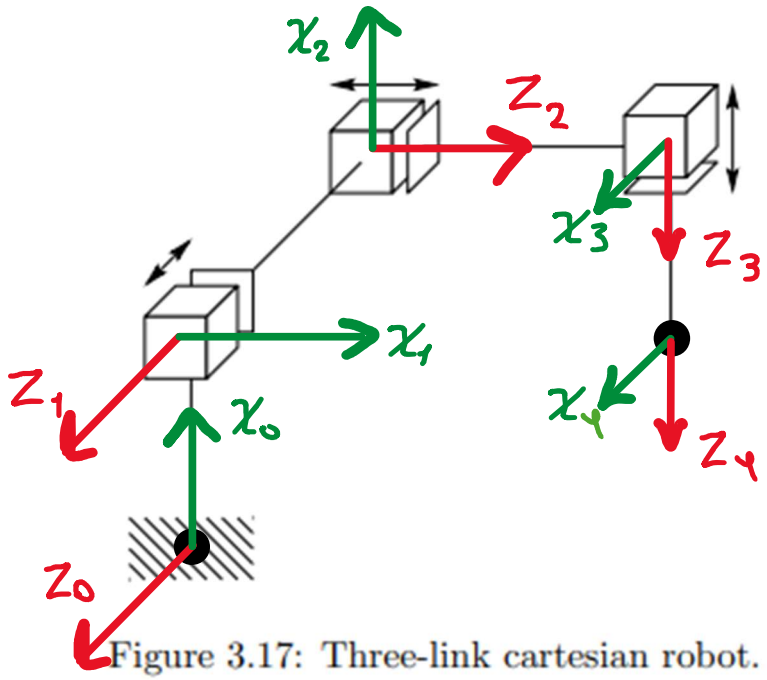

Figure 3.17: Three-link cartesian robot.

Link	a_i	α_i	d_i	θ_i
1	L_1	0	0	0
2	0	90	L_2	0
3	0	-90	L_3	90
4	0	0	L_4	0

H = A1 * A2 * A3 * A4

```
A1 =
[1, 0, 0, L1]
[0, 1, 0,  0]
[0, 0, 1,  0]
[0, 0, 0,  1]

A2 =
[1, 0,  0,  0]
[0, 0, -1,  0]
[0, 1,  0, L2]
[0, 0,  0,  1]

A3 =
[0,  0, -1,  0]
[1,  0,  0,  0]
[0, -1,  0, L3]
[0,  0,  0,  1]

A4 =
[0,  0, -1,  0]
[1,  0,  0,  0]
[0, -1,  0, L3]
[0,  0,  0,  1]

H =
[0, 0, -1, L1 - L4]
[0, 1,  0,     -L3]
[1, 0,  0,      L2]
[0, 0,  0,       1]
```

End of Assignment