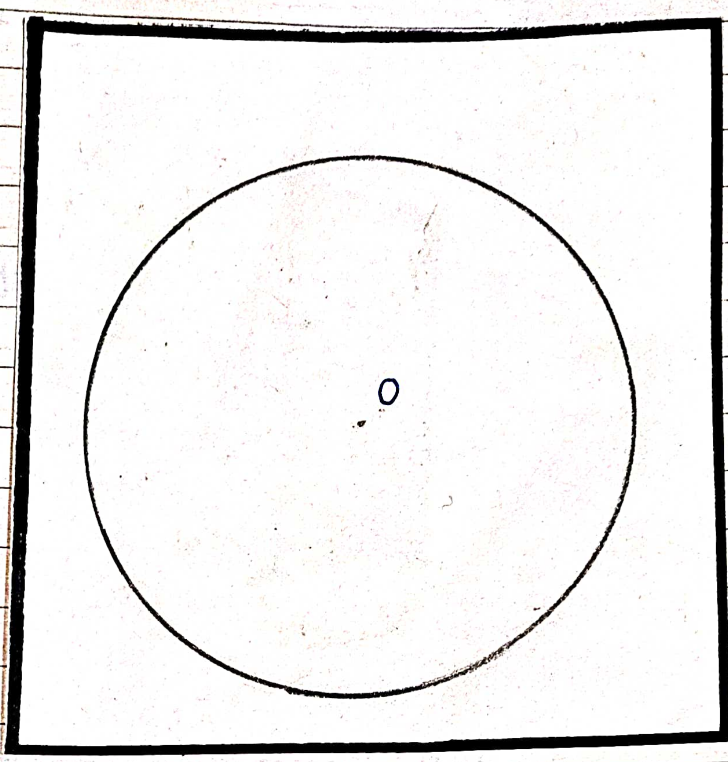
* Circle central formula *

Imagine circle with centre O.

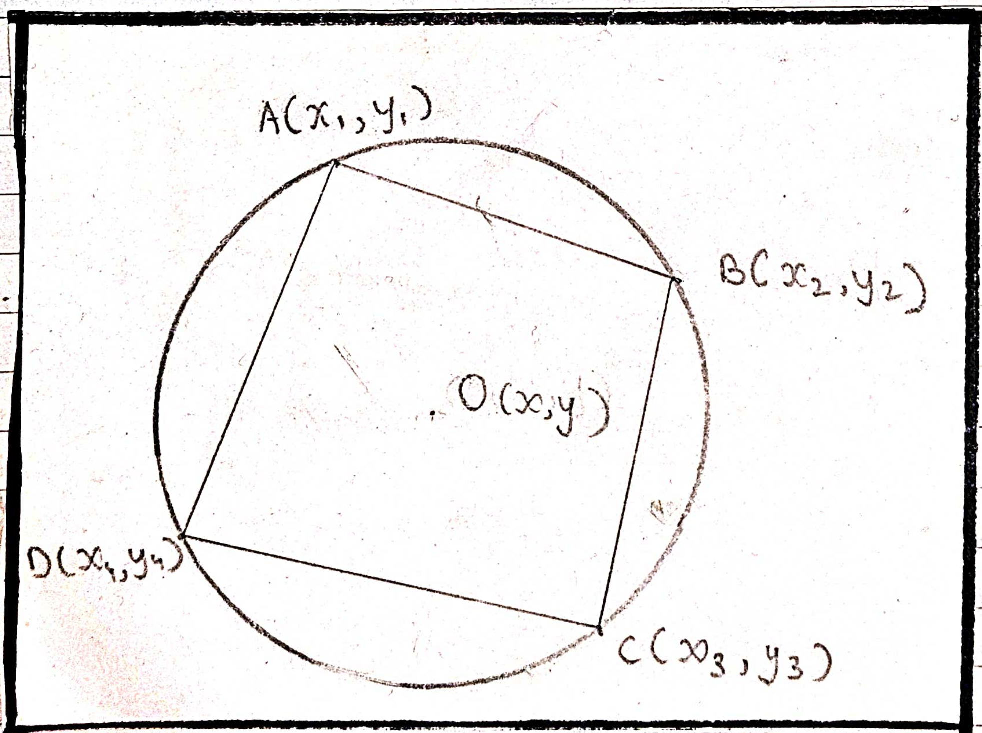
Plot points A, B, C and D on the circle.



Let,

O (x, y),
A (x_1, y_1),
B (x_2, y_2),
C (x_3, y_3)
D (x_4, y_4)

Also draw seg AB, seg BC, seg DC and seg AD.



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Find $d(O, A)$

Using distance formula:

$$d(O,A) = \sqrt{(x-x_1)^2 + (y-y_1)^2} \quad \ldots (i)$$

Also,

$$d(O,B) = \sqrt{(x-x_2)^2 + (y-y_2)^2} \quad \ldots (ii)$$

Since $d(O,A)$ and $d(O,B)$ is radii of same circle we get:

$$\sqrt{(x-x_1)^2 + (y-y_1)^2} = \sqrt{(x-x_2)^2 + (y-y_2)^2}$$

... From (i) and (ii)

$$\therefore (x-x_1)^2 + (y-y_1)^2 = (x-x_2)^2 + (y-y_2)^2$$

... squaring both sides

$$\therefore \cancel{x^2} - 2xx_1 + x_1^2 + \cancel{y^2} - 2yy_1 + y_1^2 = \cancel{x^2} - 2xx_2 + x_2^2 + \cancel{y^2} - 2yy_2 + y_2^2$$

$$\therefore x_1^2 + y_1^2 - 2xx_1 - 2yy_1 = x_2^2 + y_2^2 - 2xx_2 - 2yy_2$$

$$\therefore (x_1^2 + y_1^2) - (x_2^2 + y_2^2) = 2xx_1 - 2xx_2 + 2yy_1 - 2yy_2$$

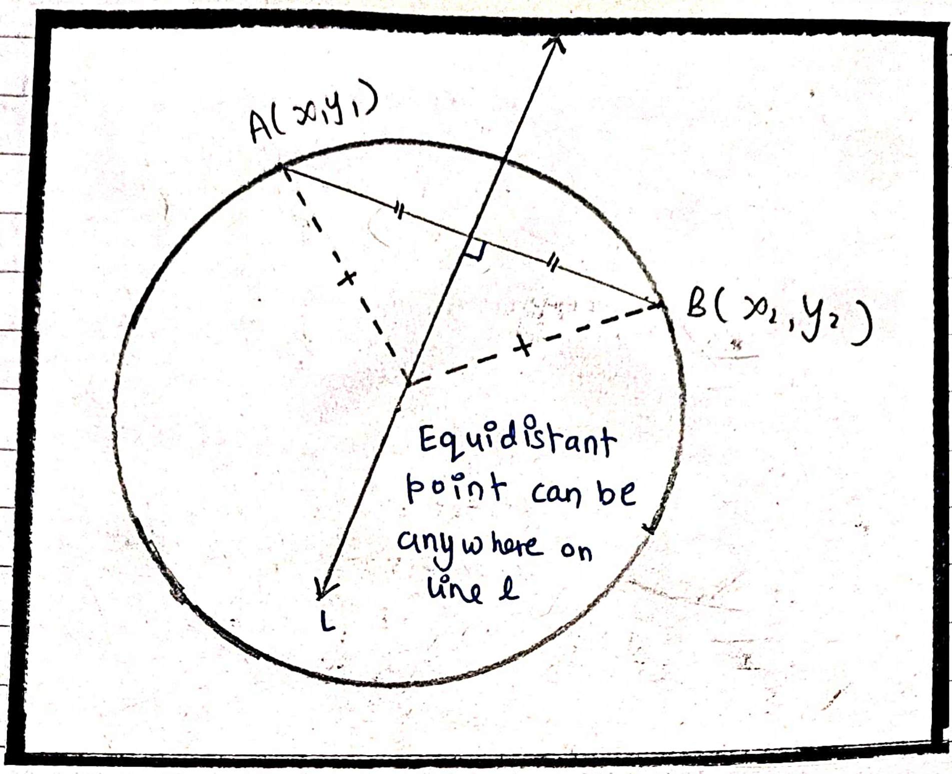
$$\therefore (x_1^2 + y_1^2) - (x_2^2 + y_2^2) = 2(x_1 - x_2)(x) + 2(y_1 - y_2)(y)$$

$\therefore$ Here in format, $ax + by = c_1$

$a_1 = 2(x_1 - x_2)$, $b_1 = 2(y_1 - y_2)$ and

$c_1 = (x_1^2 + y_1^2) - (x_2^2 + y_2^2)$

Graphically this formula can be shown as:



- The centre of the circle is on this line l

Similarly find d (O, C),

$$d(O,C) = \sqrt{(x - x_3)^2 + (y - y_3)^2} \quad \ldots (iii)$$

Since d (O, C) and d(O, B) are radii of same circle we get:

$$\sqrt{(x - x_2)^2 + (y - y_2)^2} = \sqrt{(x - x_3)^2 + (y - y_3)^2}$$

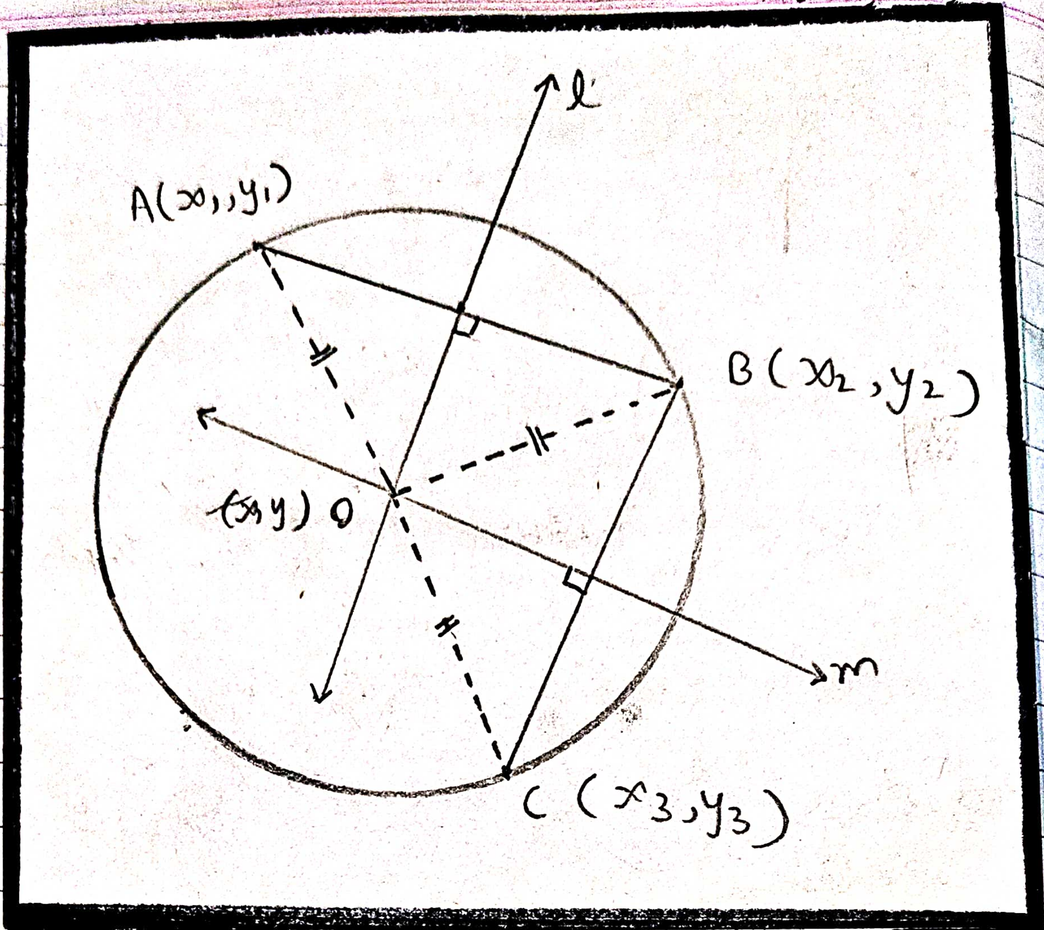
... From (ii) and (iii)

In the end of doing the operations done previously :

In format, $a_2 x + b_2 y = c_2$

$a_2 = 2(x_2 - x_3)$, $b_2 = 2(y_2 - y_3)$ and

$c_2 = (x_2^2 + y_2^2) - (x_3^2 + y_3^2)$


l
A(x1,y1)
B(x2,y2)
(x,y) O
m
C(x3,y3)

By graphically laying them out we get intersection. On this intersection, all points i.e. A, B and C are equidistant.

$\therefore$ It is the centre of the circle

$\therefore$ That point is O.

This point is inside the circle

and $d(O,A) = d(O,B) = d(O,C)$

Therefore it must be centre of the circle.

Laying the equations out:

$$[2(x_1 - x_2)]x + [2(y_1 - y_2)]y = [(x_1^2 + y_1^2) - (x_2^2 + y_2^2)]$$

$$[2(x_2 - x_3)]x + [2(y_2 - y_3)]y = [(x_2^2 + y_2^2) - (x_3^2 + y_3^2)]$$

Now by using Cramer's Rule

First $D = [4(x_1 - x_2)(y_2 - y_3)] - [4(x_2 - x_3)(y_1 - y_2)]$ ... (i)(i)

$$D_x = \left(\left\{[(x_1^2 + y_1^2) - (x_2^2 + y_2^2)] \times [2(y_2 - y_3)]\right\} - \left\{[(x_2^2 + y_2^2) - (x_3^2 + y_3^2)] \times [2(y_1 - y_2)]\right\}\right) \quad \text{...(iii)}$$

$$D_y = \left(\left\{\left[(x_2^2 + y_2^2) - (x_3^2 + y_3^2)\right] \times \left[2(x_1 - x_2)\right]\right\}\right.$$

$$\left. - \left\{\left[(x_1^2 + y_1^2) - (x_2^2 + y_2^2)\right] \times \left[2(x_2 - x_3)\right]\right\}\right) \quad \ldots \text{(iii)}$$

(iii)

$\therefore$ By Cramer's Rule,

$$x = \frac{D_x}{D}$$

$$= \frac{D_x \text{ from (ii) (ii)}}{D \text{ from (i) (i)}}$$

$$y = \frac{D_y}{D}$$

$$= \frac{D_y \text{ from (iii) (iii)}}{D \text{ from (i) (i)}}$$

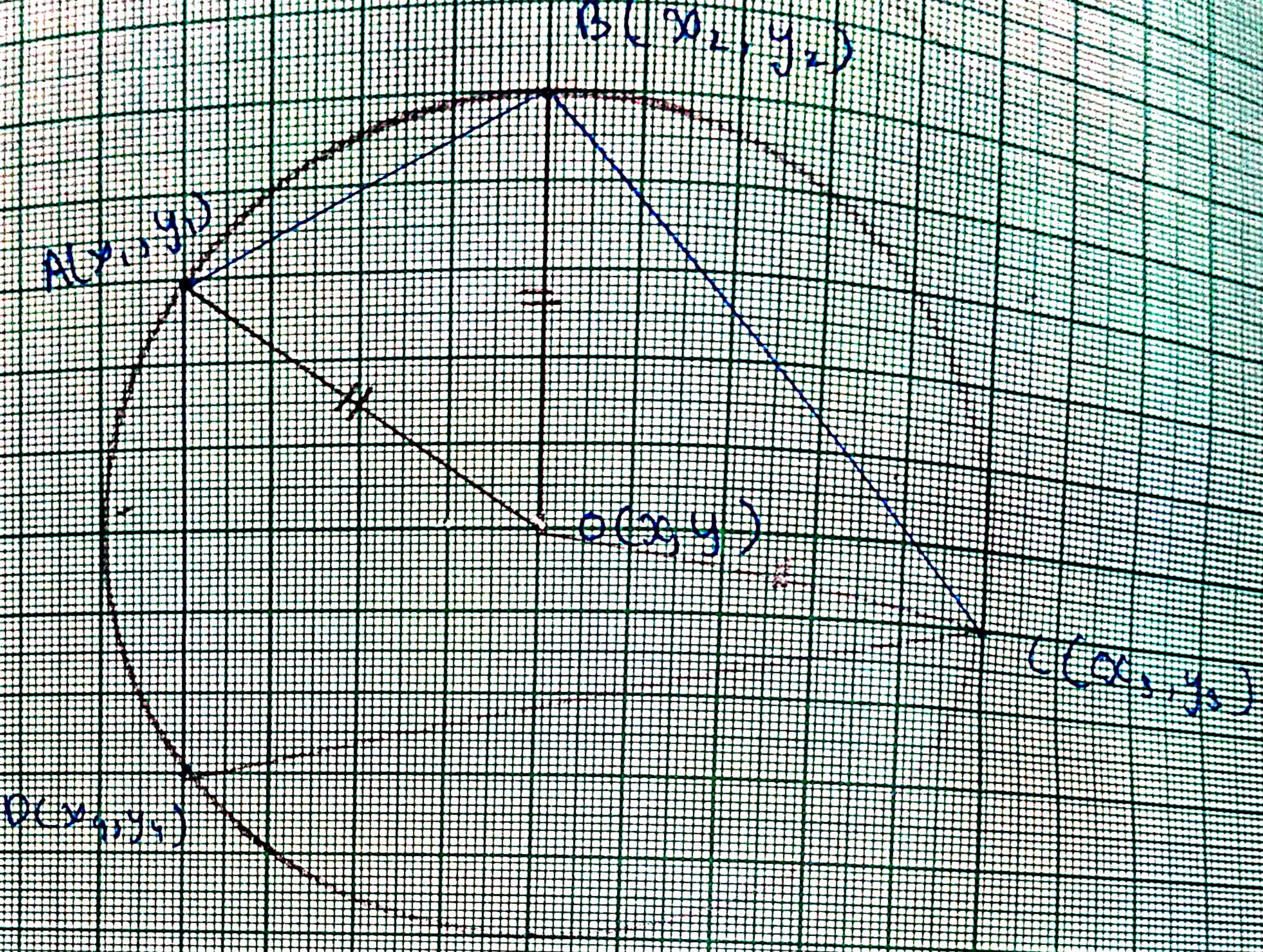
$O(x, y)$ is centre of circle.

For $O(x, y)$ (as centre of circle)

$$x = \frac{\left(\begin{array}{l} \{[(x_1^2+y_1^2) - (x_2^2+y_2^2)] \times [2(y_2-y_3)]\} \\ \quad - \{[(x_2^2+y_2^2) - (x_3^2+y_3^2)] \times [2(y_1-y_2)]\} \end{array} \right)}{\left([4(x_1-x_2)(y_2-y_3)] - [4(x_2-x_3)(y_1-y_2)] \right)}$$

$$y = \frac{\left(\begin{array}{l} \{[(x_2^2+y_2^2) - (x_3^2+y_3^2)] \times [2(x_1-x_2)]\} \\ \quad - \{[(x_1^2+y_1^2) - (x_2^2+y_2^2)] \times [2(x_2-x_3)]\} \end{array} \right)}{\left([4(x_1-x_2)(y_2-y_3)] - [4(x_2-x_3)(y_1-y_2)] \right)}$$

This is CIRCLE CENTRAL FORMULA


B(x2, y2)
A(x1, y1)
O(x, y)
C(x3, y3)
D(x4, y4)

[redacted]

[redacted]

Uses of circle central formula

Denoted by : $ccf(x)$ / $ccf(y)$

* Radius of circle

$$R = \sqrt{(x_1 - [ccf(x)])^2 + (y_1 - [ccf(y)])^2}$$

* ~~Radi~~ Area of circle

$$A(\text{circle}) = \pi \times \left\{ \sqrt{(x_1 - [ccf(x)])^2 + (y_1 - [ccf(y)])^2} \right\}^2$$

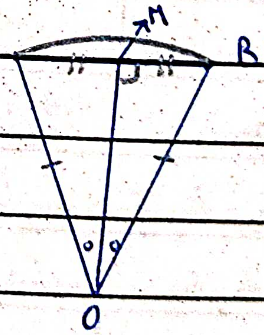
Theriotical use : To find A(sector)

First to get cc f (x) and ccf(y) ...(i)

Assume ← A & B

The d(A, B),

get $\frac{d(A,B)}{2}$



∴ we get triangle →

OB = Radius (Already recieved)

No need {

∴ Get OM by pythagoras

[redacted] [redacted]

$$MB = \frac{d(A,B)}{2}$$

$$\therefore \frac{MB}{OB} = \sin(\angle BOM)$$

$$\therefore \frac{d(A,B)}{2R} = \sin(\angle BOM)$$

$$\therefore \angle BOM = \arcsin\left(\frac{d(A,B)}{2R}\right)$$

$$\therefore 2\angle BOM = 2\arcsin\left(\frac{d(A,B)}{2R}\right)$$

$$\therefore \angle AOB = 2 \times \arcsin\left(\frac{d(A,B)}{2R}\right)$$

$$\therefore A(\text{sector}) =$$

$$\frac{2\left\{\arcsin\left[\frac{d(A,B)}{2R}\right]\right\}}{360} \times \pi R^2$$

$$= \frac{\arcsin\left[d(A,B) \div 2R\right]}{180} \times \pi R^2$$

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