



LESSON 2: Classification

Cost function, Supervised classification, Performance metrics

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$$\mathbf{X} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \dots & x_d^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \dots & x_d^{(2)} \\ \vdots & \vdots & & \vdots \\ x_1^{(n)} & x_2^{(n)} & \dots & x_d^{(n)} \end{bmatrix} = \begin{bmatrix} (\mathbf{x}^{(1)})^T \\ (\mathbf{x}^{(2)})^T \\ \vdots \\ (\mathbf{x}^{(n)})^T \end{bmatrix}$$



Agenda

Cost Function, Supervised Classification, Performance Metrics

1. Admin (afleversformat, grupper, etc.)
2. Forelæsning
 - ▶ Resumé
 - ▶ Lineær algebra og cost funktionen, J
 - ▶ Opgave: [L02/cost_function.ipynb](#)
 - ▶ Supervised binær klassifikation
 - ▶ Opgave: [L02/dummy_classifier.ipynb](#)
 - ▶ Scikit-learn fit-predict interface,
 - ▶ Scores/Performance metrics
 - ▶ Opgave: [L02/performance_metrics.ipynb](#)
3. Opgaveregning på klassen..

RESUMÉ: The toolset for ML

A list of our toolbox

- ▶ **Python:** our preferred language for ML,
- ▶ **Anaconda:** a particular distribution of python, that we will use,
- ▶ **Jupyter** notebooks: interactive coding and visualization for python (or Spider/PyCharm/VS Code),
- ▶ **NumPy, SciPy, Pandas, Matplotlib, Seaborn:** numerical computation and data visualization libraries for python,
- ▶ **Scikit-learn:** 'framework' for machine learning tools.

RESUMÉ: Jupyter Crash Course

Jupyter need-to-know:

- ▶ Ctrl+Enter: executes cell,
- ▶ Shift+Tab: help for function under cursor,
- ▶ Shift+Tab repeated: extended help,
- ▶ Tab: 'tab'-completion??

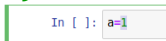
Jupyter magic commands:

- ▶ `%matplotlib inline`: pull in the matplotlib,
- ▶ `%reset -f`: reset all vars (or `-sf`),
- ▶ `%run filename.ipynb`: execute code from another notebook or python file,
- ▶ `%load filename.py`: copy contents of the file and paste into the cell,
- ▶ `! dir`: executes a shell command.

RESUMÉ: Jupyter Crash Course

Jupyter shortcuts:

- ▶ To modes: command mode (**blue**) and edit-mode (**green**),



- ▶ ESC: goto command mode (from edit mode),

Keyboard shortcuts

The Jupyter Notebook has two different keyboard input modes. **Edit mode** allows you to type code/text into a cell and is indicated by a green cell border. **Command mode** binds the keyboard to notebook level actions and is indicated by a grey cell border with a blue left margin.

Command Mode (press `Esc` to enable)

`F`: find and replace

`Ctrl-Shift-P`: open the command palette

`Enter`: enter edit mode

`Shift-Enter`: run cell, select below

`Ctrl-Enter`: run selected cells

`Alt-Enter`: run cell, insert below

`Shift-J`: extend selected cells below

`A`: insert cell above

`B`: insert cell below

`X`: cut selected cells

`C`: copy selected cells

`Shift-V`: paste cells above

RESUMÉ: Python Libraries Crash Course

A lot of modules/libraries are available for python, here we will use:

- ▶ `numpy`: numerical data representation module, for say vectors, matrices etc,
- ▶ `matplotlib`: Matplotlib is a Python 2D plotting library which produces publication quality figures.

Other libraries, typically used in ML, are:

- ▶ `pandas`: python data analysis library, a module for loading/saving and handling large data set,
- ▶ `scipy`: python library used for scientific computing and technical computing.

*but we try to stick to `numpy` in this course,
...and note that `numpy.matrix` is deprecated!*

RESUMÉ: Matplotlib Crash Course

Visualizations can be created in multiple ways:

- ▶ `matplotlib`
- ▶ `pandas`: (via `matplotlib`),
- ▶ `seaborn`: statistically-focused plotting methods.

And we will stick to `matplotlib`, don't re-invent the wheel;
find demos here

<https://matplotlib.org/gallery/index.html>



THE COST FUNCTION (LOSS)

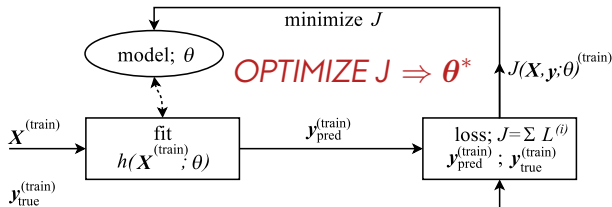
$$\mathcal{L}_2 : \quad ||\mathbf{x}||_2 = \left(\sum_{i=1}^n |x_i|^2 \right)^{1/2}$$

$$\mathcal{L}_2^2 : \quad ||\mathbf{x}||_2^2 = \mathbf{x}^\top \mathbf{x}$$

$$d(\mathbf{x}, \mathbf{y}) = ||\mathbf{x} - \mathbf{y}||_2$$

The Cost Function

Data-flow model for supervised learning



$X^{(train)}$: trænings data input,

loose notation: $X^{(train)} = X^{(i)}$ for $\forall i \in \text{train set}$

θ : model parametre,

h : hypothesis function; types of ML algos,

$y_{true}^{(train)}$: training data ground-truth,

$y_{pred}^{(train)}$: predicted (train) data output,

$L^{(i)}$: individual loss (distance),

J : loss/cost/error/objective function (summeret)

Exercise: L02/cost_function.ipynb

The Design Matrix

Say, we have d features for a given sample point. This d -sized feature column vector for a data-sample i is then given by

$$\mathbf{x}^{(i)} = \begin{bmatrix} x_1^{(i)} & x_2^{(i)} & \dots & x_d^{(i)} \end{bmatrix}^T$$

The full data matrix \mathbf{X} and target column vector \mathbf{y} are then constructed out of n samples of these feature vectors

$$\mathbf{X} = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \dots & x_d^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \dots & x_d^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{(n)} & x_2^{(n)} & \dots & x_d^{(n)} \end{bmatrix} = \begin{bmatrix} (\mathbf{x}^{(1)})^T \\ (\mathbf{x}^{(2)})^T \\ \vdots \\ (\mathbf{x}^{(n)})^T \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(n)} \end{bmatrix}$$

(and \mathbf{X} and \mathbf{y} are sometimes concatenated into a single matrix!)

Exercise: L02/cost_function.ipynb

Distance/norms

The \mathcal{L}_2 Euclidian norm for a vector of size n is defined as

$$\mathcal{L}_2 : ||\mathbf{x}||_2 = \left(\sum_{i=1}^n |x_i|^2 \right)^{1/2}$$

and thus via linear algebra and vector inner-dot product

$$\mathcal{L}_2^2 : ||\mathbf{x}||_2^2 = \mathbf{x}^\top \mathbf{x}$$

The distance between two vectors is given by

$$\begin{aligned} d(\mathbf{x}, \mathbf{y}) &= ||\mathbf{x} - \mathbf{y}||_2 \\ &= \left(\sum_{i=1}^n |x_i - y_i|^2 \right)^{1/2} \end{aligned}$$

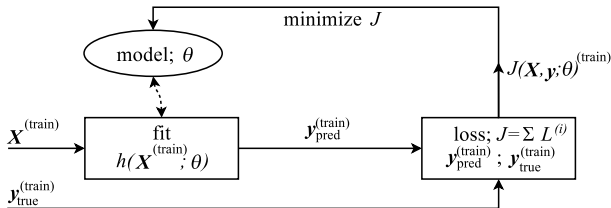
The general \mathcal{L}_p norm is given by

$$\mathcal{L}_p : ||\mathbf{x}||_p = \left(\sum_i |x_i|^p \right)^{1/p} ; \text{ norm: } \begin{cases} d(\mathbf{x}) = 0, \Rightarrow \mathbf{x} = \mathbf{0} \\ d(\mathbf{x} + \mathbf{y}) \leq d(\mathbf{x}) + d(\mathbf{y}), \\ \quad \quad \quad \text{(triangle inequality)} \\ d(\alpha \mathbf{x}) = |\alpha| d(\mathbf{x}) \end{cases}$$

• More generally, the ℓ_k norm of a vector \mathbf{v} containing n elements is defined as $||\mathbf{v}||_k = \left(|v_1|^k + |v_2|^k + \dots + |v_n|^k \right)^{1/k}$. ℓ_0 just gives the number of non-zero elements in the vector, and ℓ_∞ gives the maximum absolute value in the vector.

Exercise: L02/cost_function.ipynb

Data-flow model for supervised learning



Express J in terms of vectors and matrices using the \mathcal{L}_2^2

$$\begin{aligned} J(\mathbf{X}, \mathbf{y}_{true}; \theta) &= \frac{1}{n} \sum_{i=1}^n L^{(i)} \\ &= \frac{1}{n} \sum_{i=1}^n d(h(\mathbf{X}^{(i)}) - \mathbf{y}_{true}^{(i)})^2 \\ &= \frac{1}{n} \|\mathbf{h}(\mathbf{X}) - \mathbf{y}_{true}\|_2^2 \\ &= \frac{1}{n} \|\mathbf{y}_{pred} - \mathbf{y}_{true}\|_2^2 \end{aligned}$$

Equation 2-1. Root Mean Square Error (RMSE)

$$\text{RMSE}(\mathbf{X}, h) = \sqrt{\frac{1}{m} \sum_{i=1}^m (h(\mathbf{x}^{(i)}) - y^{(i)})^2}$$

arriving at a J proportional to the MSE or \mathcal{L}_2 metric

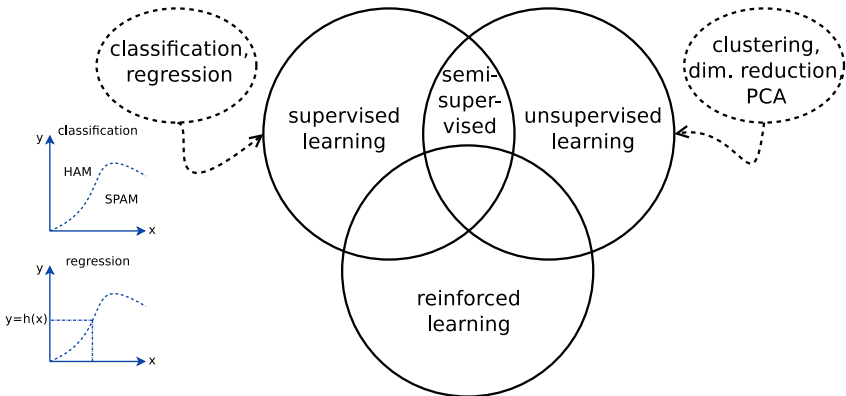
cost function: $J(\mathbf{X}, \mathbf{y}_{true}; \theta) \propto \frac{1}{2} \|\mathbf{y}_{pred} - \mathbf{y}_{true}\|_2^2 \propto \text{MSE}$

CLASSIFICATION

5 0 4 1 9 2 1 3 1
3 5 3 6 1 7 2 8 6
4 0 9 1 1 2 4 3 2
3 8 6 9 0 5 6 0 7
1 8 1 9 3 9 8 5 9
3 0 7 4 9 8 0 9 4
4 4 6 0 4 5 6 1 0

Machine learning taksonomi

Machine learning læringstyper

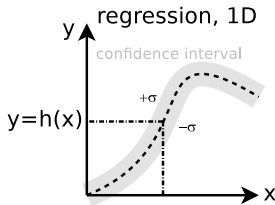
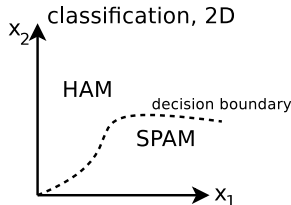


Classification vs. Regression

Given the following hypothesis function

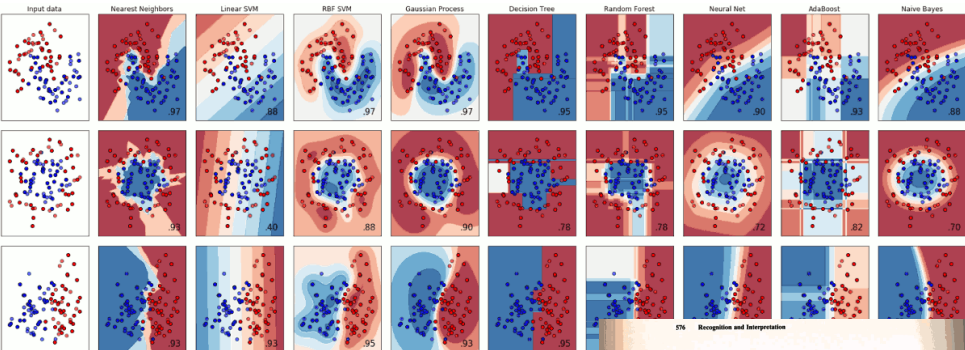
$$h(\mathbf{x}) \rightarrow y$$

- ▶ if y is *discrete/categorical* variable, then this is **classification** problem.
- ▶ if y is *real number/continuous*, then this is a **regression** problem.



Classification

Decision Boundaries for different Models and Datasets



576 Recognition and Interpretation

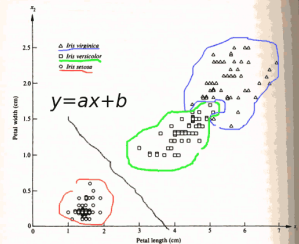


Figure 9.2 Two measurements performed on three types of iris. (Adapted from Duda et al. (2001).)

Binary Classification



Figure 3-1. A few digits from the MNIST dataset

Training a Binary Classifier

Let's simplify the problem for now and only try to identify one digit—for example, the number 5. This “5-detector” will be an example of a *binary classifier*, capable of distinguishing between just two classes, 5 and not-5. Let's create the target vectors for this classification task:

```
y_train_5 = (y_train == 5) # True for all 5s, False for all other digits.  
y_test_5 = (y_test == 5)
```

=> converting from 10-class to BINARY classifier!

Multiclass/Multinomial Classification

And Introduction to Multilabel Classification

- ▶ Many classifiers are binary (HAM/SPAM)
- ▶ What to do for say a three category, like CAT/DOG/TURTLE problem?
- ▶ Divide into CAT/NON-CAT, etc, binary classifiers and solve for each three classes!
- ▶ Aka.: one-vs-rest/one-vs-all (OvA), one-against-all (OAA).
- ▶ Or the one-vs-one (OvO) method.
- ▶ NOTE: Multilabel classification is yet again different, it can categorize item into more classes, say both CAT and DOG!
- ▶ ...and Multioutput/multilabel multiclass classification.



The Scikit-learn Fit-Predict Interface



Supervised Classification in practice

*The API has one predominant object: **the estimator**.*

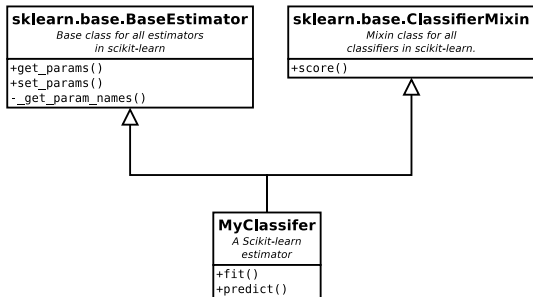


An estimator is an object that fits a model based on some training data and is capable of inferring some properties on new data. It can be, for instance, a classifier or a regressor.

All estimators implement the fit method: `estimator.fit(X,y)` All built-in estimators also have a `set_params` method, which sets data-independent parameters (overriding previous parameter values passed to `__init__`).

All estimators in the main scikit-learn codebase should inherit from `sklearn.base.BaseEstimator`.

The Scikit-learn Fit-Predict Interface



Python module and class function and member encapsulation:

- ▶ module private: one underscore
- ▶ class-private: two underscores

via mangled names.

...NOTE: no `virtual void fit() = 0`; declaration in python!

...for modules, private funcs can still be accessed via a hack?!

...src file: `/opt/anaconda3/pkgsrc/.../sklearn/base.py`

The Scikit-learn Fit-Predict Interface



Demo..

Implementing an estimator via a python class as simple as

```
1 class ParadoxClassifier(BaseEstimator, ClassifierMixin):
2     def fit(self, X, y=None):
3         pass
4     def predict(self, X):
5         assert X.ndim==2
6         return np.ones(X.shape[0],dtype=bool)
```

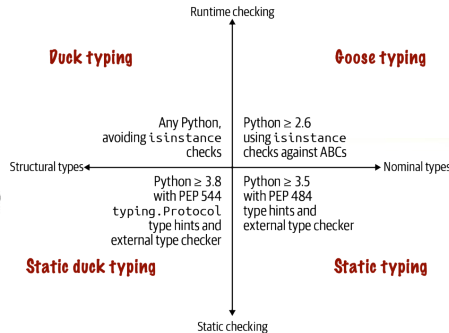
But will the following also be an estimator?

```
1 class ParadoxClassifierModified:
2     def fit(self, X, y):
3         pass
4     def predict(self, X):
5         assert X.ndim==2
6         return np.ones(X.shape[0],dtype=bool)
7     def score(self, y_true, y_pred):
8         # add some nice preconditions..
9         return 1 # this is good!
```

Python Duck Typing

"Duck Typing"

If it looks like a duck, swims like a duck, and quacks like a duck, then it is probably a duck!



[<https://algorystcorner.hashnode.dev/what-is-gradual-typing-in-python>]

Exercise: L02/dummy_classifier.ipynb

A dummy classifier for the fit-predict interface,
plus intro to a Stochastic Gradient Decent method (SGD)
and introduction to the accuracy-paradox.

The screenshot shows a Jupyter Notebook interface with the title 'dummy_classifier'. The code cell contains the following text:

```
In [ ]: # TODO: add your code here..  
assert False, "TODO: solve Qb, and remove me.."
```

Below the code cell, the text reads:

Qc Implement a dummy binary classifier

Now we will try to create a Scikit-learn compatible estimator implemented via a python class. Follow the code found in [HOML], p84, but name you estimator `DummyClassifier` instead of `Never5Classifier`.

Here our Python class knowledge comes into play. The estimator class hierarchy looks like

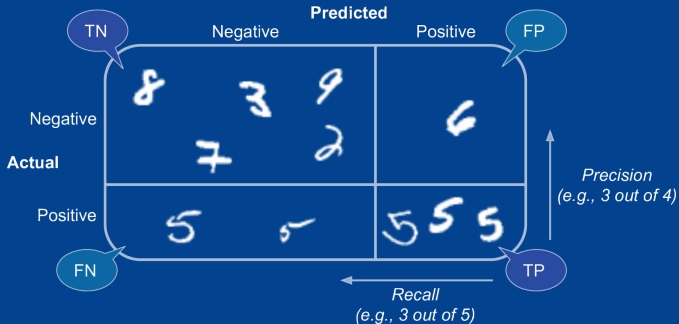
```
graph BT
    Base[sklearn.base.BaseEstimator] --> MyClassifier[MyClassifier]
    Mixin[sklearn.base.ClassifierMixin] --> MyClassifier
```

The diagram illustrates the class hierarchy for a Scikit-learn estimator. It shows three classes:

- sklearn.base.BaseEstimator**: Base class for all estimators in scikit-learn. Methods: `+get_params()`, `+set_params()`, `- get_param_names()`.
- sklearn.base.ClassifierMixin**: Mixin class for all classifiers in scikit-learn. Method: `+score()`.
- MyClassifier**: A Scikit-learn estimator. Methods: `+fit()`, `+predict()`.

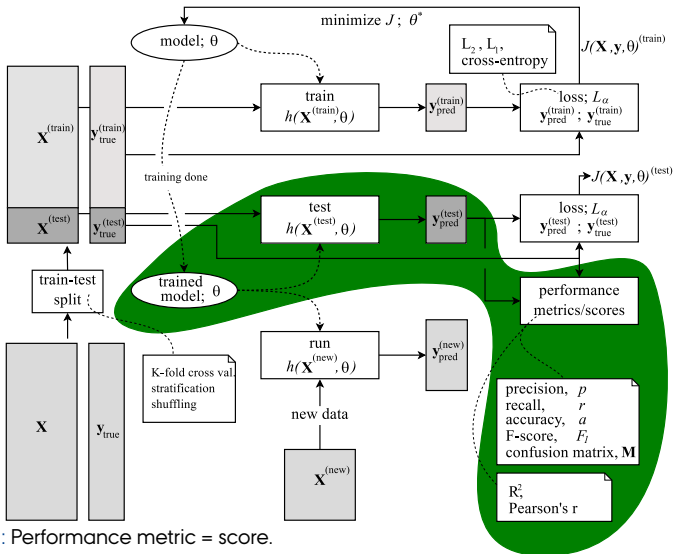
Arrows indicate that `MyClassifier` inherits from both `BaseEstimator` and `ClassifierMixin`.

PERFORMANCE METRICS (SCORES)



Evaluér på test-data: Performance metrics

Kort intro til konceptet *performance metrics*.



NOTE₀: Performance metric = score.

NOTE₁: 'Performance measure' begreb bruges ikke, kun score eller perf. metric.

NOTE₂: Loss er ML algo'ens 'performance mål', score er vores evalueringsmål.

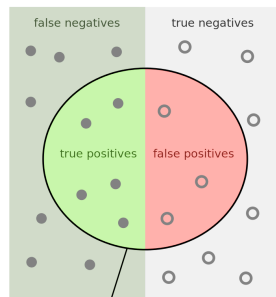
Exercise: L02/performance_metrics.ipynb

Nomenclature

For a binary classifier

NAME	SYMBOL	ALIAS
true positives	TP	
true negatives	TN	
false positives	FP	type I error
false negatives	FN	type II error

and $N = N_P + N_N$ being the total number of samples and the number of positive and negative samples respectively.



[https://en.wikipedia.org/wiki/Precision_and_recall]

Exercise: L02/performance_metrics.ipynb

Precision, recall and accuracy, F_1 -score, and confusion matrix

precision, $p = \frac{TP}{TP+FP}$

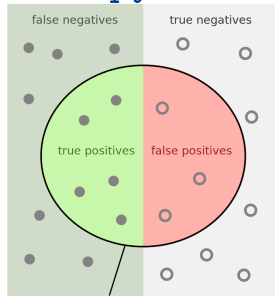
recall (or sensitivity), $r = \frac{TP}{TP+FN}$

accuracy, $a = \frac{TP+TN}{TP+TN+FP+FN}$

F_1 -score, $F_1 = \frac{2pr}{p+r}$

Confusion Matrix, $M_{\text{confusion}} =$

	actual true	actual false
predicted true	TP	FP
predicted false	FN	TN



Precision = $\frac{\text{green semi-circle}}{\text{green semi-circle} + \text{red semi-circle}}$

Recall = $\frac{\text{green semi-circle}}{\text{green semi-circle} + \text{green rectangle}}$

NOTE₀: you can compare precision... F_1 -score, but not necessarily the cost, J .

NOTE₁: beware of matrix transpose and interpretation of 'TP/TN'!

Exercise: L02/performance_metrics.ipynb

Nomenclature for the Confusion Matrix

		True condition			
		Condition positive	Condition negative	Prevalence $= \frac{\Sigma \text{Condition positive}}{\Sigma \text{Total population}}$	Accuracy (ACC) = $\frac{\Sigma \text{True positive} + \Sigma \text{True negative}}{\Sigma \text{Total population}}$
Predicted condition	Predicted condition positive	True positive, Power	False positive, Type I error	Positive predictive value (PPV), Precision = $\frac{\Sigma \text{True positive}}{\Sigma \text{Predicted condition positive}}$	False discovery rate (FDR) = $\frac{\Sigma \text{False positive}}{\Sigma \text{Predicted condition positive}}$
	Predicted condition negative	False negative, Type II error	True negative	False omission rate (FOR) $= \frac{\Sigma \text{False negative}}{\Sigma \text{Predicted condition negative}}$	Negative predictive value (NPV) = $\frac{\Sigma \text{True negative}}{\Sigma \text{Predicted condition negative}}$
		True positive rate (TPR), Recall, Sensitivity, probability of detection $= \frac{\Sigma \text{True positive}}{\Sigma \text{Condition positive}}$	False positive rate (FPR), Fall-out, probability of false alarm $= \frac{\Sigma \text{False positive}}{\Sigma \text{Condition negative}}$	Positive likelihood ratio (LR+) $= \frac{\text{TPR}}{\text{FPR}}$	Diagnostic odds ratio (DOR) $= \frac{\text{LR+}}{\text{LR-}}$ F1 score = $\frac{2}{\frac{1}{\text{Recall}} + \frac{1}{\text{Precision}}}$
		False negative rate (FNR), Miss rate $= \frac{\Sigma \text{False negative}}{\Sigma \text{Condition positive}}$	True negative rate (TNR), Specificity (SPC) $= \frac{\Sigma \text{True negative}}{\Sigma \text{Condition negative}}$	Negative likelihood ratio (LR-) $= \frac{\text{FNR}}{\text{TNR}}$	

Mr. Itmal



prevalence, positive predictive value, etc. not important to know at all!

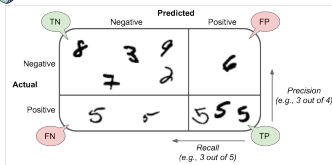


Figure 3-2. An illustrated confusion matrix

Exercise: L02/performance_metrics.ipynb

Accuracy Paradox...

```
1 class ParadoxClassifier(BaseEstimator, ClassifierMixin):
2     def fit(self, X, y=None):
3         pass
4     def predict(self, X):
5         assert X.ndim==2
6         return np.ones(X.shape[0], dtype=bool)
```

Test via the breast cancer Wisconsin dataset..

```
1 X_train, X_test, y_train, y_test =
2     train_test_split(
3         X, y_true, test_size=0.2, shuffle=True, random_state=42
4     )
5
6 clf = ParadoxClassifier()
7 clf.fit(X_train, y_train)
8 y_pred = clf.predict(X_test)
9
10 acc = accuracy_score(y_test, y_pred)
11 print(f' acc={acc}, N={y_pred.shape[0]}')
12 score = clf.score(X_test, y_test)
13 print(f' clf.score()={score} (same as accuracy_score)')
```

prints: acc=0.6228070175438597,
N=114

NOTE₀: for MNIST, a dum classify as '5' $\sim a = 10\%$

NOTE₁: for MNIST, a dum classify not-as '5' $\sim a = 90\%$

Exercise: L02/performance_metrics.ipynb

More on metrics, oh-so-many!

[<https://scikit-learn.org/stable/modules/classes.html#sklearn-metrics-metrics>]

Classification metrics

See the [Classification metrics](#) section of the user guide for further details.

<code>metrics.accuracy_score(y_true, y_pred[, ...])</code>	Accuracy classification score.
<code>metrics.auc(x, y[, reorder])</code>	Compute Area Under the Curve (AUC) using the trapezoidal rule
<code>metrics.average_precision_score(y_true, y_score)</code>	Compute average precision (AP) from prediction scores
<code>metrics.balanced_accuracy_score(y_true, y_pred)</code>	Compute the balanced accuracy
<code>metrics.brier_score_loss(y_true, y_prob[, ...])</code>	Compute the Brier score.
<code>metrics.classification_report(y_true, y_pred)</code>	Build a text report showing the main classification metrics
<code>metrics.cohen_kappa_score(y1, y2[, labels, ...])</code>	Cohen's kappa: a statistic that measures Inter-annotator agreement.
<code>metrics.confusion_matrix(y_true, y_pred[, ...])</code>	Compute confusion matrix to evaluate the accuracy of a classification
<code>metrics.f1_score(y_true, y_pred[, labels, ...])</code>	Compute the F1 score, also known as balanced F-score or F-measure
<code>metrics.fbeta_score(y_true, y_pred, beta[, ...])</code>	Compute the F-beta score
<code>metrics.hamming_loss(y_true, y_pred[, ...])</code>	Compute the average Hamming loss.
<code>metrics.hinge_loss(y_true, pred_decision[, ...])</code>	Average hinge loss (non-regularized)
<code>metrics.jaccard_similarity_score(y_true, y_pred)</code>	Jaccard similarity coefficient score
<code>metrics.log_loss(y_true, y_pred[, eps, ...])</code>	Log loss, aka logistic loss or cross-entropy loss.
<code>metrics.matthews_corrcoef(y_true, y_pred[, ...])</code>	Compute the Matthews correlation coefficient (MCC)
<code>metrics.precision_recall_curve(y_true, ...)</code>	Compute precision-recall pairs for different probability thresholds
<code>metrics.precision_recall_fscore_support(...)</code>	Compute precision, recall, F-measure and support for each class

