MA731: Robust HW

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April 25, 2011

Our system is as follows:

$$\dot{x} = Ax + Bu + Dw \tag{1}$$

$$y = Cx + Ew \tag{2}$$

$$z = Hx + Gu \tag{3}$$

with

$$\text{A} = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}, \qquad \text{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \text{C} = \begin{bmatrix} 3 & 1 \end{bmatrix}, \qquad \text{E} = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{4}$$

$$\text{D} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \qquad \text{H} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad \text{G} = \begin{bmatrix} 4 \end{bmatrix}. \tag{5}$$

We also define an uncertainty term $\omega = (w, x(0))$ to be measured by

$$\|\omega\|^2 = \|x(0)\|_X^2 + \int_0^3 w(t)^T w(t) dt \tag{6}$$

Also let

$$\|\eta\|^2 = \|x(3)\|_Y^2 + \int_0^3 z(t)^T z(t)dt \tag{7}$$

In the above equations, $\|x(0)\|_X^2 = x(0)^T X x(0)$ and $\|x(3)\|_Y^2 = x(3)^T Y x(3)$, where X and Y are both the identity matrix of the appropriate size.

Our stated goal is to obtain values γ that are both greater than and less than $\hat{\kappa}$, where $\hat{\kappa}$ is the infinum of values κ_μ satisfying $\|\eta\| \leq \kappa_\mu \|\omega\|$. From the source provided, we see that if values of κ_μ satisfy the above inequality, then the Riccati equations, *Eq*. 8 and *Eq*. 10, have solutions $\Sigma(t)$ and $P(t)$, respectively, on the given time interval for the given κ_μ value such that $\forall t \in [0, T]$, $\rho(\Sigma(t)P(t)) < \kappa_\mu^2$, where $\rho(Mat)$ is the maximum magnitude eigenvalue of Mat.

$$\dot{\Sigma} + A\Sigma + \Sigma A^T - (\Sigma C^T + L^T)N^{-1}(C\Sigma + L) + \gamma^{-2}\Sigma Q \Sigma + M = 0 \tag{8}$$

$$\Sigma(0) = Y^{-1} \tag{9}$$

$$\dot{P} + PA + A^T P - (PB + S)R^{-1}(B^T P + S^T) + \gamma^{-2} PMP + Q = 0 \tag{10}$$

$$P(T) = X \tag{11}$$

where

$$\begin{bmatrix} H^t H & H^T G \\ G^T H & G^T G \end{bmatrix} = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \tag{12}$$

and

$$\begin{bmatrix} DD^T & DE^T \\ ED^T & EE^T \end{bmatrix} = \begin{bmatrix} M & L^T \\ L & N \end{bmatrix}. \tag{13}$$

Since Assumption A ($R > 0$, R^{-1} bounded, $N > 0$, N^{-1} bounded) and Assumption B (the pairs (A, D) and (A^T, H^T) are stabilizable) hold, then Theorem 3 may be utilized. Thus, the ARE forms of *Eq*. 10 and *Eq*. 8 were solved and then the condition $\rho(\Sigma^* P^*) < \kappa_\mu^2$ was checked, where Σ^* and P^* were the ARE solutions to the respective Riccati equations. The Theorem states that in addition to this, there exists some critical value of κ_μ such that numbers below κ_μ do not satisfy the conditions stated above, while numbers above κ_μ do. This critical value corresponds to the sought after value, $\hat{\kappa}$. Thus, using the statements in Theorem 3, the AREs were solved for various κ values and the corresponding P^* and Σ^* matrices were used to check the required conditions. In doing so, it was found that the critical value, $\hat{\kappa}$, was somewhere between 5.4 and 5.2. Additionally, it was found that sufficiently small κ values resulted in unsolvable Riccati equations due to singularities. Thus, κ values such as 4 and 2 resulted in solvable Riccati equations that did not meet the above requirements, while κ values such as 7 and 20 resulted in solvable AREs with corresponding Σ^* and P^* matrices that did satisfy the required conditions.