Homework Assignment, Spring'11

Consider this homework to have been assigned on Tuesday, March 15.

You have the dynamical system
$$x'' + 4x = 2u \tag{1}$$
The control effort is bounded by $|u| \leq 1$. You start at $x(0) = 3.3$, $x'(0) = 1.1$. You wish to steer to the origin in minimal time T and have zero velocity when you arrive there. Thus $x(T) = 0$ and $x'(T) = 0$.

1. Set up the necessary conditions.

2. Determine the nature and general properties of the time optimal control u.

3. Follow the procedures in class

   (a) Determine the trajectories in (x, x') space for constant u. Graph the trajectories on one graph for relevant values of u. Show the direction of motion.

   (b) Determine how many switches there are.

   (c) Find the switching times and the final time. Show your work including equations to be solved. State what numerical method you use to solve the nonlinear equations.

4. Graph the optimal trajectory in the (x, x') domain.