MA731: HW7

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Our system is as follows:

$$\dot{x} = Ax + Bu \tag{1}$$

$$J(x(t)) = \frac{1}{2}x(T)^T S(T)x(T) + \frac{1}{2}\int_0^T x^T Qx + u^T Rudt \tag{2}$$

with

$$\text{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \qquad \text{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad \text{S(T)} = \begin{pmatrix} 10 & 1 \\ 1 & 20 \end{pmatrix},$$

$$\text{Q} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}, \qquad \text{R} = 2, \qquad \text{x(0)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

We first found $S(\infty)$, as a means to obtaining $K(\infty)$, which is the solution to the algebraic Riccati equation constructed from the above system:

$$-\dot{S} = A^T S + SA + Q - SBR^{-1}B^T S \tag{1}$$

In the case of the ARE, we would set $\dot{S}$ equal to zero and solve to obtain $S(\infty)$. For the given system, however, we instead simulated S backwards in time as a boundary value problem, given $S(T)$, on some sufficiently large time interval, $[0, T]$, where T was chosen such that the value of $S(t)$ reached some positive, symmetric, constant matrix. This resulting matrix approximates $S(\infty)$ for our system. In doing so, with $T = 20$, we obtained the following:

$$S(\infty) = \begin{pmatrix} 4.37 & 4.28 \\ 4.28 & 9.53 \end{pmatrix}, \qquad K(\infty) = \begin{pmatrix} 4.32 & 6.91 \end{pmatrix}$$

For two different T values, $T_1 = 1$ and $T_2 = 10$, we found the optimal, time dependent feedback, $K(t)$, the trajectory resulting from driving the system in Eq. 1 with the optimal feedback, and the cost, as defined above, associated with the system. We compared these trajectories and costs to those obtained from driving the system using $K(\infty)$. The comparison plots for each of these cases are shown below. The costs for T_1 were 6.87362 ($K(\infty)$ cost) and 5.31856, and the costs for T_2 were 2.71506 ($K(\infty)$ cost) and 2.71496. Note that the optimal cost for T_1 was found to be significantly smaller than the suboptimal cost, relative to the difference in costs for T_2. Given the comparison plots for controls and trajectories, this was to be expected, as these plots for T_2 show that the optimal and suboptimal systems are almost indistinguishable for the system trajectories, and nearly indistinguishable for the controls until around $t = 8$, while the plots for T_1 show drastically different behaviors in comparison. Therefore, for long durations, long being relative to the settling time for $S(\infty)$, we can expect similar overall behavior from the suboptimal, constant control and the optimal, time-varying control. The implementation of a constant, stable control in practice would seem to be much easier than using a time-varying control if the outcomes from both were similar enough for the task.

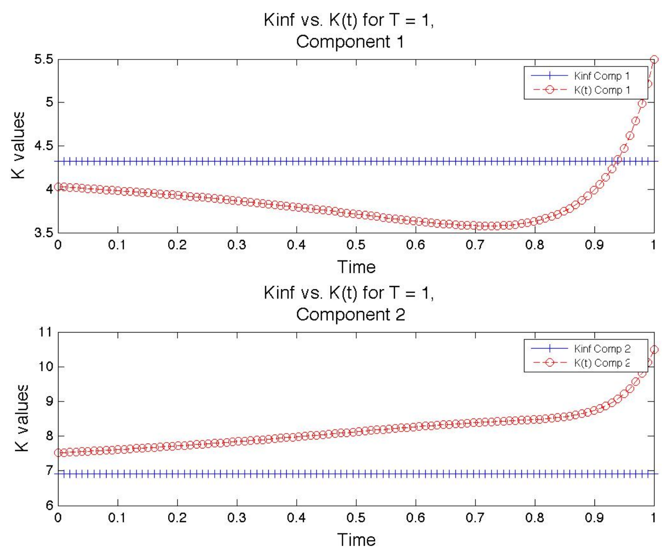

Figure 1: Comparison of K's for T_1

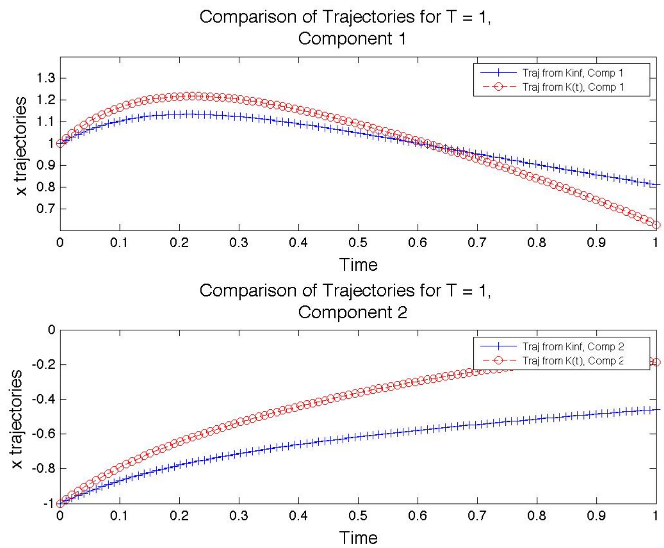

Figure 2: Comparison of trajectories for T_1

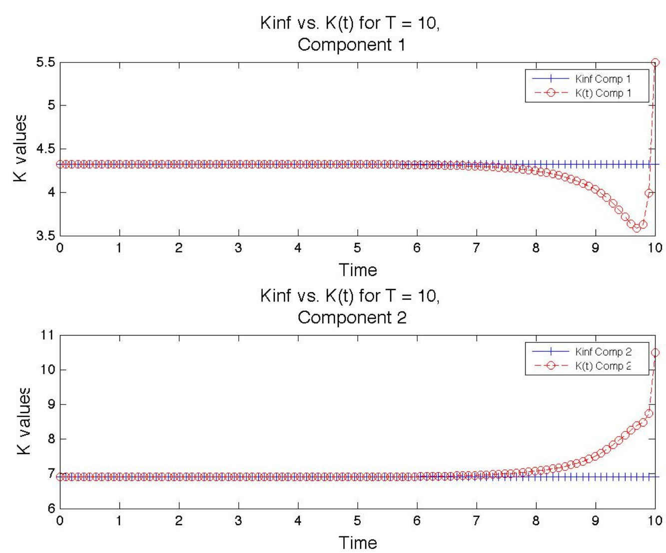

Figure 3: Comparison of K's for T_2

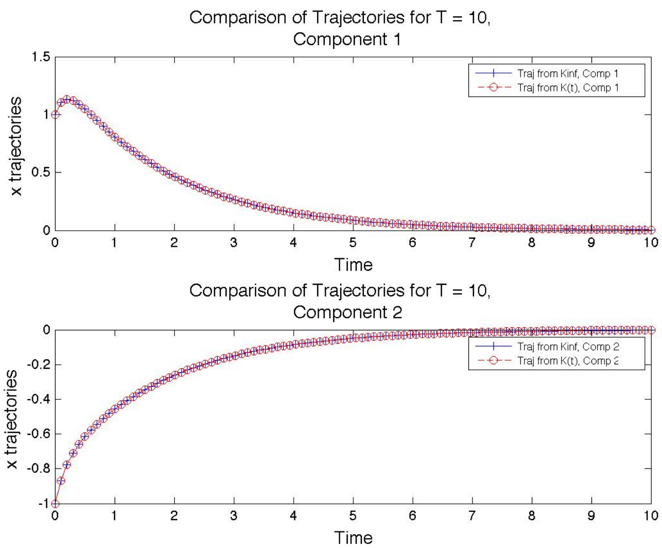

Figure 4: Comparison of trajectories for T_2