# OR/MA/ST 706: Nonlinear Programming
# Course Project
## Assigned: April 19, 2011    Due: May 11, 2011

We have already learned some basic solution methods for solving unconstrained nonlinear programming problems. This project intends to let you understand more about the potential difficulties that you may encounter in solving nonlinear programming problems. It also trains you to learn new solution techniques by yourself.

When you are working on the project, please follow the rules listed below:

1. DO NOT use any of the so-called soft-computing/meta-heuristics techniques (such as Genetic Algorithms, Tabu Search, Simulated Annealing, Neural Networks, Ant Colony, Particle Swam Optimization, EM).

2. You may use Matlab to draw the picture of each problem for better intuition, but DO NOT use any pre-known information about the value and position of any global optimal solution in your algorithm.

3. Only use Calculus based (such as the first, second, and third order information) solution techniques.

4. Chapters 6-9 of our textbook (D. G. Luenberger's *Linear and Nonlinear Programming* 2nd Edition) may help a lot. It is a time for self learning.

5. Be sure to turn in your computer runs with clear input/output data and computational efforts to support your findings.

**Problems**

1. Consider the optimization problem

$$\begin{array}{ll} \min & \displaystyle\sum_{k=1}^{5} k\sin((k+1)x+k) \\ s.t. & -10 \le x \le 10 \end{array}$$

(1a). Draw the picture of this function using Matlab.

(1b). From the picture, we know the function is a unimodal function on the interval $[6, 6.8]$. Use any one of the line search methods we learned to find the minimum point over this interval. Then, starting at $x = 6$, use the Steepest Decent Method and Newton Method to find a local minimal solution. Compare the numbers of iterations used in the three methods involved. Also, draw a table to show how many gradients and Hessian matrices you computed using each method.

(1c). Try to find all of the local minimal solutions over $[-10, 10]$.

(1d). Try to find all of the global minimums over $[-10, 10]$.

2. Consider the following optimization problems.

$$(P1) \quad \min \quad \left\{ \sum_{k=1}^{5} k \cos[(k+1)x_1 + k] \right\} \cdot \left\{ \sum_{k=1}^{5} k \cos[(k+1)x_2 + k] \right\} + \left[ (x_1 + 1.42513)^2 + (x_2 + 0.80032)^2 \right]$$
$$\text{s.t.} \quad -10 \le x_1 \le 10$$
$$-10 \le x_2 \le 10$$

$$(P2) \quad \min \quad \left\{ \sum_{k=1}^{5} k \cos[(k+1)x_1 + k] \right\} \cdot \left\{ \sum_{k=1}^{5} k \cos[(k+1)x_2 + k] \right\}$$
$$\text{s.t.} \quad -10 \le x_1 \le 10$$
$$-10 \le x_2 \le 10$$

$$(P3) \quad \min \quad -\cos x_1 \cdot \cos x_2 \cdot \exp\left[ -\frac{(x_1 - \pi)^2}{100} - \frac{(x_2 - \pi)^2}{100} \right]$$
$$\text{s.t.} \quad -100 \le x_1 \le 100$$
$$-100 \le x_2 \le 100$$

(2a). Design an algorithm to find a global optimal solution.

(2b). How many iterations does your algorithm take to find one optimal solution?

(2c). How many gradients and Hessian matrices are computed in finding this optimal solution?

(2d). Re-design your algorithm to find all global optimal solutions.

(2e). How many global optimal solutions can you find?

(2f). How many gradients and Hessian matrices are computed in finding this optimal solution?

3. After you have done Problems 1 and 2, please think over and answer the following question:

(3a). List all major difficulties you encountered in conducting this project and the way you handled each of them.

(3b). What do you learn from this project?

[Hint 1]: You can use “mesh” and “meshgrid” commands to draw the three dimensional picture of a function in Matlab.

[Hint 2]: The keyword of “filled function” in literature may help you jump out the trap of sitting at a local minimal solution if you cannot figure out any other ways.