King Fahd University of Petroleum and Minerals
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EE 417 Digital Communication

Course Project

Prepared for

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I hereby declare that all codes, results, information, discussion, and comments included in this report are solely performed by me and my partners, and I have received no help by any means in preparing this report.

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Introduction

Quadrature amplitude modulation QAM combines the merits of amplitude and phase modulation (i.e., AM and PM). It increases the bandwidth efficiency by utilizing the amplitude and phase components to provide modulation. QAM can be used to transmit different number of symbols whereas the bandwidth efficiency increases as the number of symbols increase. QAM signal can be described mathematically as a weighted sum of two sinusoids with phase difference of 90° to ensure orthogonality:

$$S_i(t) = a_i \cos(wt + \theta_i) + b_i \cos(wt + \theta_i - 90°) \quad (1)$$

For simplicity we can consider cosine and sine signals as they are orthogonal.

$$S_i(t) = a_i \cos(wt) + b_i \sin(wt) \quad (2)$$

To facilitate the analysis of the signals, signal space is used to deal with signals independently of time. It can be shown that sinusoids with same frequency and phase shift of 90°, form a vector space. Therefore, we can express different signals in term of a collection of basis functions. To illustrate, a time domain signal $S_i(t)$ in (2) can be expressed as a vector $\boldsymbol{S_i}$ in the signal space where $\cos(wt)$ and $\sin(wt)$ are the basis functions.

$$\boldsymbol{S_i} = \begin{bmatrix} a_i \\ b_i \end{bmatrix} \quad (3)$$

However, when transmitting thru AWGN channel these signals become corrupted and hence it becomes a challenge to decode them and extract the information. The received signal will be the original signal with a noise added on top of it. The noise is modeled as a vector of Gaussian random variables (RVs) $\boldsymbol{W_i}$, where its elements are independent RV that have zero mean and variance equal to $\frac{N_0}{2}$.

$$\boldsymbol{X_i} = \boldsymbol{S_i} + \boldsymbol{W_i} = \begin{bmatrix} a_i + w_{i1} \\ b_i + w_{i2} \end{bmatrix} \quad (4)$$

Due to noise, the received signals $\boldsymbol{S_i}$ will be a cloud of points centered around the points (a_i, b_i) on the signal space and the size of the cloud increases as the noise power (i.e., variance) increases. Luckily, the signal space allows us to utilize the Euclidean distance for signal detection. One of the methods is the minimum distance detector (MD). MD detector method, calculate the distance between the received signals and the coordinates of the different symbols and draw a decision region for each symbol as shown in Fig.2. If the cloud size of the received signals is not large enough to spell to the decision region of the adjacent symbol, the MD detector will decode the received signal $\boldsymbol{S_i}$ correctly to the symbol m_i.

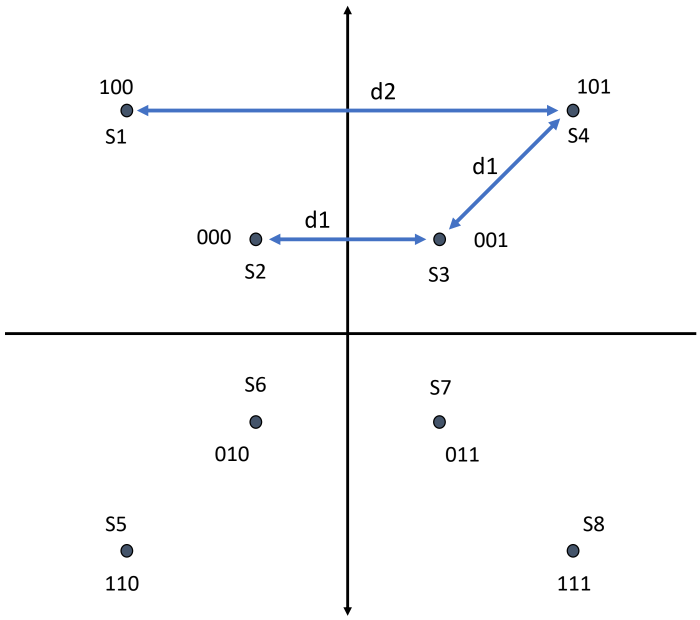

Figure 1. Constellation diagram considered in the project.

Methodology

Utilizing MATLAB software, the simulation results and theoretical bound will be produced. The simulation will vary the SNR per bit in steps of 2 dB, ranging from 0 dB to an SNR value that yields a BER of 10^{-4}. First, 1000 random binary sequences are generated with equal numbers of 1's and 0's for each SNR value, and it will use Gray mapping to map every three bits onto a single 2-D vector (complex number) in order to produce the signal constellations shown in figure 1. The code then adds an independent Gaussian noise sample to each signal. The distance between the received vector and each of the signal vectors, s_1 to s_8, will be calculated at the receiver in order to implement ML detection. The error vector with the minimum distance will determine the originally transmitted signal. The code will continue doing the steps until there are more than 100-bit errors, at which point it will compute the SER and BER. Moreover, based on the minimum distance the constellation diagram will be split to regions for each signal as shown in Figure.2.

Error Probability Analysis

In this section the analysis and the derivation of the error bound probability are presented. The task now is to relate the distances d_1 and d_2.

Relationship between d_1& d_2

The relationship between d1 and d2 to make signals the furthest apart while maintaining most signals apart by the min distance d1 can be found as follows:

$$\frac{(d_2)^2}{2} = \left(d_1 + \frac{\sqrt{2}}{2} d_1\right)^2 \tag{4}$$

$$d_2 = \sqrt{2}\left(d_1 + \frac{\sqrt{2}}{2} d_1\right) \tag{5}$$

$$d_2 = d_1\left(1 + \sqrt{2}\right) \tag{6}$$

Average energy per symbol

Our objective is to find the average energy per symbol E_s in terms of the distances d_1 and d_2. It is calculated as follows:

$$E_S = \frac{1}{8}\left(4 \times \left(\frac{\sqrt{2}d_1}{2}\right)^2 + 4 \times \left(\frac{\sqrt{2}d_2}{2}\right)^2\right) \tag{7}$$

$$E_S = \frac{1}{8}\left(4 \times \frac{1}{2} \times d_1^2 + 4 \times \frac{1}{2} \times d_2^2\right) \tag{8}$$

$$E_S = \frac{1}{4}(d_1^2 + d_2^2) \tag{9}$$

Error upper bound

This section presents the derivation of the error bound for the constellation diagram given in Fig.1.

1- Union Bound

First, the error bound for the probability of error P_e for M symbol is given by:

$$P_e \leq \sum_{i=1}^{M} \sum_{\substack{k=1 \\ k \neq i}}^{M} p_i \times P(S_i, S_k) \tag{10}$$

Where, $P(S_i, S_k)$ is the probability of mistakenly decoding the symbol m_i for a symbol m_k. Assuming equiprobable symbols, we have:

$$P_e \leq \frac{1}{M}\sum_{i=1}^{M}\sum_{\substack{k=1\\k\neq i}}^{M} P(S_i, S_k) \tag{11}$$

It can be easily shown that

$$P(S_i, S_k) = \frac{1}{2} erfc\left(\frac{d_{ik}}{2\sqrt{N_0}}\right) \tag{12}$$

Where $erfc(\,.\,)$ is the complementary error function which can be written in terms of the Q function. Hence eq.8 becomes:

$$P_e \leq \frac{1}{2M}\sum_{i=1}^{M}\sum_{\substack{k=1\\k\neq i}}^{M} erfc\left(\frac{d_{ik}}{2\sqrt{N_0}}\right) \tag{13}$$

$$P_e \leq \frac{1}{M}\sum_{i=1}^{M}\sum_{\substack{k=1\\k\neq i}}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right) \tag{14}$$

Considering the minimum distance d_{min} and that the Q function is monotonically decreasing:

$$P_e \leq \frac{M-1}{M}\sum_{i=1}^{M} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = (M-1) \times Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) \tag{15}$$

2- Immediate Neighbors

We can classify the constellation into two groups of symbols, the inner square (s_2, s_3, s_6, and s_7) and the outer square (s_1, s_4, s_5, and s_8). For the inner square, each symbol has three immediate neighbors, and for the outer square each symbol has one immediate neighbor. The upper bound can be found as follows:

$$P_s(e|s_i) \leq \sum_{s_i \in N_j} Q\left(\sqrt{\frac{d_{ij}^2}{2N_o}}\right) \tag{16}$$

Where N_j is the set of immediate neighbors of s_i.

Now without loss of generality, let's consider the inner square by calculating the conditional probability of s_2:

$$P_s(e|s_2) \leq \sum_{s_i = s_1, s_7, s_3} Q\left(\sqrt{\frac{d_{ij}^2}{2N_o}}\right) = 3Q\left(\sqrt{\frac{d_1^2}{2N_o}}\right) \tag{17}$$

Let's consider the outer square by calculating the conditional probability of s_1:

$$P_s(e|s_1) \leq \sum_{s_i = s_2} Q\left(\sqrt{\frac{d_{ij}^2}{2N_o}}\right) = Q\left(\sqrt{\frac{d_1^2}{2N_o}}\right) \tag{18}$$

The average error probability by considering the remaining symbols is given by:

$$P_e \leq \frac{1}{8}\left((4 \times 3)Q\left(\sqrt{\frac{d_1^2}{2N_o}}\right) + (4 \times 1)Q\left(\sqrt{\frac{d_1^2}{2N_o}}\right)\right) = 2Q\left(\sqrt{\frac{d_1^2}{2N_o}}\right) \tag{19}$$

For the bit error rate bound, we assume all symbol error are equiprobable, hence the upper bound is given by:

$$BER \leq \frac{M}{2(M-1)} P_e = \frac{2^{k-1}}{(2^k - 1)} P_e \tag{20}$$

Decision Regions

The decision regions for the ML detectors are shown in Figure 2. It was calculated based on the given distances d_1 and d_2. d_1 was chosen to be two $d_1 = 2$ hence $d_2 = 4.83$.

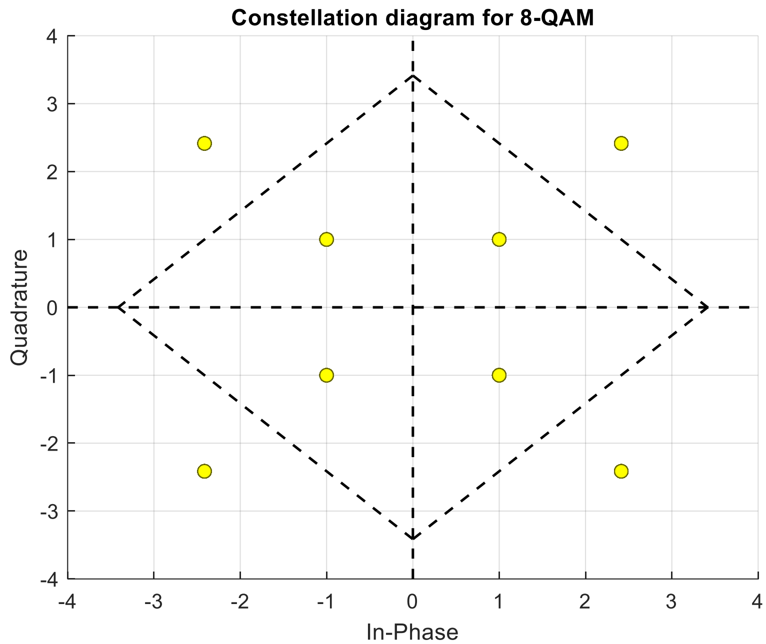

Figure 2. The decision regions for the MD detection.

Simulation Results

The simulation results after running the code is shown in Figure 3.

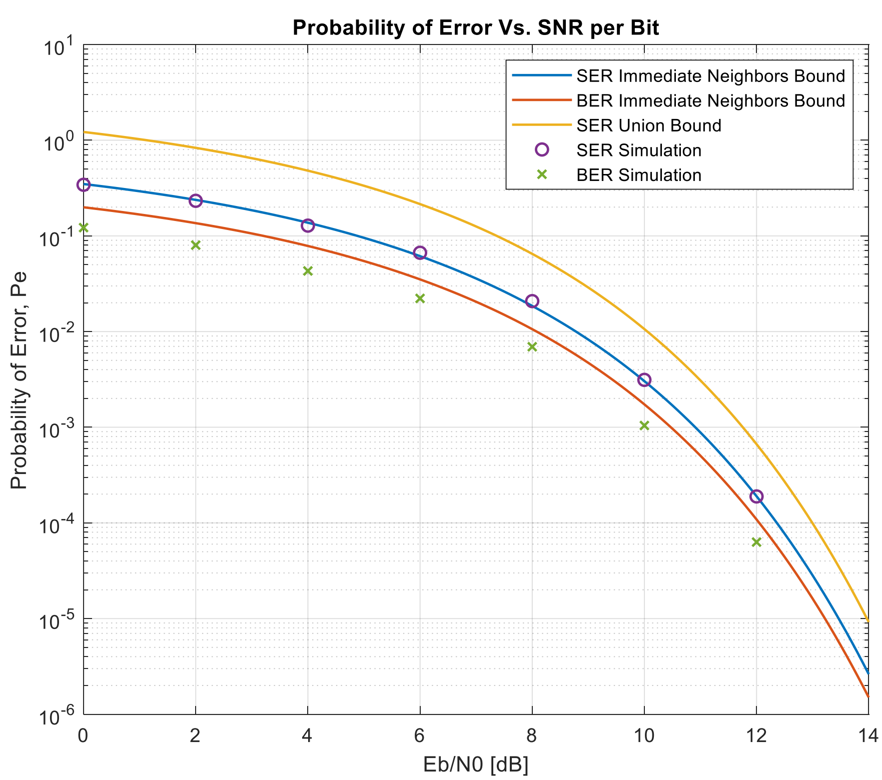

Figure 3. Probability of Error Vs. SNR per bit.

As seen in the above figure, the theoretical bound and the simulation results both exhibit the waterfall expected trend. Due to the overlap between the decision regions, the union bound is less tight than the immediate neighbors bound. The simulated BER is equal to 7.8509882×10^{-5} 7. at $12\ dB$, while the bound for the immediate neighbors is equal to 1.8064910×10^{-4} .

The probability of receiving a vector that exceeds the decision region decreases as the SNR value increases and the received cloud noise become smaller. Figure 4 shows the constellation at a relatively high SNR ($16\ dB$). We see that all the received message points are decoded correctly.

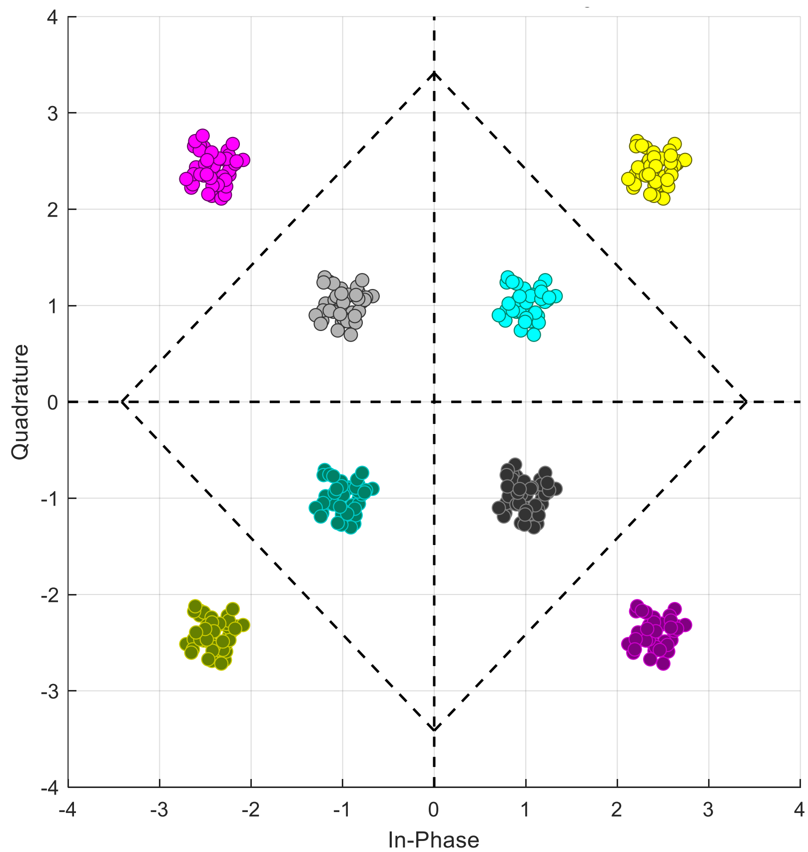

Figure 4: Received Signals Constellation at 16 dB SNR.

Conclusions

In this project we simulated an 8-QAM signaling scheme in the presence of (AWGN). The constellation diagram was given and not chosen. The simulated scheme was tested with different SNR values to observe the probability of error. In addition, minimum distance detector (MD) is employed for signal decoding. Moreover, a comparison between the simulation results and the theoretical error bound was presented in the report.