METHOD TO ESTIMATE THE MEAN AND VARIANCE OF THE SPE PEAK

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SUMMARY OF PAPER

- Take the mean and the variance of the ADC spectrum of the PMT with the laser light on and off
- Estimate the mean number of laser-induced photoelectrons produced in each trigger (referred to as the occupancy in the paper)

THEORY SUMMARY

$$E[\psi] = \frac{E[T]-E[B]}{E[L]}$$

$$V[\psi] = \frac{V[T]-V[B]-E^2[\psi]V[L]}{E[L]}$$

- $E[T]$ and $V[T]$ = mean and variance signal distribution (i.e the distribution with the laser on)
- $E[B]$ and $V[B]$ = mean and variance of the background distribution (i.e the distribution with the laser off)
- $E[L]$ and $V[L]$ = mean and variance of the number of photoelectrons produced by the laser light

THEORY PART 1/3

- The probability distribution of the total integrated charge (T(q)) is the convolution of the background distribution (B(q)) and the signal distribution (S(q))

$$T(q) = (B * S)(q)$$

- Moments of distribution (additive):

Mean: $E[T] = E[B] + E[S]$

Variance: $V[T] = V[B] + V[S]$

- Signal charge distribution is summation of each signal charge distribution corresponding to p=(0,1,2..) photoelectrons ($S_p(q)$) and the discrete probability distribution of the photoelectrons produced by a laser light pulse ($L(p)$)

THEORY PART 2/3: SINGLE PHOTO-ELECTRON MEAN

- The signal charge distribution for p=1: $S_1(q) = \psi(q)$
- The signal charge distribution for p: $S_p(q) = (\psi(q))^p$
- The mean and variance of $S_p(q)$ is:

$$E[\psi^p] = pE[\psi]$$

$$V[\psi^p] = pV[\psi]$$

- Therefore the signal charge distribution moments are:

$$E[s] = E[\psi]E[L]$$

$$V[s] = V[\psi]E[L] + E^2[\psi]V[L]$$

THEORY PART 3/3: PARAMETER ESTIMATION

- $E[T]$ and $V[T]$ come as a the mean and variance of the signal distribution (i.e the distribution with the laser on)
- $E[B]$ and $V[B]$ come as a the mean and variance of the background distribution (i.e the distribution with the laser off)
- $E[L]$ and $V[L]$ represent the mean and variance of the number of photoelectrons produced by the laser light. If it is assumed that $L(p)$ is represented by a Poisson distribution (which is valid for autenuated light) then:

$$L(p) = \frac{\lambda^p e^{-\lambda}}{p!} \quad \lambda = E[L] = V[L]$$

- The occupancy can be estimated by the number of sample triggers 0-pe trigger over the total number of triggers

PARAMETER ESTIMATION 1/2: T AND B

spectrum with laser light turned off.

mV

Signal spectrum with laser light turned on.

mV