A generalized-means choice model for regularity violations

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THE REGULARITY ASSUMPTION

The probability of choosing an alternative does not increase when other alternatives are added to a choice set

Violation of regularity

- Choice complexity
  - Redelmeier and Shafir (1995), Iyengar and Lepper (2000)

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- Menu dependence, herd behavior, chooser dependence, responsibility
  - Kant (1788), Adam Smith (1790), Banerjee (1992), Sen (1993), Redelmeier and Shafir (1995), Sen (1997)

IS THE GLASS HALF FULL OR HALF EMPTY?

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- Prospect theory : V defined
  - (i) on deviations from a reference point x_j^*
  - (ii) concave for gains and convex for losses
  - (iii) steeper for losses than gains

where:

- $x_j, j = 1, \ldots, m$ are product attributes
- $x_j^*, j = 1, \ldots, m$ are attributes references
- β_j attribute weights

Objectives

- Propose a generalized mean value function
- Show how to use it for modeling choice
- Show how it captures violation of regularity
- Illustrate the model on empirical choice data

The value function

- Let the value of attribute j be

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THE VALUE FUNCTION

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- Then the generalized mean value of an alternative is:

$$V_r = \left(\frac{1}{m} \sum_{j=1}^{m} (v_j)^r \right)^{\frac{1}{r}}$$

where r is a parameter that governs the averaging process

SPECIAL CASES

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$r = 1 :$ $V_1 = \frac{1}{m} \sum_{j=1}^{m} v_j$ Arithmetic mean

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r = 0: &\quad V_0 = \textstyle\prod_{j=1}^{m} v_j^{\frac{1}{m}} &&\text{Geometric mean} \\
r \to -\infty: &\quad V_{-\infty} = \min(v_j) &&\text{Minimum}
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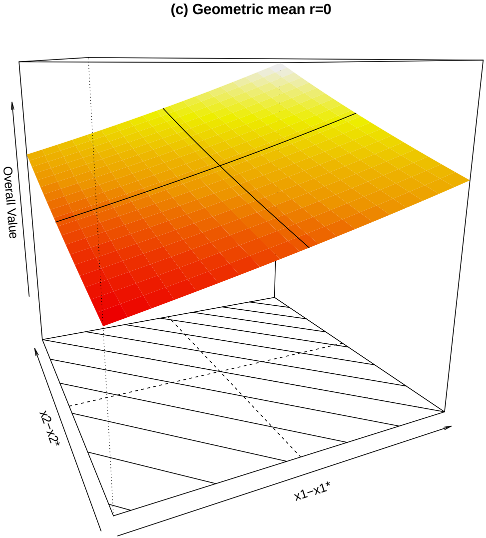
r \to +\infty: &\quad V_{+\infty} = \max(v_j) && \text{Maximum}
\end{aligned}$$

VALUE SURFACE



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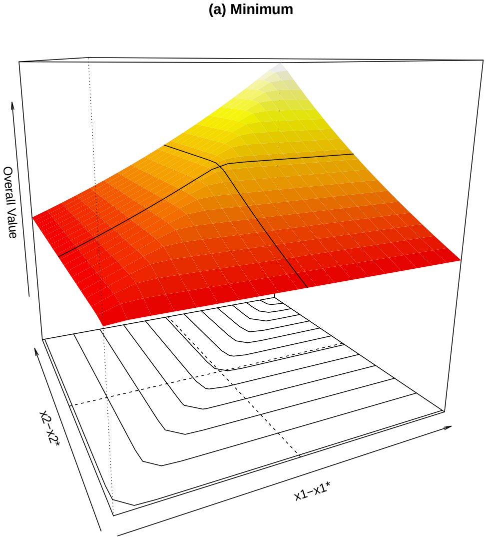

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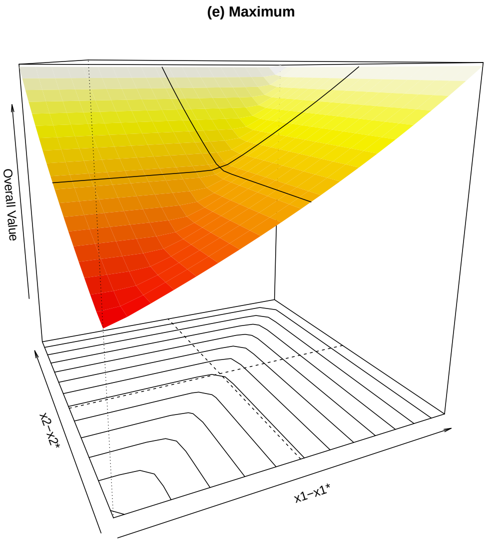

THE CHOICE MODEL

$$U_r(i) = V_r(i)e^{\epsilon}$$

- $i \in C = \{1 \ldots n\}$ denotes alternatives
- V_r is the systematic value
- $v_{ij} = e^{\beta_j(x_{ij} - x_j^*)}$
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$$\mathbb{P}(\mathrm{i} \in C|r) = \frac{V_r(i)}{V_r(1) + \cdots + V_r(n)}$$

MNL IS A SPECIAL CASE WHEN $r \to 0$ (GEOMETRIC MEAN)

$$V_r(i) = \left(\frac{1}{m} \sum_{j=1}^{m} e^{r\beta_j (x_{ij} - x_j^*)} \right)^{\frac{1}{r}} \xrightarrow{r \to 0} \prod_{j=1}^{m} e^{\frac{\beta_j}{m}(x_{ij} - x_j^*)} = e^{\sum_{j=1}^{m} \frac{\beta_j}{m}(x_{ij} - x_j^*)}$$

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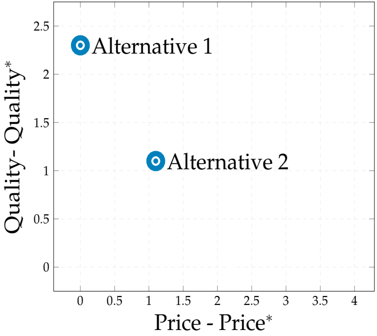
The Logit model is equivalent to a geometric averaging process:

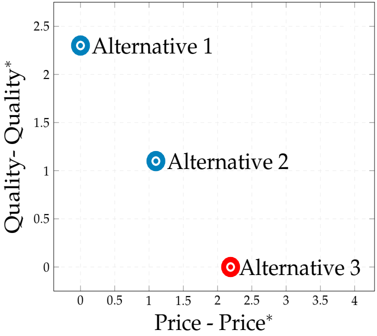
$$\mathbb{P}(\mathrm{i} \in \mathrm{C} | r = 0) = \frac{e^{\sum_{j=1}^{m} \tilde{\beta}_j x_{ij}}}{e^{\sum_{j=1}^{m} \tilde{\beta}_j x_{1j}} + \dots + e^{\sum_{j=1}^{m} \tilde{\beta}_j x_{nj}}}$$

where $\tilde{\beta}_j \leftarrow \frac{\beta_j}{m}$

ILLUSTRATING VIOLATION OF REGULARITY

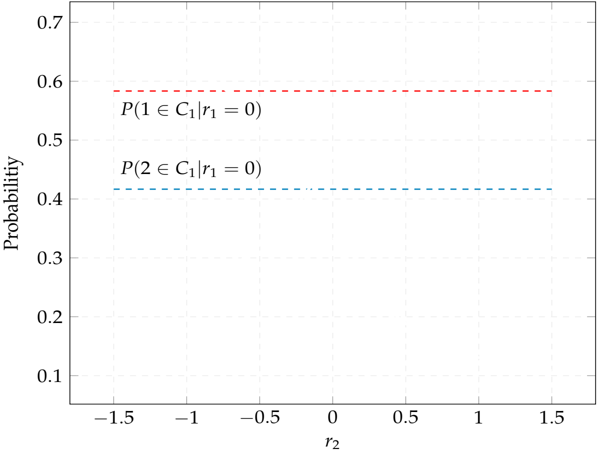


$r_1 = 0$



r_2

CHOICE PROBABILITIES



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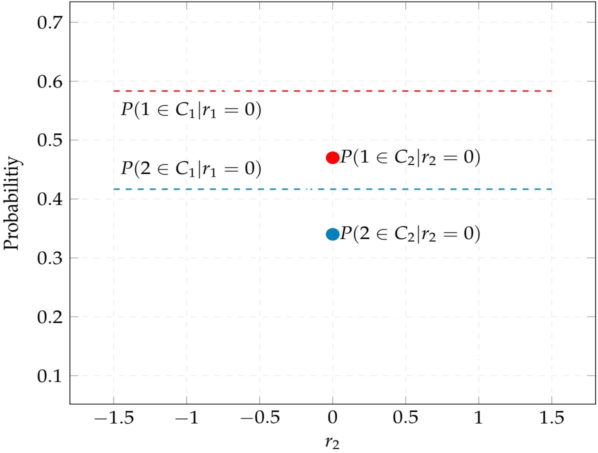
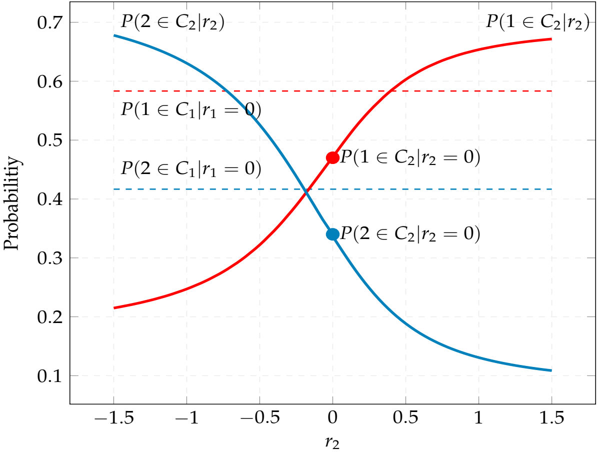


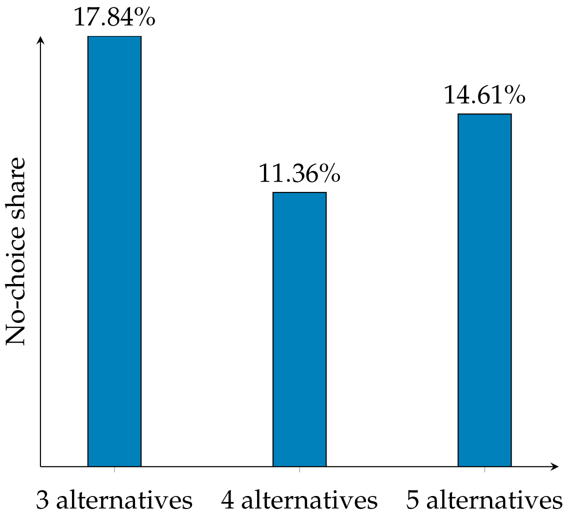
Choice probabilities



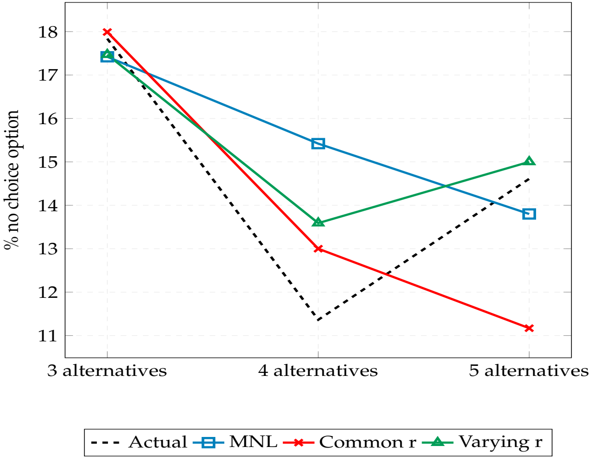
Empirical illustration

- Data on choice of digital cameras (Rooderkerk et. al. (2011)) of 154 subjects
- Independent variables include picture quality (from 2MP to 6MP) and optical zoom (from $\times 2$ to $\times 10$)
- Choice sets varying in size from three to five alternatives plus a no-choice option

Violation of regularity



Actual and predicted shares of the no-choice option



Model performance statistics

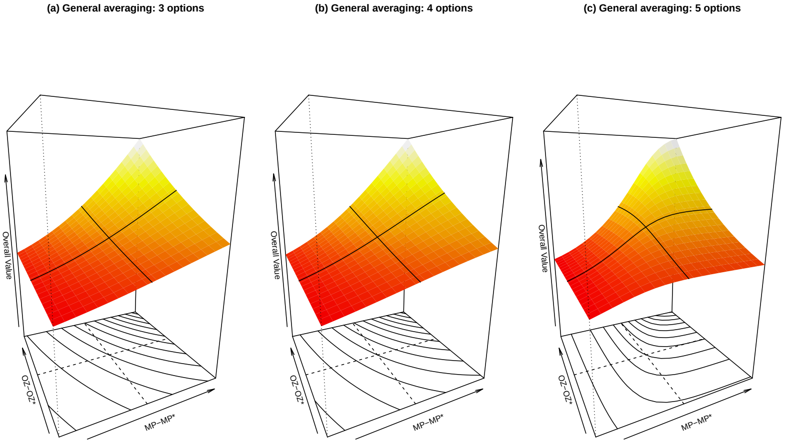
Model	WAIC	LOO
Multinomial Logit	6553	-3279
Common r	3904	-1963
Varying r	**3882**	**-1956**

HYPER PARAMETERS ESTIMATES

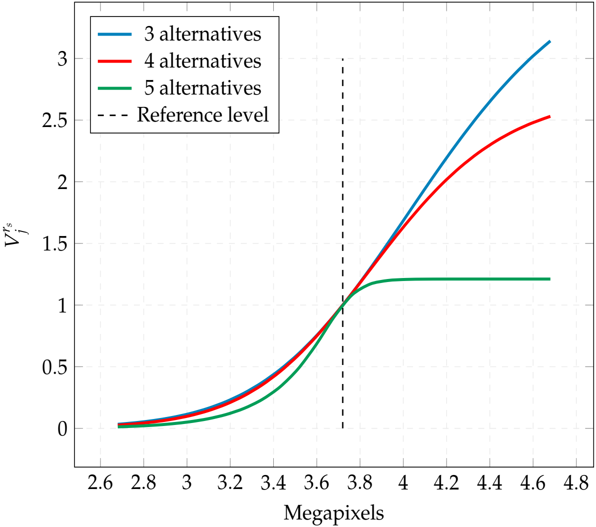
Parameter	Label	Multinomial logit	Constant averaging	General averaging
No choice	β_0	**10.71** (9.91,11.52)		
Reference points				
Picture quality	PQ^*		**3.77** (3.61,3.90)	**3.73**(3.58,3.89)
Optical zoom	OZ^*		**3.45** (3.24,3.84)	**3.45**(3.14,3.76)
Sensitivities				
Megapixels	β_1	**1.82** (1.70,4.56)	**4.56**(4.02,5.16)	**4.42**(4.00,4.92)
Optical zoom	β_2	**0.67** (0.62,0.73)	**2.51**(2.18,2.89)	**2.43**(2.15,2.72)
Averaging rules				
Invariant	r		**-0.43**(-0.59,-0.31)	
3 alternatives	r_3			**-0.51**(-0.71,-0.39)
4 alterantives	r_4			**-0.69**(-1.48,-0.37)
5 alternatives	r_5			**-3.62**(-6.66,-0.88)

Notes. Significant coefficients are highlighted in boldface ($p \leq 0.05$).
The 95% posterior intervals for the parameters are shown in parentheses.

FITTED VALUE SURFACES

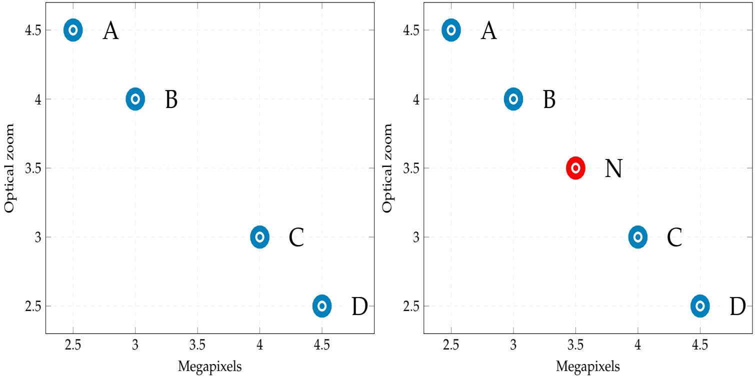


Fitted value function



Optical zoom is fixed at its reference point

Implications



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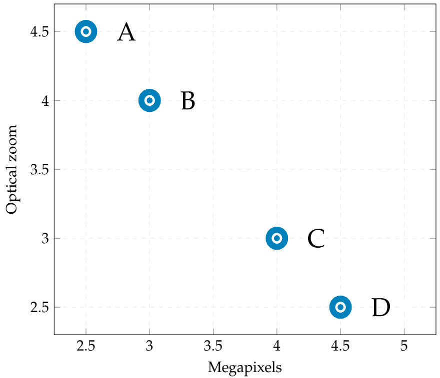

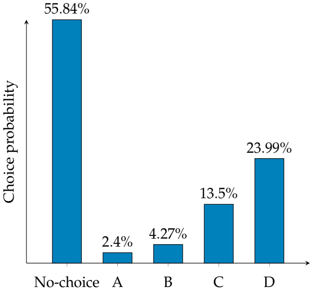

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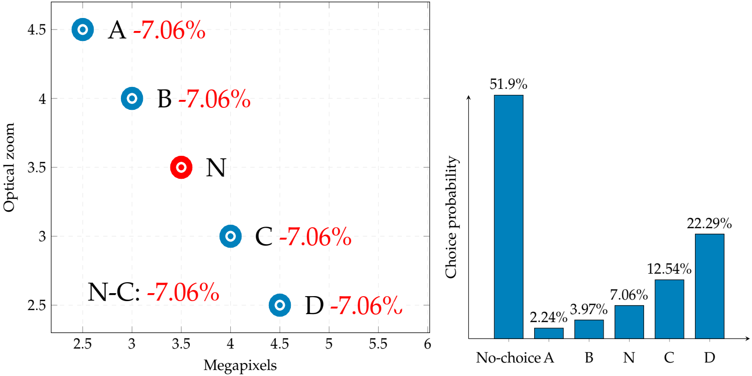
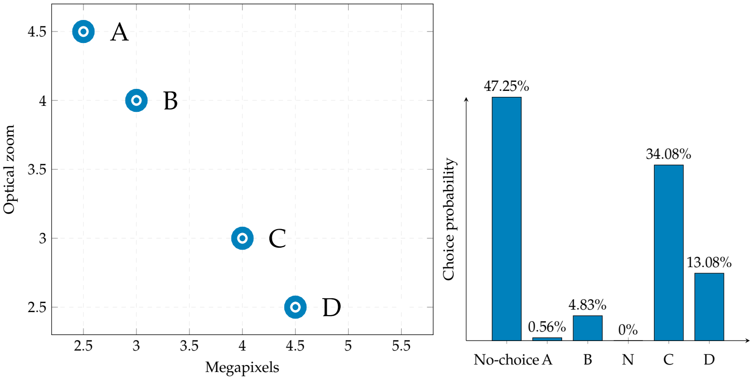


Implications: Generalized-means model



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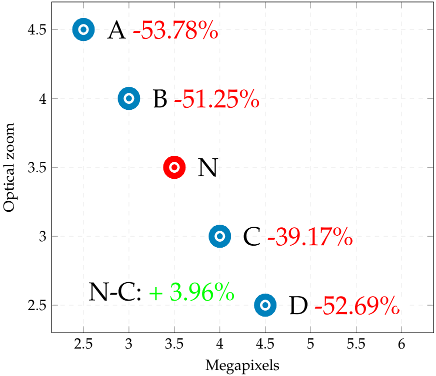

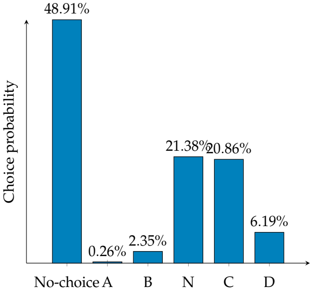

Conclusion

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- Illustrated the model on empirical choice data

Is the glass half full or half empty?

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- Optimists: Half-full
- Pessimists: Half-Empty
- $V_r(\text{Glass}) = \left(\frac{v^r_{\text{Full}} + v^r_{\text{Empty}}}{2} \right)^{\frac{1}{r}}$

Expected in-sample and out-of-sample hit rates by choice-set size

	3 options		4 options		5 options	
Model	In	Out	In	Out	In	Out
Multinomial Logit	0.46	0.45	0.39	0.38	0.38	0.38
Common r	**0.71**	**0.67**	0.67	0.62	0.65	0.62
Varying r	**0.71**	**0.67**	**0.68**	**0.63**	**0.69**	**0.63**