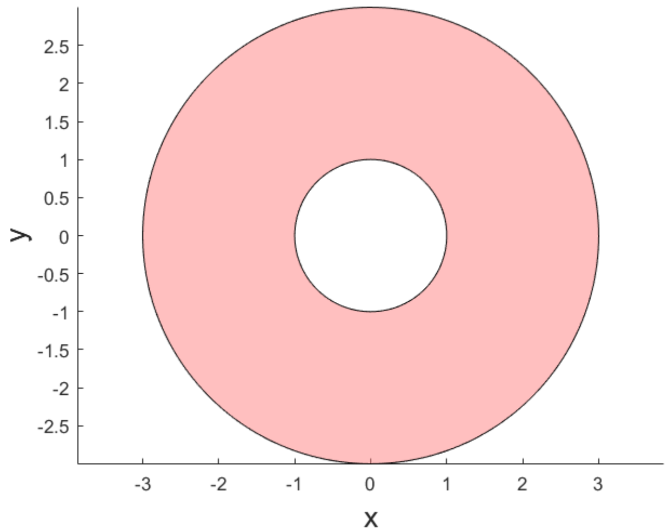
The following coursework is to be handed in to the ETO online dropbox by **16:00 on Thursday the 17th of February 2022**. This is **coursework number 1 of 2**. Each coursework is worth 20% of the final mark.

You should submit the solution to this coursework in the form of a short report on Learn. This report must be typed, with at least 2cm margins, a font size of 11pt and line spacing of at least 1, and must not be longer than 8 pages. The title page does not count towards the page limit.

The marks for this coursework will be awarded for the correctness of the solution as well as for the clarity and logical progression of the solution. Details on laying out mathematical calculations and general guidelines for the coursework are given on Learn.

1. Consider an insulated pipe whose cross section is given by the following figure. [50]



The temperature distribution in the red pipe can be described by the nondimensional Laplace equation in cylindrical coordinates which is given by

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

The boundary conditions on the inside and outside of the pipe are given by

$$u(1,\theta) = 0, \quad 0 \le \theta \le 2\pi \tag{1}$$

$$\frac{\partial u}{\partial r}(3,\theta) = \begin{cases} 0, & 0 \le \theta < \pi \\ 1, & \pi \le \theta \le 2\pi \end{cases} \tag{2}$$

(a) Describe the heat transfer situation given by the boundary conditions 1 and 2 and bring it into the context of a physical problem. [8]

(b) Calculate the analytical solution of the Laplace equation on the annulus with the given boundary conditions. [32]

(c) Plot the analytical solution and discuss the convergence of the Fourier series. [10]

Remarks:

- Have a look at the exercise sheets and examples to get started.
- The combined solution needs to fulfil the boundary conditions.

2. Using the `Grid` class defined in the LAPLACE-POLAR Jupyter notebook, complete the function `PolarLaplaceSolver` to produce a numerical solution to the coursework PDE using the bi-conjugate gradient solver from the SCIPY library. [50]

   You are reminded that your PYTHON code will need to assemble both the A matrix and the $\vec{b}$ RHS vector so that the coefficients are those for the discretised version of the nondimensional Laplace equation in cylindrical coordinates. You are also reminded that their is a periodic boundary condition $u(r, 0) = u(r, 2\pi)$ which has implications for the structure of the A matrix.

   Write a short report in which you need to explain what you are doing, compare the solution against the analytical solution, perform a grid convergence analysis and plot the solution.

   Your report should include the derivation of the stencil.

   It should not include the whole PYTHON code, but must include code fragments implementing the periodic boundary, the Neumann boundary condition at $r = 3$ and showing how the coefficients are calculated for the computational stencil needed for the cylindrical coordinate version of the Laplace equation.