$\mathcal{S}$HEBA

Experiential Learning For Non Compensatory Choice

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Research Questions

1. How is learning under non-compensatory decision rules different from learning under compensatory rules?
2. What are the different sources of uncertainty that consumers learn about over time?
3. When and how does learning occur?
4. Can we make inference about the learning process even though we do not observe learning signals?

Airline Choice

Alternative	Brand 1	Brand 2	On Time
American Airlines	1	0	1
United Airlines	0	1	0

- Two types of choice rules:
  - Compensatory: Logit, Probit, Nested Logit

$$u_j = \sum_{k=1}^{m} \beta_k x_{jk} + \epsilon_j$$

  - Non-compensatory: EBA, Lexicographic rule, Disjunctive decision rule, Conjunctive decision rule

Airline Choice

Alternative	Brand 1	Brand 2	On Time
American Airlines	1	0	?
United Airlines	0	1	?

- Two types of uncertainties:
  - The attribute importance
  - The attribute value

Literature

- Consumers face repeated uncertain choice situations
- Consumers learn from their consumption experiences about
  - their own preferences for the attributes
  - the products' attributes
- Literature on Learning under compensatory choice rules
  - Erdem and Keane (1996), Jeuland (1978), Guadani and Little (1983), Papatla and Krishnamurthi (1992), Robert and Urban(1988), Meyer and Sathi (1985), Eckstein et al (1988)

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- No learning models for non compensatory choice rules

Objectives

Learning Under Compensatory Rules

Elimination By Aspect (EBA)

Simulated EBA (SEBA)

Empirical Application (SHEBA)

Conclusion

Learning Under Compensatory Rules

Learning Under Compensatory Rules

$$u_j = \sum_{k=1}^{m} \beta_k x_{jk} + \epsilon_j$$

- Two types of learning:
  - The attribute importance: $\beta_k(t) \longrightarrow \tilde{\beta}_k$
  - The attribute value: maximizing expected utility

$$\mathbb{E}(u_j|t) = \sum_{k=1}^{m} \beta_k \mathbb{E}(x_{jk}|t)$$

- Cannot distinguish both learnings simultaneously because of the multiplicative form $\beta_k x_{jk}$

Elimination By Aspect (EBA)

What is EBA?

- In this theory, each alternative is viewed as a set of aspects. At each stage in the process, an aspect is selected (with probability proportional to its weight), and all the alternatives that do not include the selected aspect are eliminated. The process continues until all alternatives but one are eliminated (Tversky 1972)
- It is a generalization of Logit, Nested Logit and Lexicographic decision rules (Kohli and Jedidi 2015)

What is EBA?

Alternatives	Brand 1	Brand 2	Brand 3	On Time	Probability
American	1	0	0	0	$\frac{W_1}{W_1+W_2+W_3+W_4}$
United	0	1	0	1	$\frac{W_2}{W_1+W_2+W_3+W_4} + \frac{W_4}{W_1+W_2+W_3+W_4} \cdot \frac{W_2}{W_2+W_3}$
Spirit	0	0	1	1	$\frac{W_3}{W_1+W_2+W_3+W_4} + \frac{W_4}{W_1+W_2+W_3+W_4} \cdot \frac{W_3}{W_2+W_3}$

where

- W_1, W_2 and W_3 are the importance weights of the brands
- W_4 is the importance weight of the aspect On Time

Challenge

- Learning about the W_k is similar to that in compensatory models
- Under EBA we assume that all the aspects are **known with certainty**
- For uncertain aspects, one cannot use the expected utility of an alternative

Uncertain Aspect

Alternative	Brand 1 (W_1)	Brand 2 (W_2)	On Time (W_3)
American Airlines	1	0	?
United Airlines	0	1	?

Uncertain Aspect

Alternative	Brand 1 (W_1)	Brand 2 (W_2)	On Time (W_3)
American Airlines	1	0	?
United Airlines	0	1	?

Scenario 1 (1,1)

Alternative	Brand 1	Brand 2	On Time
American	1	0	1
United	0	1	1

Scenario 2 (1,0)

Alternative	Brand 1	Brand 2	On Time
American	1	0	1
United	0	1	0

Scenario 3 (0,1)

Alternative	Brand 1	Brand 2	On Time
American	1	0	0
United	0	1	1

Scenario 4 (0,0)

Alternative	Brand 1	Brand 2	On Time
American	1	0	0
United	0	1	0

Probabilities of Choice Under Each Scenario

	(1,1)	(1,0)	(0,1)	(0,0)
Alternative	Scenario 1	Scenario 2	Scenario 3	Scenario 4
$Pr(\text{American})$	$\frac{W_1}{W_1+W_2}$	$\frac{W_1+W_3}{W_1+W_2+W_3}$	$\frac{W_1}{W_1+W_2+W_3}$	$\frac{W_1}{W_1+W_2}$
$Pr(\text{United})$	$\frac{W_2}{W_1+W_2}$	$\frac{W_2}{W_1+W_2+W_3}$	$\frac{W_2+W_3}{W_1+W_2+W_3}$	$\frac{W_2}{W_1+W_2}$

Simulated EBA (SEBA)

Model

- The attributes importance W_k are known to the consumer
- At time t, each alternative j has one uncertain aspect $X_{jt} \in \{0, 1\}$ that is only observed after consumption
- X_{jt} is generated randomly on each consumption occasion

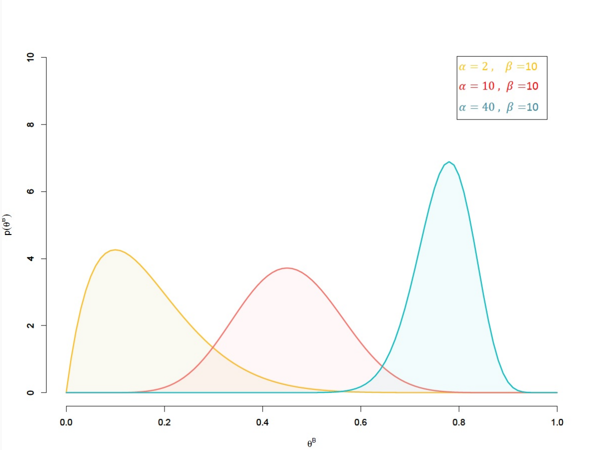
$$X_{jt} \sim Bernouilli(\theta_j)$$

- θ_j represents the true probability of the aspect taking the value 1 for brand j

Prior Belief about θ_j

- The individual has a subjective belief about the location of the probability θ_j

$$\pi_t(\theta_j^B) = Beta(\alpha_{jt}, \beta_{jt})$$



Predictive Probability of X_j

For each alternative j

- Given the θ_j^B's the individual has predictive beliefs about the values X_{jt}^B of the uncertain aspect

$$Pr(X_{jt}^B = 1) = \frac{\alpha_{jt}}{\alpha_{jt} + \beta_{jt}} = \mathbb{E}(\theta_j^B)$$

- Simply

$$X_{jt}^B \sim BetaBernoulli(\alpha_{jt}, \beta_{jt})$$

Simulating a Scenario

- The consumer has a belief about some scenario S_t
- The uncertain aspect can be irrelevant under that scenario
- The consumer makes a choice using EBA

SEBA: Learning Process

Prior Belief
$\pi_t(\theta^B)$

$\longrightarrow$ $\Pr(X^B|\theta^B)$ $\longrightarrow$

Scenario Simulation
$\Pr(S_t = s)$

$\downarrow$

EBA Choice
$\Pr(j|S_t = s)$

$\downarrow$

Bayesian Updating
$\pi_{t+1}(\theta^B|X_t) \propto \pi_t(\theta^B)\,\Pr(X_t|\theta^B)$

$\uparrow$

Experience
$X_t \sim Berouilli(\theta)$

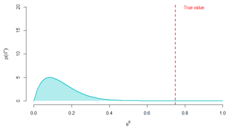
$\longleftarrow$

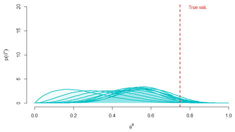
Posterior Belief
$\pi_{t+1}(\theta^B|X_t)$

$\uparrow$

Next time step
$t \leftarrow t+1$

$\uparrow$





Learning

- Thanks to conjugacy

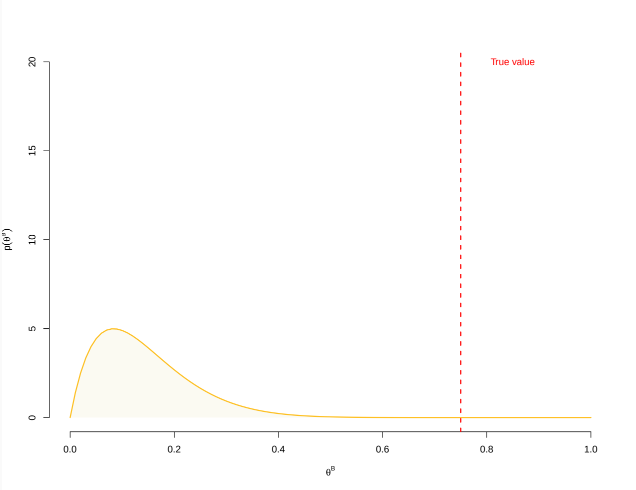
$$X^B_{jt+1} \sim BetaBernouilli(\alpha_{jt+1}, \beta_{jt+1})$$

- Simply

$$Pr(X^B_{jt} = 1) = \frac{\alpha_{0j} + \sum_{\ell=1}^{t} X_{jt}}{\alpha_{j0} + \beta_{0j} + t}$$

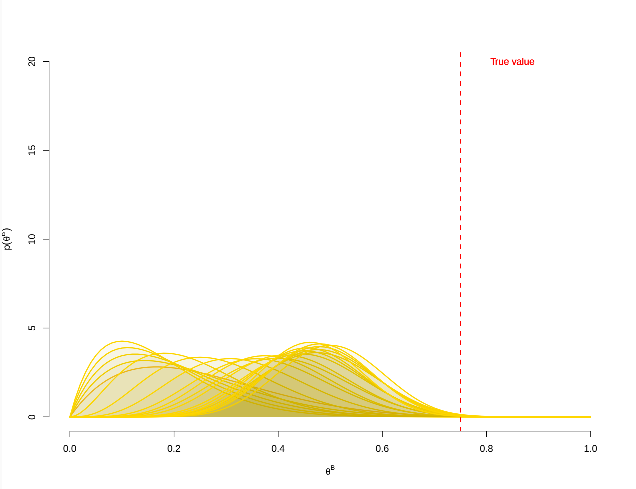
Visualizing Learning

$$t = 0 \text{ and } \pi_0(\theta_j^B) = Beta(\alpha_{j0}, \beta_{j0})$$



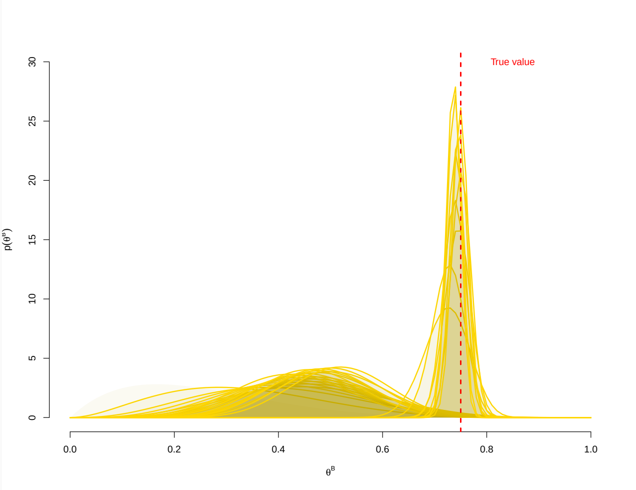
Visualizing Learning

$$t = 20 \text{ and } \pi_t(\theta_j^B) = Beta(\alpha_{jt}, \beta_{jt})$$



Visualizing Learning

$$t = 100 \text{ and } \pi_t(\theta_j^B) = Beta(\alpha_{jt}, \beta_{jt})$$



Unobserved Signals $\sum_{\ell=1}^{t} X_{jt}$

$$Pr(X_{jt}^B = 1) = \frac{\alpha_{0j} + \sum_{\ell=1}^{t} X_{jt}}{\alpha_{j0} + \beta_{0j} + t}$$

Unobserved Signals $\sum_{\ell=1}^{t} X_{jt}$

$$Pr(X_{jt}^{B} = 1) = \frac{\alpha_{0j} + \sum_{\ell=1}^{t} X_{jt}}{\alpha_{j0} + \beta_{0j} + t}$$

- $\sum_{\ell=1}^{t} X_{jt} \sim Binomial(t, \theta_j)$

$$Pr(X_{jt}^{B} = 1) = \frac{\alpha_{0j} + t\theta_j}{\alpha_{j0} + \beta_{0j} + t}$$

Choice Probability

$$Pr_t(j) = \sum_{s=1}^{4} \underbrace{Pr(S_t = s)}_{\alpha_{j0},\beta_{j0},\theta_j} \underbrace{Pr(j|S_t = s)}_{W_k}$$

where:

- $Pr(S_t = s)$ is the probability of simulating scenario s
- $Pr(j|S_t = s)$ is the EBA probability of choosing j under scenario s

Empirical Application (SHEBA)

SHEBA : Data

- Discrete choice panel data of canned tuna of 536 households:
  - Brand 1: Chicken of the sea oil
  - Brand 2: Chicken of the sea water
- Four observed aspects: brand name, price, feature, display
- One unobserved uncertain aspect
- We estimate a SHEBA (Simulated Hierarchical EBA) with heterogeneous experiences for each individual i

$$W_{ik}, \alpha_{ij0}, \beta_{ij0}, \theta_{ij}$$

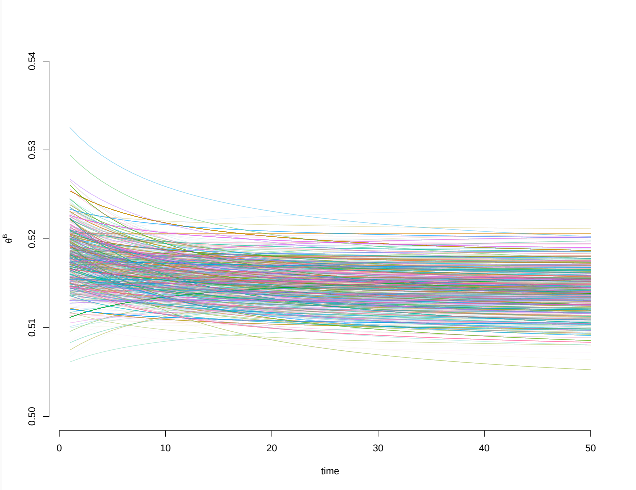
Results

Parameter	SHEBA Posterior mean	Mixed Logit Posterior mean	Mixed EBA Posterior Mean
Chicken of the sea – Oil	0	0	0
Chicken of the sea – Water	1.15	2.36	1.11
Cheaper	2.78	1.90	2.71
Feature	2.10	1.07	1.87
Display	0.57	1.38	-0.04
The uncertain aspect	-3.27	–	–
θ_1	0.51	–	–
θ_2	0.75	–	–
α_{01}	5.53	–	–
α_{02}	5.98	–	–
β_{01}	5.14	–	–
β_{02}	8.19	–	–
In sample Hit	84%	83%	83%
WAIC	–	–	–
LOO	–	–	–

- The uncertain aspect was relevant 49% of the times
- The uncertain aspect had a probability of 0.13% to be selected first

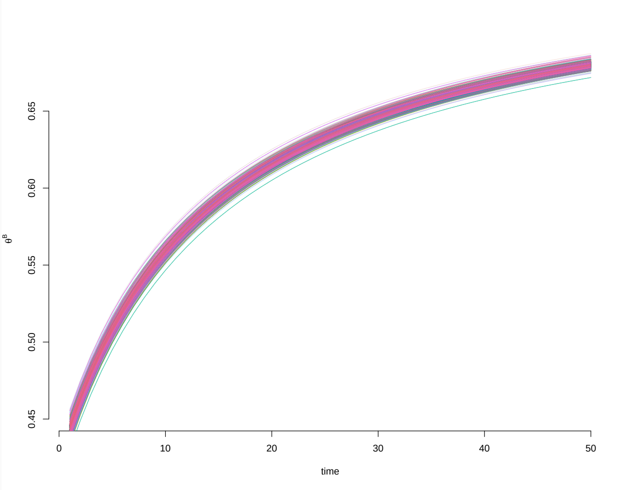
Results

Figure 1: Learning about θ_1



Results

Figure 2: Learning about θ_2



Conclusion

Conclusion

- Contribution:
  - A model of learning under non compensatory choice rule
  - Inference can be made with unobserved signals
  - Both types of learning can be allowed
- Limitations:
  - Complexity of EBA probabilities
  - Curse of dimensionality 2^n scenarios
- Future research:
  - Learning on multiple attributes
  - Estimate the model on other data sets
  - Capture the two types of learning simultaneously

Probabilistically Simulated Scenarios S_t

Scenario 1 (1,1)

Alternative	Brand 1	Brand 2	On Time	$Pr(j\|S_t = 1)$	$Pr(S_t = 1)$
A	1	0	1	$\frac{W_1}{W_1+W_2}$	$\mathbb{E}(\theta_1^B)\mathbb{E}(\theta_2^B)$
B	0	1	1	$\frac{W_2}{W_1+W_2}$	

Scenario 2 (1,0)

Alternative	Brand 1	Brand 2	On Time	$Pr(j\|S_t = 2)$	$Pr(S_t = 2)$
A	1	0	1	$\frac{W_1+W_3}{W_1+W_2+W_3}$	$\mathbb{E}(\theta_1^B)(1-\mathbb{E}(\theta_2^B))$
B	0	1	0	$\frac{W_2}{W_1+W_2+W_3}$	

Scenario 3 (0,1)

Alternative	Brand 1	Brand 2	On Time	$Pr(j\|S_t = 3)$	$Pr(S_t = 3)$
A	1	0	0	$\frac{W_1}{W_1+W_2+W_3}$	$(1-\mathbb{E}(\theta_1^B))\mathbb{E}(\theta_2^B)$
B	0	1	1	$\frac{W_2+W_3}{W_1+W_2}$	

Scenario 4 (0,0)

Alternative	Brand 1	Brand 2	On Time	$Pr(3\|S_t = 4)$	$Pr(S_t = 4)$
A	1	0	0	$\frac{W_1}{W_1+W_2}$	$(1-\mathbb{E}(\theta_1^B))(1-\mathbb{E}(\theta_2^B))$
B	0	1	0	$\frac{W_2}{W_1+W_2}$	