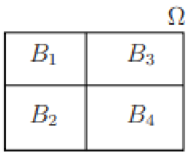
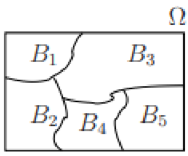
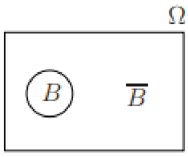
Definition: A partition of Ω is a collection of mutually exclusive events whose union is Ω.

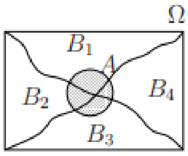
Examples:







Partitioning an event A Any set A can be partitioned: if $B_1, \ldots, B_k$ form a partition of Ω, then $(A \cap B_1), \ldots, (A \cap B_k)$ form a partition of A.

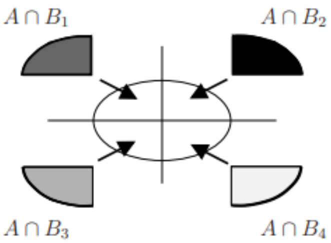


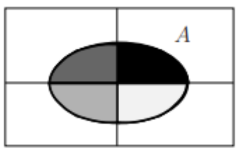
Partition Theorem (Law of Total Probability) Let $B_1, \ldots, B_m$ form a partition of Ω. Then for any event A,

$$P(A) = \sum_{i=1}^{m} P(A \cap B_i) = \sum_{i=1}^{m} P(A \mid B_i) P(B_i)$$

Both formulations of the Partition Theorem are very widely used.

The Partition Theorem is easy to understand because it simply states that "the whole is the sum of its parts."





$$\mathbb{P}(A) = \mathbb{P}(A \cap B_1) + \mathbb{P}(A \cap B_2) + \mathbb{P}(A \cap B_3) + \mathbb{P}(A \cap B_4).$$

Bayes' Theorem (or Bayes' Rule) is a very famous theorem in statistics. It was originally stated by the Reverend Thomas Bayes.

If we have two events A and B, and we are given the conditional probability of A given B, denoted P(A | B), we can use Bayes' Theorem to find P(B | A), the conditional probability of B given A.

Bayes' Theorem: $\mathrm{P(B \mid A)} = \dfrac{\mathrm{P(A \mid B)P(B)}}{\mathrm{P(A \mid B)P(B)} + \mathrm{P}\left(\mathrm{A} \mid \mathrm{B}'\right)\mathrm{P}\left(\mathrm{B}'\right)}$ where $\mathrm{P}\left(\mathrm{B}'\right)$ is the probability of B not occurring.

Genralized Bayes Theorem Statement: Let $E_1, E_2, \ldots, E_n$ be a set of events associated with a sample space S, where all the events $E_1, E_2, \ldots, E_n$ have nonzero probability of occurrence and they form a partition of S. Let A be any event associated with S, then according to Bayes theorem,

$$P(E_i \mid A) = \frac{P(E_i) P(A \mid E_i)}{\sum_{k=1}^{n} P(E_k) P(A \mid E_k)}$$

for any $k = 1, 2, 3, \ldots, n$

Example: In a factory there are two machines manufacturing bolts. The first machine manufactures 75% of the bolts and the second machine manufactures the remaining 25%. From the first machine 5% of the bolts are defective and from the second machine 8% of the bolts are defective. A bolt is selected at random, what is the probability the bolt came from the first machine, given that it is defective?

Problem 1

. It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?