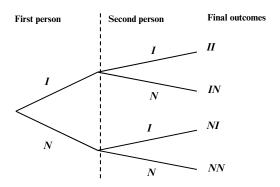
Chapter 4 Probability

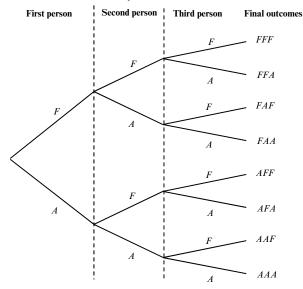
Section 4.1

- An **experiment** is a process that, when performed, results in one and only one of many observations. An **outcome** is the result of the performance of an experiment. The collection of all outcomes for an experiment is called a **sample space**. A **simple event** is an event that includes one and only one of the final outcomes of an experiment. A **compound event** is a collection of more than one outcome of an experiment.
- 4.3 The experiment of selecting two items from the box without replacement has the following six possible outcomes: AB, AC, BA, BC, CA, CB. The sample space is written as $S = \{AB, AC, BA, BC, CA, CB\}$.
- Let I = person owns an iPad and N = person does not own an iPad. The experiment has four outcomes: II, IN, NI, and NN.



2 Chapter 4

4.7 Let F = person selected is in favor of tax increase and A = person selected is against tax increase. The experiment of selecting three persons has eight outcomes: FFF, FFA, FAF, FAA, AFF, AFA, AAF, and AAA. The sample space is written as $S = \{FFF, FFA, FAF, FAA, AFF, AFA, AAF, AAA\}$.



- **4.9 a.** {*IN*, *NI*}; a compound event
 - **b.** {*II, IN, NI*}; a compound event
 - **c.** {*NN*, *NI*, *IN*}; a compound event
 - **d.** $\{IN\}$; a simple event

Section 4.2

- **4.11** 1. The probability of an event always lies in the range zero to 1, that is: $0 \le P(E_i) \le 1$ and $0 \le P(A) \le 1$
 - 2. The sum of the probabilities of all simple events for an experiment is always 1, that is:

$$\sum P(E_i) = P(E_1) + P(E_2) + P(E_3) + \dots = 1$$

4.13 1. Classical probability approach: Let E_i be a simple event and let A be a compound event for an experiment with equally likely outcomes:

$$P(E_i) = \frac{1}{\text{Total number of outcomes for the experiment}}$$

$$P(A) = \frac{\text{Number of outcomes favorable to A}}{\text{Total number of outcomes for the experiment}}$$

For example, the probability of observing a head when a fair coin is tossed once is 1/2.

- 2. **Relative frequency approach:** If an event A occurs f times in n repetitions of an experiment, then P(A) is approximately f/n. As the experiment is repeated more and more times, f/n approaches P(A). For example, if 50 of the last 5000 cars off the assembly line are lemons, the probability that the next car is a lemon is approximately P(lemon) = f/n = 50/5000 = 0.01
- 3. **Subjective probability approach:** Probabilities are assigned based on subjective judgment, experience, information and belief. For example, a teacher might estimate the probability of a student earning an A on a statistics test to be 1/6 based on previous classes.
- 4.15 The values 2.4, -0.63, 9/4, and -2/9 cannot be probabilities of events because the probability of an event can never be less than zero or greater than one.
- **4.17** This is a case of subjective probability because the given probability is based on the president's hunch.
- **4.19 a.** P(marble selected is red) = 18/40 = 0.45
 - **b.** P(marble selected is green) = 22/40 = 0.55
- 4.21 P(adult selected has shopped on the internet) = 1320/2000 = 0.66
- 4.23 P(Caribbean country selected) = 4/6. This is an example of the classical approach since all six balls are equally likely to be chosen.
- 4.25 P(company selected offers free health fitness center on the company premises) = 130/400 = 0.325

Number of companies that do not offer free health fitness center on the company premises = 400 - 130 = 270

P(company selected does not offer free health fitness center on the company premises) = 270/400 = 0.675

Yes, the sum of the probabilities is 1.0 because this experiment has only these two outcomes.

4.27

Income	Frequency	Relative Frequency
Less than \$40,000	50	50/500 = 0.10
\$40,000 to \$80,000	180	180/500 = 0.36
More than \$80,000	270	270/500 = 0.54

- **a.** P(income is less than \$40,000) = 0.10
- **b.** P(income is more than \$80,000) = 0.54

Section 4.3

- **Marginal probability** is the probability of a single event without consideration of any other event. **Conditional probability** is the probability that an event will occur given that another event has already occurred. For example, when a single die is rolled, the marginal probability of a number less than 4 is 1/2; the conditional probability of an odd number given that a number less than 4 has occurred is 2/3.
- 4.31 Two events are **independent** if the occurrence of one event does not affect the probability of the occurrence of the other event. Two events are **dependent** if the occurrence of one event affects the probability of the occurrence of the other event. If two events A and B satisfy the condition P(A|B) = P(A), or P(B|A) = P(B), they are independent; otherwise they are dependent.
- **4.33** a. P(A) = 4/11 and P(A|B) = 1/4. Since these probabilities are not equal, A and B are dependent. P(A) = 4/11 and P(A|C) = 0. Since these probabilities are not equal, A and C are dependent.
 - **b.** Events A and B are not mutually exclusive since they have the element "j" in common.

Events A and C are mutually exclusive because they have no elements in common.

Events B and C are not mutually exclusive since they have the element "c" in common.

- **c.** $\overline{A} = \{a, c, f, g, h, i, k\}; P(\overline{A}) = 7/11 = 0.636$ $\overline{B} = \{b, d, e, g, h, i, k\}; P(\overline{B}) = 7/11 = 0.636$ $\overline{C} = \{a, b, d, e, f, h, i, j\}; P(\overline{C}) = 8/11 = 0.727$
- **4.35** a. i. P(likes chocolate ice cream) = 207/700 = 0.2957
 - ii. P(is a woman) = 400/700 = 0.5714
 - iii. P(likes vanilla ice cream | is a woman) = 143/400 = 0.3575
 - iv. $P(\text{is a man} \mid \text{likes chocolate ice cream}) = 98/207 = 0.4734$
 - **b.** The events "is a man" and "likes vanilla ice cream" are not mutually exclusive because P("is a man" and "likes vanilla ice cream) is not zero. The events "likes vanilla ice cream" and "likes chocolate ice cream" are mutually exclusive because they cannot occur together, at least for this particular sample.
 - **c.** P(is a woman) = 400/700 = 0.571 and $P(\text{is a woman} \mid \text{likes chocolate ice cream}) = 103/207 = 0.5266.$

Since these probabilities are not equal, the events "is a woman" and "likes chocolate ice cream" are dependent.

4.37 P(is a man) = 290/500 = 0.58 and P(is a man | coffee without sugar) = 130/200 = 0.65. Since these probabilities are not equal, the events "is a man" and "coffee without sugar" are dependent.

- 4.39 The experiment involving two tosses of a coin has four outcomes: HH, HT, TH, and TT, where H denotes the event that a head is obtained and T that a tail is obtained on any toss. Events A and B contain the following outcomes: $A = \{HH, HT, TH\}$ and $B = \{TT\}$.
 - **a.** Since events A and B do not contain any common outcomes, they are mutually exclusive events.

P(A) = 3/4 = 0.750 and P(A|B) = 0. Since these two probabilities are not equal, A and B are not independent events.

b. Events A and B are complementary events because they do not contain any common outcomes and, taken together, they contain all the outcomes for this experiment.

$$P(B) = 1/4 = 0.250$$
 and $P(A) = 1 - P(B) = 1 - 0.250 = 0.750$

4.41 The two complementary events are that the graduate received a job offer and that the graduate did not receive a job offer.

P(received an offer) = 0.3 and P(did not receive an offer) = 1 - 0.3 = 0.7

Section 4.4

- **4.43** The **intersection** of two events is the collection of all outcomes that are common to both events. For example, if $A = \{1, 2, 3\}$ and $B = \{1, 3, 5\}$, then the intersection of A and B is the event $\{1, 3\}$.
- Unlike the rule for independent events, the rule for dependent events requires a conditional probability. Thus, if A and B are dependent, then P(A and B) = P(A)P(B|A). If A and B are independent events, then P(A and B) = P(A)P(B).
- **4.47 a.** P(A and B) = P(A)P(B|A) = (0.36)(0.87) = 0.3132 **b.** P(A and B) = P(B and A) = P(B)P(A|B) = (0.53)(0.22) = 0.1166
- **4.49 a.** P(A and B and C) = P(A)P(B)P(C) = (0.81)(0.49)(0.36) = 0.1429 **b.** P(A and B and C) = P(A)P(B)P(C) = (0.02)(0.03)(0.05) = 0.0000
- **4.51** P(B) = P(A and B) / P(A|B) = 0.33/0.44 = 0.750
- **4.53 a.** P(yes and woman) = 106/600 = 0.1767 **b.** P(no opinion and man) = 58/600 = 0.0967
- **4.55 a.** i. $P(\text{man and vanilla}) = P(\text{man})P(\text{vanilla}|\text{man}) = \left(\frac{300}{700}\right)\left(\frac{79}{300}\right) = 0.1129$ ii. $P(\text{other and woman}) = P(\text{other})P(\text{woman}|\text{other}) = \left(\frac{271}{700}\right)\left(\frac{148}{271}\right) = 0.2114$
 - **b.** P(chocolate and other) = 0 because these two events are mutually exclusive.

4.57 Let A = first candidate selected is a woman, B = first candidate selected is a man, C = second candidate selected is a woman, and D = second candidate selected is a man

Then $P(\text{both candidates selected are women}) = P(A \text{ and } C) = P(A)P(C|A) = \left(\frac{5}{8}\right)\left(\frac{4}{7}\right) = 0.3571.$

The probability that a student does not have loans to pay off is 1 - 0.60 = 0.40. Let B = first student selected does not have student loans to pay off and D = second student selected does not have student loans to pay off. Because students are independent,

P(neither student selected has loans to pay off) = P(B and D) = P(B)P(D) = (0.4)(0.4) = 0.16

- **4.61 a.** Let A_1 = adult 1 believes the housing bubble will occur in the next 4 to 6 years, A_2 = adult 2 believes the housing bubble will occur in the next 4 to 6 years, and A_3 = adult 3 believes the housing bubble will occur in the next 4 to 6 years. Using independence, $P(A_1 \text{ and } A_2 \text{ and } A_3) = (0.40)(0.40)(0.40) = 0.064$.
 - **b.** Let B_1 = adult 1 does not believe the housing bubble will occur in the next 4 to 6 years, B_2 = adult 2 does not believe the housing bubble will occur in the next 4 to 6 years, and B_3 = adult 3 does not believe the housing bubble will occur in the next 4 to 6 years. Using independence, $P(B_1 \text{ and } B_2 \text{ and } B_3)$ = (1 0.40)(1 0.40)(1 0.40) = 0.216.
- 4.63 Let F = employee selected is a female and M = employee selected is married. Since P(F) = 0.36 and P(F and M) = 0.19, $P(M \mid F) = P(F \text{ and } M) / P(F) = 0.19/0.36 = 0.5278$.
- 4.65 Let A = adult in small town lives alone and Y = adult in small town has at least one pet. Since P(A) = 0.20 and P(A and Y) = 0.08, P(Y|A) = P(A and Y) / P(A) = 0.08/0.20 = 0.400

Section 4.5

- When two events are mutually exclusive, their joint probability is zero and is dropped from the formula. So, if A and B are mutually nonexclusive events, then P(A or B) = P(A) + P(B) P(A and B). However, if A and B are mutually exclusive events, then P(A or B) = P(A) + P(B) 0 = P(A) + P(B).
- The formula P(A or B) = P(A) + P(B) is used when A and B are mutually exclusive events. For example, Let $A = \text{an odd number on both rolls and } B = \text{the sum is an odd number. Since } A \text{ and } B \text{ are mutually exclusive, } P(A \text{ or } B) = P(A) + P(B) = \frac{9}{36} + \frac{18}{36} = \frac{27}{36} = 0.75.$

4.71 a.
$$P(A \text{ or } B) = P(A) + P(B) = 0.71 + 0.03 = 0.74$$

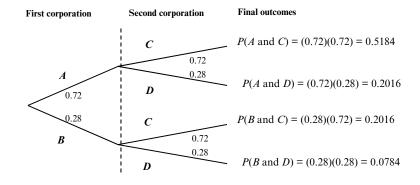
b.
$$P(A \text{ or } B) = P(A) + P(B) = 0.44 + 0.38 = 0.82$$

4.73 Let M = male, F = female, Y = this adult has shopped on the internet, and N = this adult has never shopped on the internet.

a.
$$P(N \text{ or } F) = P(N) + P(F) - P(N \text{ and } F) = \frac{1200}{2000} + \frac{800}{2000} - \frac{500}{2000} = 0.750$$

b.
$$P(M \text{ or } Y) = P(M) + P(Y) - P(M \text{ and } Y) = \frac{1200}{2000} + \frac{800}{2000} - \frac{500}{2000} = 0.750$$

- c. Since Y and N are mutually exclusive events, $P(Y \text{ or } N) = P(Y) + P(N) = \frac{800}{2000} + \frac{1200}{2000} = 1.0$. In fact, these two events are complementary.
- 4.75 Let T = vehicle selected ticketed and V = vehicle selected vandalized. Then, P(T or V) = P(T) + P(V) P(T and V) = 0.35 + 0.15 0.10 = 0.40
- 4.77 Let F = teacher selected is a female and S = teacher selected holds a second job. Then, P(F or S) = P(F) + P(S) - P(F and S) = 0.68 + 0.38 - 0.29 = 0.77
- 4.79 Let C = adult describes the U.S. health-care system in a state of crisis and M = adult describes the U.S. health-care system as having major problems. Since C and M are mutually exclusive events, P(C or M) = P(C) + P(M) = 0.15 + 0.55 = 0.70. This probability is not equal to 1.0 because some adults describe the U.S. health-care system as neither in crisis nor having major problems.
- 4.81 Let A = first selected corporation makes charitable contributions, B = first selected corporation does not make charitable contributions, C = second selected corporation makes charitable contributions, and D = second selected corporation does not make charitable contributions.



P(at most one corporation makes charitable contributions) = P(A and D) + P(B and C) + P(B and D) = 0.2016 + 0.2016 + 0.0784 = 0.4816

Section 4.6

4.83 Total outcomes for four rolls of a die = $6 \times 6 \times 6 \times 6 = 1296$

4.87 Total outcomes = $4 \times 8 \times 5 \times 6 = 960$

4.89
$${}_{25}C_2 = \frac{25!}{2!(25-2)!} = \frac{25!}{2!23!} = 300; \ {}_{25}P_2 = \frac{25!}{(25-2)!} = \frac{25!}{23!} = 600$$

4.91
$${}_{20}C_6 = \frac{20!}{6!(20-6)!} = \frac{20!}{6!14!} = 38,760; {}_{20}P_6 = \frac{20!}{(20-6)!} = \frac{20!}{14!} = 27,907,200$$

4.93
$$_{15}C_5 = \frac{15!}{5!(15-5)!} = \frac{15!}{5!10!} = 3003$$

Supplementary Exercises

- 4.95 Let M = adult selected is a male, F = adult selected is a female, A = adult selected prefers watching sports, and B = adult selected prefers watching drama shows.
 - **a.** i. P(B) = 49/250 = 0.1960
 - ii. P(M) = 120/250 = 0.4800
 - iii. P(A|F) = 95/130 = 0.7308
 - iv. P(M|A) = 106/201 = 0.5274

v.
$$P(F \text{ and } B) = P(F)P(B|F) = \left(\frac{130}{250}\right)\left(\frac{35}{130}\right) = 0.1400$$

vi.
$$P(A \text{ or } M) = P(A) + P(M) - P(A \text{ and } M) = \frac{201}{250} + \frac{120}{250} - \frac{106}{250} = 0.8600$$

- **b.** P(F) = 130/250 = 0.5200 and P(F|A) = 95/201 = 0.4726. Since these two probabilities are not equal, the events "female" and "prefers watching sports" are dependent. The events "female" and "prefers watching sports" are not mutually exclusive because they can occur together.
- 4.97 Let A = student selected is an athlete, B = student selected is a nonathlete, F = student selected favors paying college athletes, and N = student selected is against paying college athletes.
 - **a.** i. P(F) = 300/400 = 0.750
 - ii. P(F|B) = 210/300 = 0.700

iii.
$$P(A \text{ and } F) = P(A)P(F|A) = \left(\frac{100}{400}\right)\left(\frac{90}{100}\right) = 0.225$$

iv.
$$P(B \text{ or } N) = P(B) + P(N) - P(B \text{ and } N) = \frac{300}{400} + \frac{100}{400} - \frac{90}{400} = 0.775$$

- **b.** P(A) = 100/400 = 0.250 and P(A|F) = 90/300 = 0.300Since these two probabilities are not equal, the events "student athlete" and "should be paid" are dependent. The events "student athlete" and "should be paid" are not mutually exclusive because they can occur together.
- 4.99 Let G_1 = first car selected has a GPS system and G_2 = second car selected has a GPS system.

$$P(G_1 \text{ and } G_2) = P(G_1)P(G_2|G_1) = \left(\frac{28}{44}\right)\left(\frac{27}{43}\right) = 0.3996$$

- 4.101 P(both machines are not working properly)= P(first machine is not working properly)P(second machine is not working properly)
 - = (0.08)(0.06) = 0.0048

Advanced Exercises

- **4.103 a.** There are 26 possibilities for each letter and 10 possibilities for each digit. Hence, there are $26^3 \times 10^3 = 17,576,000$ possible different license places.
 - **b.** There are 2 possibilities for the second letter, 26 possibilities for the third letter, 10 possibilities for each of the two missing numbers, and 1 possibility for the last number. There are $1 \times 2 \times 26 \times 10 \times 10 = 5200$ license plates which fit the description.
- **4.105 a.** Each draw is independent of the previous draw. P(sixth marble is red) = 10/20 = 0.5000
 - **b.** Each draw is dependent on the previous draw(s) and after five draws, there are 5 red marbles and 15 total marbles left in the box. P(sixth marble is red) = 5/15 = 0.3333

- **c.** The probability of obtaining a head on the sixth toss is 0.5, since each toss is independent of the previous outcomes. Tossing a coin is mathematically equivalent to the situation in part a. Each drawing in part a is independent of previous drawings and the probability of drawing a red marble is 0.5 each time.
- **4.107 a.** $P(\text{you must pay gambler}) = P(\text{gambler rolls at least one 6 in four tries}) = 1 P(\text{she rolls no 6 in four tries}) = 1 (5/6)^4 = 1 0.4823 = 0.5177$
 - **b.** Let: E = you obtain at least one double six in 24 rolls,

 A_1 = you obtain a double six on the first roll,

 A_2 = you obtain a double six on the second roll,

 A_{24} = you obtain a double six on the 24th roll

Then, $P(E) = P(A_1 \text{ or } A_2 \text{ or } \cdots \text{ or } A_{24})$. However, A_1, A_2, \ldots, A_{24} are not mutually exclusive, since it is possible to obtain a double 6 on more than one roll. Thus, we cannot find $P(A_1 \text{ or } A_2 \text{ or } \ldots \text{ or } A_{24})$ by simply adding $P(A_1) + P(A_2) + \ldots + P(A_{24})$ as the gambler does to obtain (24)(1/36) = 2/3. To find P(E) we may use complementary events, as in part a. The probability of failing to roll a double six on any one attempt is 1 - 1/36 = 35/36.

Hence, P(E) = 1 - P(you roll no double six in 24 tries) = 1 -

$$\left(\frac{35}{36}\right)^{24} = 1 - 0.5086 = 0.4914$$

Since your chance of winning is less than 50%, the gambler has the advantage and you should not accept his proposition.

4.109 a. Let E = neither topping is anchovies, A = customer's first selection is not anchovies, and B = customer's second selection is not anchovies. For A to occur, the customer may choose any of 11 toppings from the 12 available. Thus P(A) = $\frac{11}{12}$. For B to occur, given that A has occurred, the customer may choose any of

10 toppings from the remaining 11 and $P(B|A) = \frac{10}{11}$. Therefore, $P(E) = P(A \text{ and } B|A) = \frac{10}{11}$.

$$B) = P(A)P(B|A) = \left(\frac{11}{12}\right)\left(\frac{10}{11}\right) = 0.8333$$

- **b.** Let C = pepperoni is one of the toppings. Then \overline{C} = neither topping is pepperoni. By a similar argument used to find P(E) in part a, we obtain $P(\overline{C}) = \left(\frac{11}{12}\right)\left(\frac{10}{11}\right) = 0.8333$. So, $P(C) = 1 P(\overline{C}) = 1 0.8333 = 0.1667$.
- 4.111 Let W_1 = the first machine on the first line works, W_2 = the second machine on the first line works, W_3 = the first machine on the second line works, W_4 = the second machine on the second line works, D_1 = the first machine on the first line does not work, D_2 = the second machine on the first line does not work, D_3 = the first machine on the second line does not work, and D_4 = the second machine on the second line does not work.

- **a.** $P(W_1W_2W_3W_4) = P(W_1)P(W_2)P(W_3)P(W_4) = (0.98)(0.96)(0.98)(0.96) = 0.8851$
- **b.** P(at least one machine in each production line is not working properly)= $P(W_1D_2W_3D_4) + P(W_1D_2D_3W_4) + P(D_1W_2W_3D_4) + P(D_1W_2D_3W_4) + P(W_1D_2D_3D_4) + P(W_1D_2D_3D_4) + P(D_1W_2D_3D_4) + P(D_1D_2W_3D_4) + P(D_1D_2D_3W_4) + P(D_1D_2D_3W_4) + P(D_1D_2D_3D_4) = (0.98)(0.04)(0.98)(0.04) + (0.98)(0.04)(0.02)(0.96) + (0.02)(0.96)(0.98)(0.04) + (0.02)(0.96)(0.02)(0.96) + (0.98)(0.04)(0.02)(0.04) + (0.02)(0.96)(0.02)(0.04) + (0.02)(0.04)(0.02)(0.04) + (0.02)($

P(not successful and not successful) = P(not successful) * P(not successful) = [1 - P(successful)][1 - P(successful)] = [1 - (0.98)(0.96)][1 - (0.98)(0.96)] = 0.0035

Self-Review Test

0.0035

- **1.** a
- 3. c
- **5.** a
- **7.** c
- **9.** b
- **11.** b
- 13. a. P(job offer selected is from the insurance company) = 1/3 = 0.3333
 - **b.** P(job offer selected is not from the accounting firm) = 2/3 = 0.6667
- 15. P(female or out-of-state) = P(female) + P(out-of-state) P(female and out-of-state)
 - = P(female) + P(out-of-state) P(female) P(out-of-state|female)

$$= \frac{110}{200} + \frac{125}{200} - \left(\frac{110}{200}\right) \left(\frac{70}{110}\right) = 0.825$$

Chapter 4

- 17. Let F_1 = first adult selected has experienced a migraine headache, N_1 = first adult selected has never experienced a migraine headache, F_2 = second adult selected has experienced a migraine headache, and N_2 = second adult selected has never experienced a migraine headache. Note that the two adults are independent.

 From the given information: $P(F_1) = 0.35$ and $P(F_2) = 0.35$. Hence, $P(N_1) = 1 0.35 = 0.65$ and $P(N_2) = 1 0.35 = 0.65$. Then $P(N_1) = P(N_2) P(N_2) = 1 0.35 = 0.65$.
 - From the given information: $P(F_1) = 0.35$ and $P(F_2) = 0.35$. Hence, $P(N_1) = 1 0.35 = 0.65$ and $P(N_2) = 1 0.35 = 0.65$. Then, $P(N_1 \text{ and } N_2) = P(N_1)P(N_2) = (0.65)(0.65) = 0.4225$.
- 19. Let M = male, F = female, W = works more than 10 hours, and N = does not work more than 10 hours.
 - **a.** P(M and W) = P(M)P(W) = (0.45)(0.62) = 0.279
 - **b.** P(F or W) = P(F) + P(W) P(F and W) = 0.55 + 0.62 (0.55)(0.62) = 0.829