731 Homework Assignment, Spring'11

Consider this homework to have been assigned on Thursday, April 14. The accompanying reference is [1].

You have the dynamical system

$$x' = Ax + Bu + Dw \tag{1}$$
$$y = Cx + Ew \tag{2}$$
$$z = Hx + Gu \tag{3}$$

where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 3 & 1 \end{bmatrix}, E = \begin{bmatrix} 0 & 1 \end{bmatrix}, H = \begin{bmatrix} 0 & 1 \end{bmatrix}, G = [4].$$

Suppose the time interval is [0 3]. Measure $x(0)$ and $x(3)$ using the usual Euclidian norm. Thus the uncertainty $\omega = (w, x(0))$ is measured by

$$\|\omega\|^2 = x(0)^T x(0) + \int_0^3 w(\tau)^T w(\tau) d\tau. \tag{4}$$

Let

$$\|\eta\|^2 = x(3)^T x(3) + \int_0^3 y(\tau)^T y(\tau) d\tau \tag{5}$$

Suppose that u is given by an output feedback law. For a given feedback law μ let κ_μ be the smallest number such that

$$\|\eta\| \leq \kappa_\mu \|\omega\| \text{ for all } \omega. \tag{6}$$

Let

$$\hat{\kappa} = \inf_\mu \kappa_\mu$$

Determine several numbers γ which are greater than $\hat{\kappa}$ and several numbers γ which are strictly less than $\hat{\kappa}$. You need not find the feedbacks μ.

References

[1] Pierre Bernhard, *Survey of Linear Quadratic Robust Control*, Macroeconomic Dynamics, 6 (2001), 19–39.