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**Partial Differential Equations 3 – Static PDE Coursework**  
By : Khalid Hersafril (s2016510)

**Question 1**

a) The boundary condition on the inside and outside of the pipe are given by:

$$u(1, \theta) = 0, \quad 0 \leq \theta \leq 2\pi \quad (1)$$
$$\frac{\partial u}{\partial r}(3, \theta) = \begin{cases} 0, & 0 \leq \theta < \pi \\ 1, & \pi \leq \theta \leq 2\pi \end{cases} \quad (2)$$

The first boundary condition where  $u = 0$ , tells us that the temperature throughout the whole boundary of the inside pipe at radius = 1, is 0.

The second boundary conditions where  $\frac{\partial u}{\partial r} = 0$ , from 0 to less than  $\pi$ , informs us that the boundary of the outside wall of the upper part of the pipe is a perfectly insulated wall where there is no heat gained or loss throughout the whole boundary whereas when  $\frac{\partial u}{\partial r} = 1$ , from  $\pi$  to  $2\pi$  tells us that the bottom part of the outside wall of the pipe is losing heat flux where the direction of the heat flux is normal to the domain.

b) We know that the basic solutions of cylindrical coordinate is given by:

$$\mu = 0 \quad u(r, \theta) = (A\theta + B)(C \ln r + D) \quad (3)$$
$$\mu^2 < 0 \quad u(r, \theta) = (A \cosh(\mu\theta) + B \sinh(\mu\theta))(C r^{-\mu} + D r^{\mu}) \quad (4)$$
$$\mu^2 > 0 \quad u(r, \theta) = (A \cos(\mu\theta) + B \sin(\mu\theta))(C r^{-\mu} + D r^{\mu}) \quad (5)$$

Since we are applying methods of separation variable, we can generally write equation [3], [4] and [5] as

$$u(r, \theta) = \Phi(\theta)R(r) \quad (6)$$

Solving for equation [3] by applying the boundary conditions given from equation [1] and [2], we can see that;

$$\Phi(\theta) = A\theta + B \quad (7)$$

Since our solution needs to be periodic, and the relationship given from equation [7] will give a linear relationship for increasing  $\theta$ , hence we have to let  $A = 0$ , hence we get  $\Phi(\theta) = B$ .

$$R(1) = C \ln 1 + D = 0 \quad (8)$$

Applying the [1] boundary conditions, for equation [8], we get that the expression,  $C \ln 1 = 0$ , and this gives us that  $D = 0$ , however, we should not simply drop down the C term and make it equal to

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