DATA-DRIVEN MIXED PRECISION SPARSE MATRIX VECTOR MULTIPLICATION FOR GPUS

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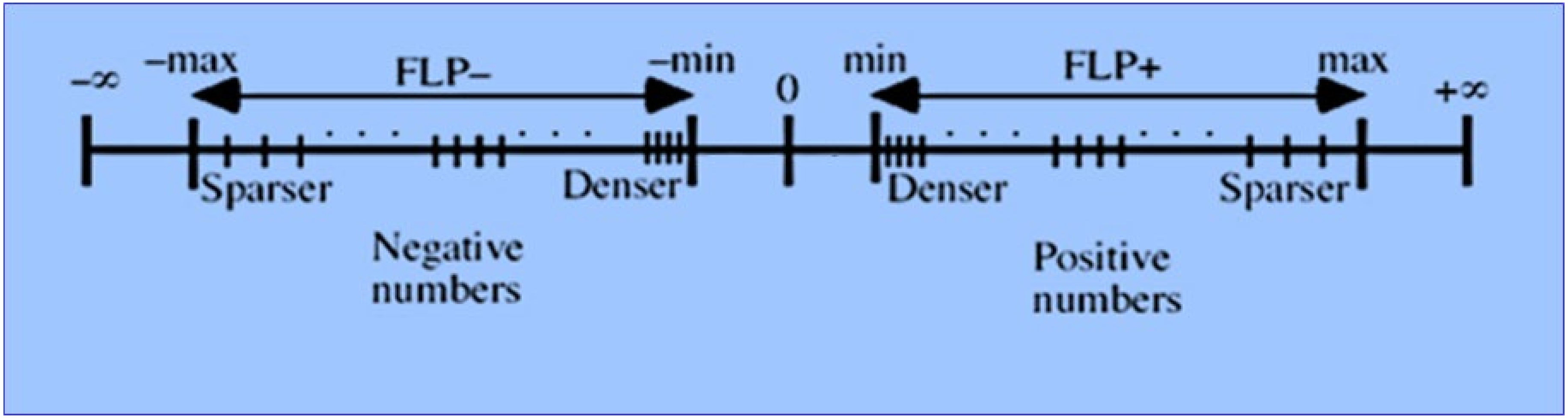
Motivation

- Scientific applications use double precision for higher accuracy
- Downcasting precision leads to intolerable inaccuracies
- MpSpMV
  - alternative data-driven approach
  - lowers precision based on nonzero values

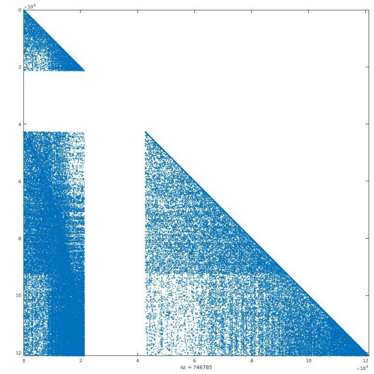
Values Closer to Zero are Well Represented



IEEE 754 floating point representation

Matrix Split

Input Matrix [cop20k_A.mtx]

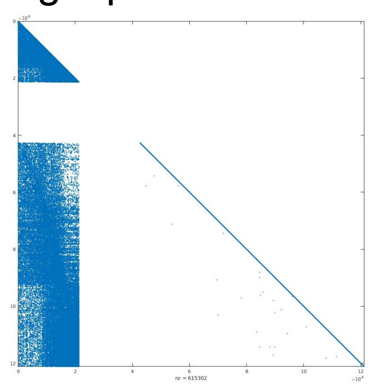


(a) NNZ=1,362,087

< |2|

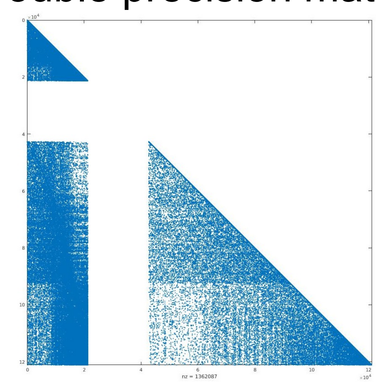
> |2|

Single precision matrix



(b) NNZ=746,785

Double precision matrix



(c) NNZ=615,302

Contributions

- ↓ execution time
- ↑ parallelism
- ↓ data movement
- ↑ accuracy compared to single precision

Results

- Average speedup of 1.06X
- Maximum speedup over double precision of 2.61X
- On average one decimal digit more accurate than single precision

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Compressed Sparse Row (CSR)

Dense Representation

0.1	0.2	0.3	0.4	0	0
0	1.2	1.3	0	0	0
0	0	2.3	2.4	2.5	2.6
0	0	0	3.4	3.5	0
0	0	0	0	0	4.6
0	0	0	0	0	5.6

NxN

Sparse Representation

A	0.1	0.2	0.3	0.4	1.2	1.3	2.3	2.4	2.5	2.6	3.4	3.5	4.6	5.6

Auxiliary Data Structures

col	0	1	2	3	1	2	2	3	4	5	3	4	5	5

index (row pointer)	0	4	6	10	12	13	14

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Sequential SpMV CSR Computation

```
for (i=0; i<N; i++) {
    for (j=index[i]; j<index[i+1]; j++) {
        y[i] += A[j] * x[col[j]];
    }
}
```

CUSP SpMV CSR Vector Implementation

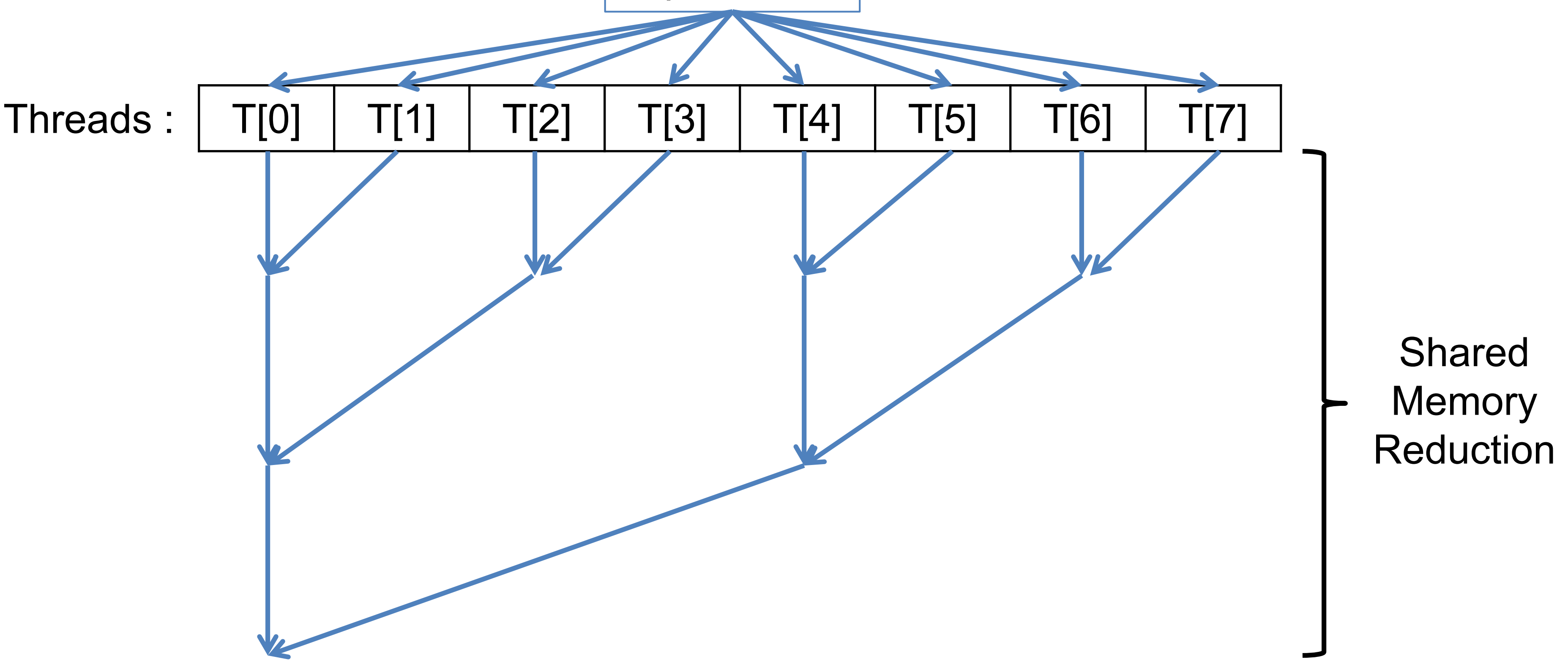
```
for (i=0; i < n; i++)
    for (j=index[i]; j<index[i+1]; j++){
        T[j] =a[j]*x[col[j]];
        y[i]  += T[j]
}
```

Reduction Sum

CUSP SpMV CSR Vector Implementation

```
for (i=0; i < n; i++)
    for (j=index[i]; j<index[i+1]; j++){
        T[j] =a[j]*x[col[j]];
        y[i]  += T[j]
}
```

Product Expression is scalar expanded



SpMV Kernel Difference

Kernel Section / Precision	Single	Mixed	Double
Prototype	A, x, y	As, x_s, Ad, x_d, y	A, x, y
Partial Products	Just one set	Two sets: single precision; double precision	Just one set
Reduction	One reduction for all		

Mixed precision is profitable:
2N + 1 < NNZ_s

* Kernel code available in the paper

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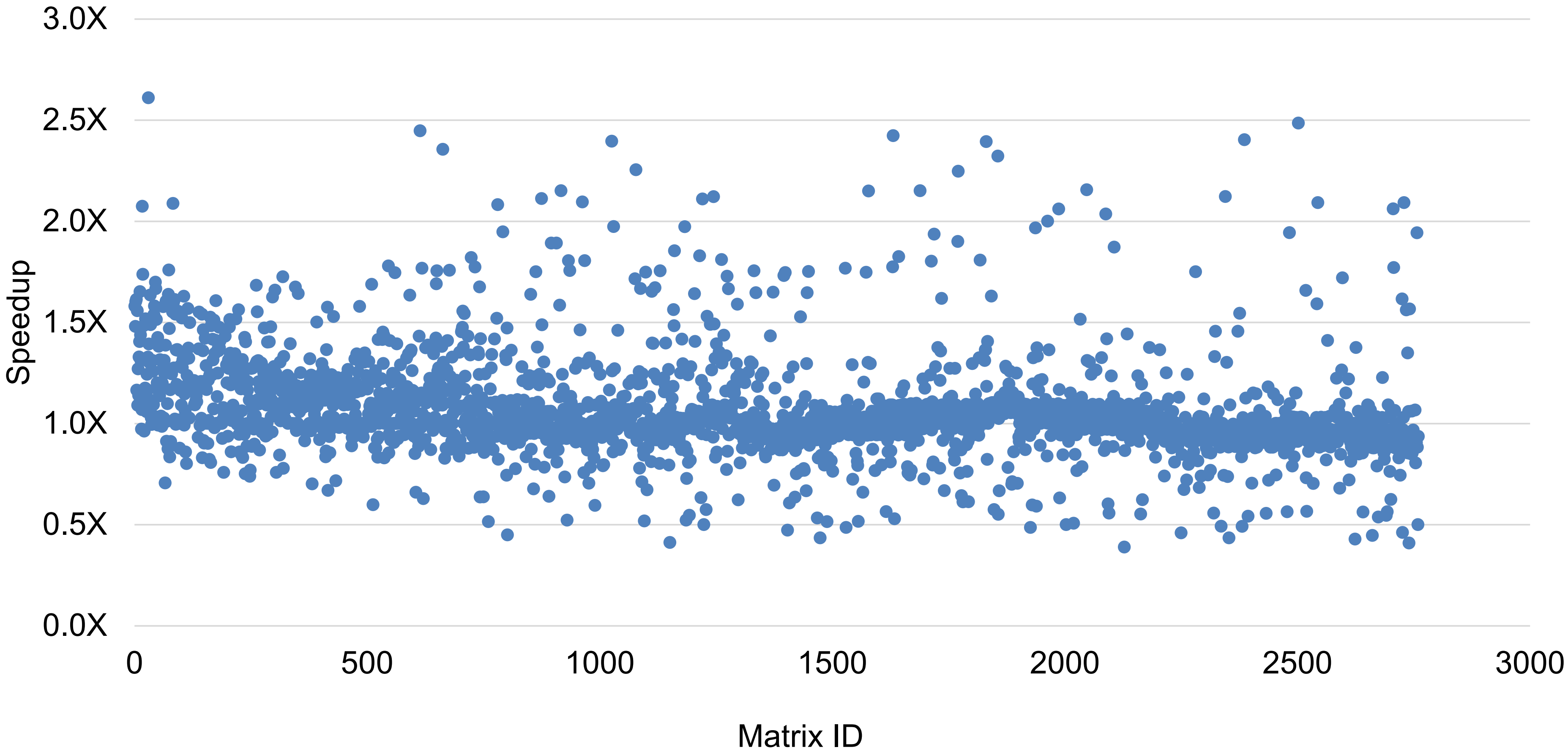
Sparse Matrix Collection



1359 out of the 2202 matrices showed speedup using mixed precision

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Experimental Evaluation

- Methodology
  - Select representative subset of matrices
  - Nvprof metrics collected
- Results
  - V100 and P100
  - Average of 500 SpMV runs recorded

Selecting Representative Matrices

- K Dimension Algorithm
- We used 4 dimensions to select a representative subset of matrices:
  - MpSpMV speedup over DpSpMV
  - Number of non zeros in a sparse matrix
  - Density of the sparse matrix
  - Ratio of nonzero values inside the range

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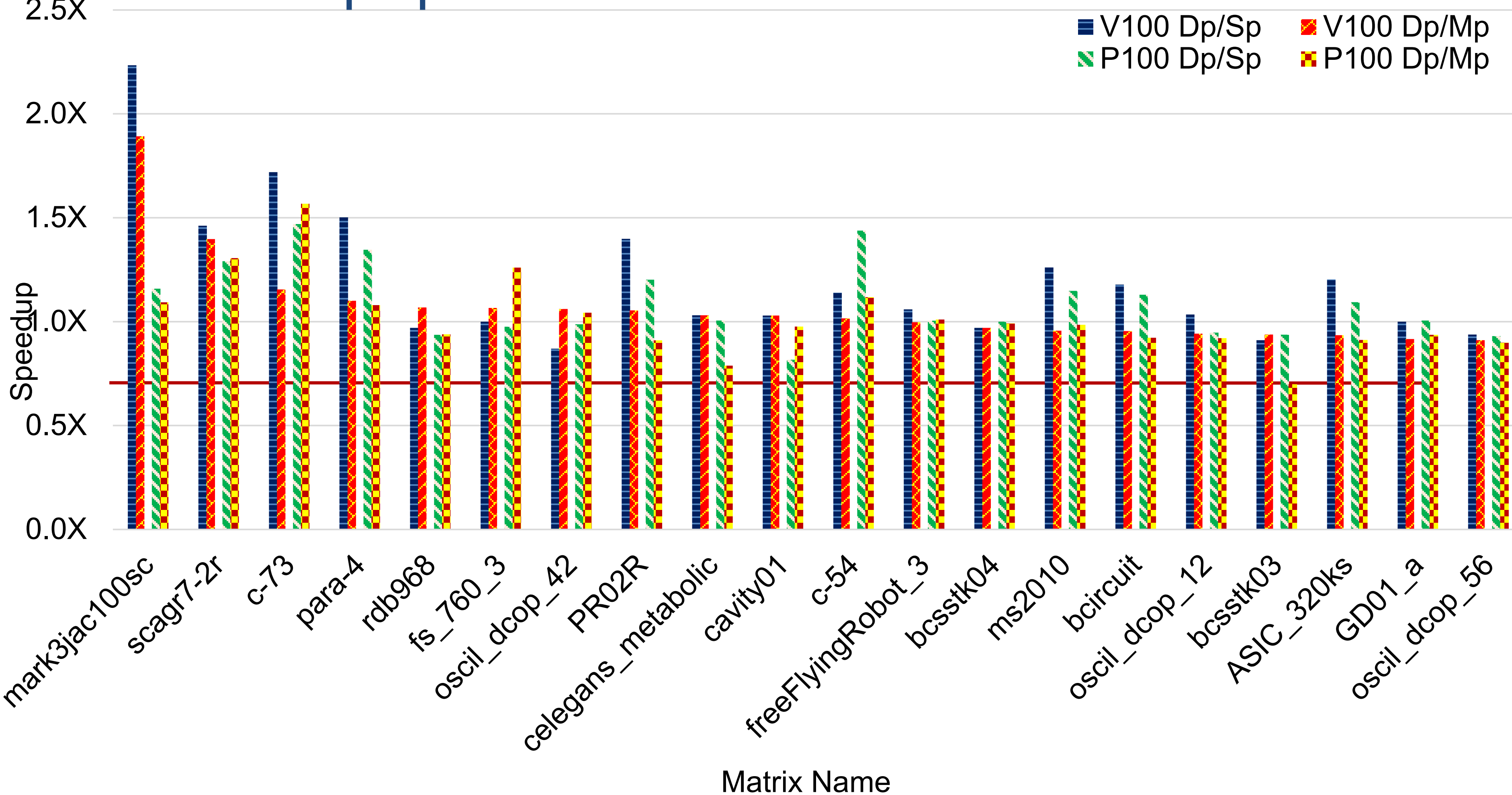
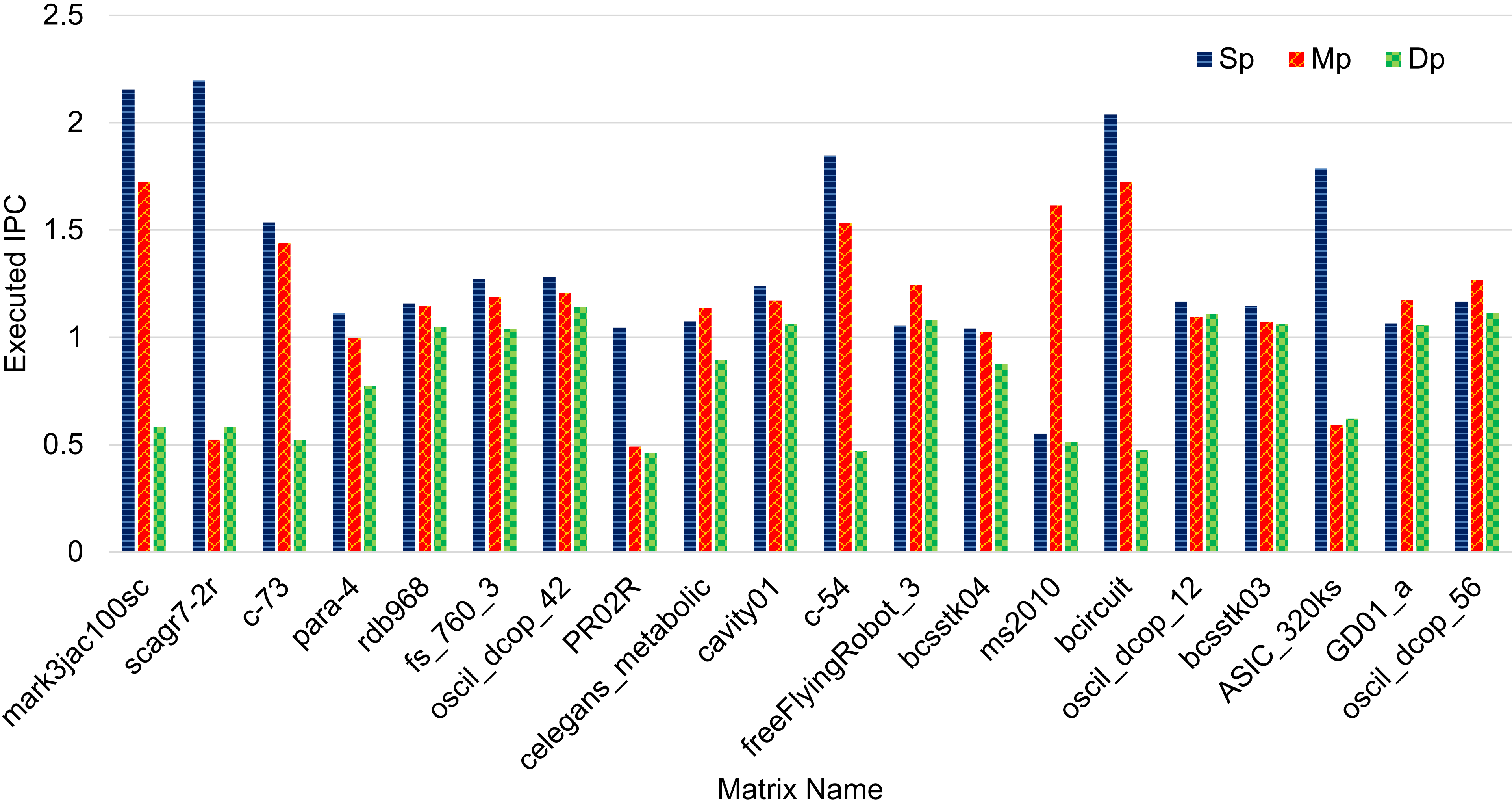
Speedup Comparison of MpSpMV and SpSpMV over DpSpMV on V100 and P100 GPUs



11 matrices show speedup using mixed precision.
Performance for rest of matrices → mixed precision ≈ double Precision.

Executed IPC



Higher instructions per cycle than double precision

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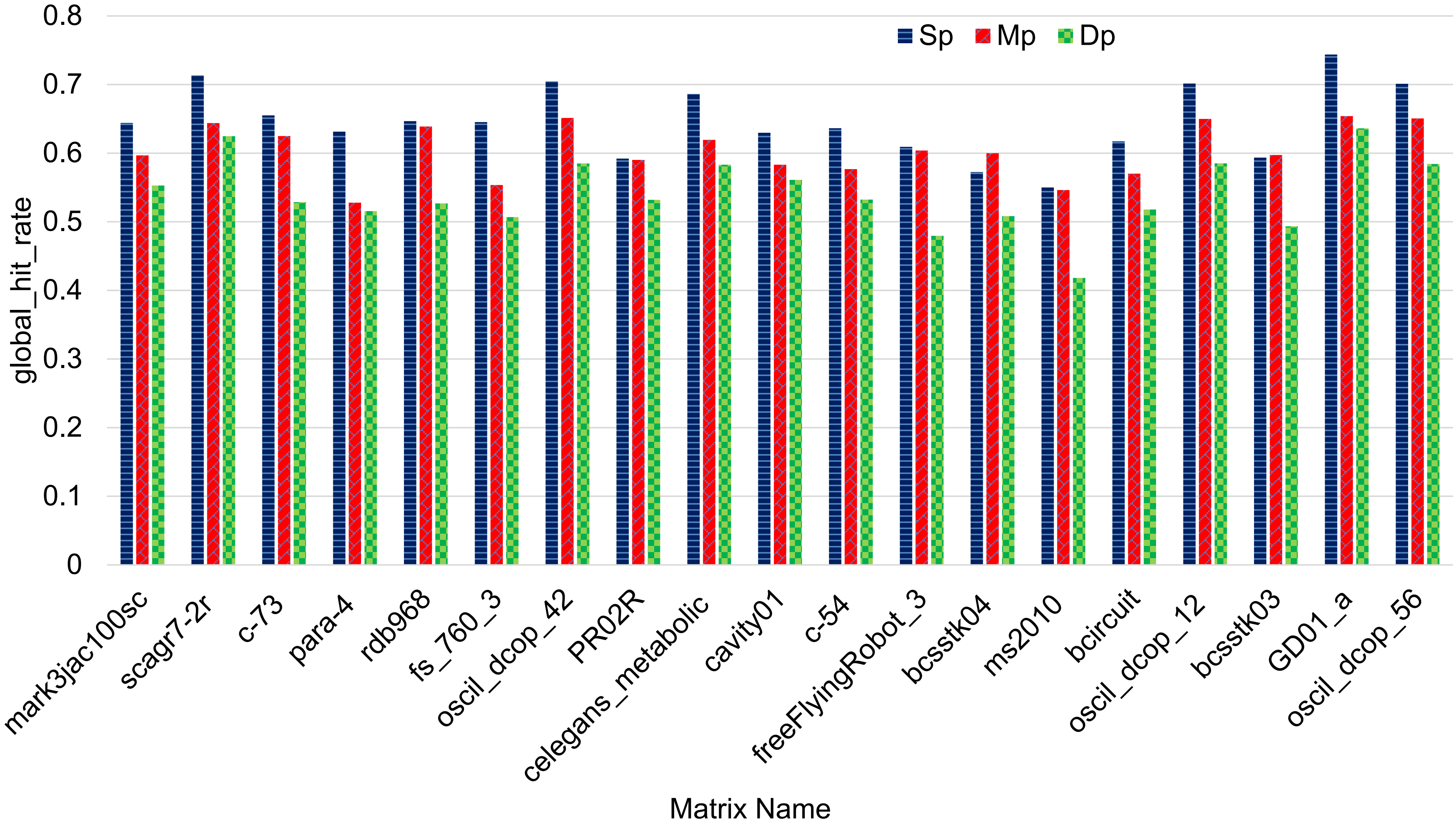
Global Hit Rate



Higher hit rates in L1 cache compared to double precision

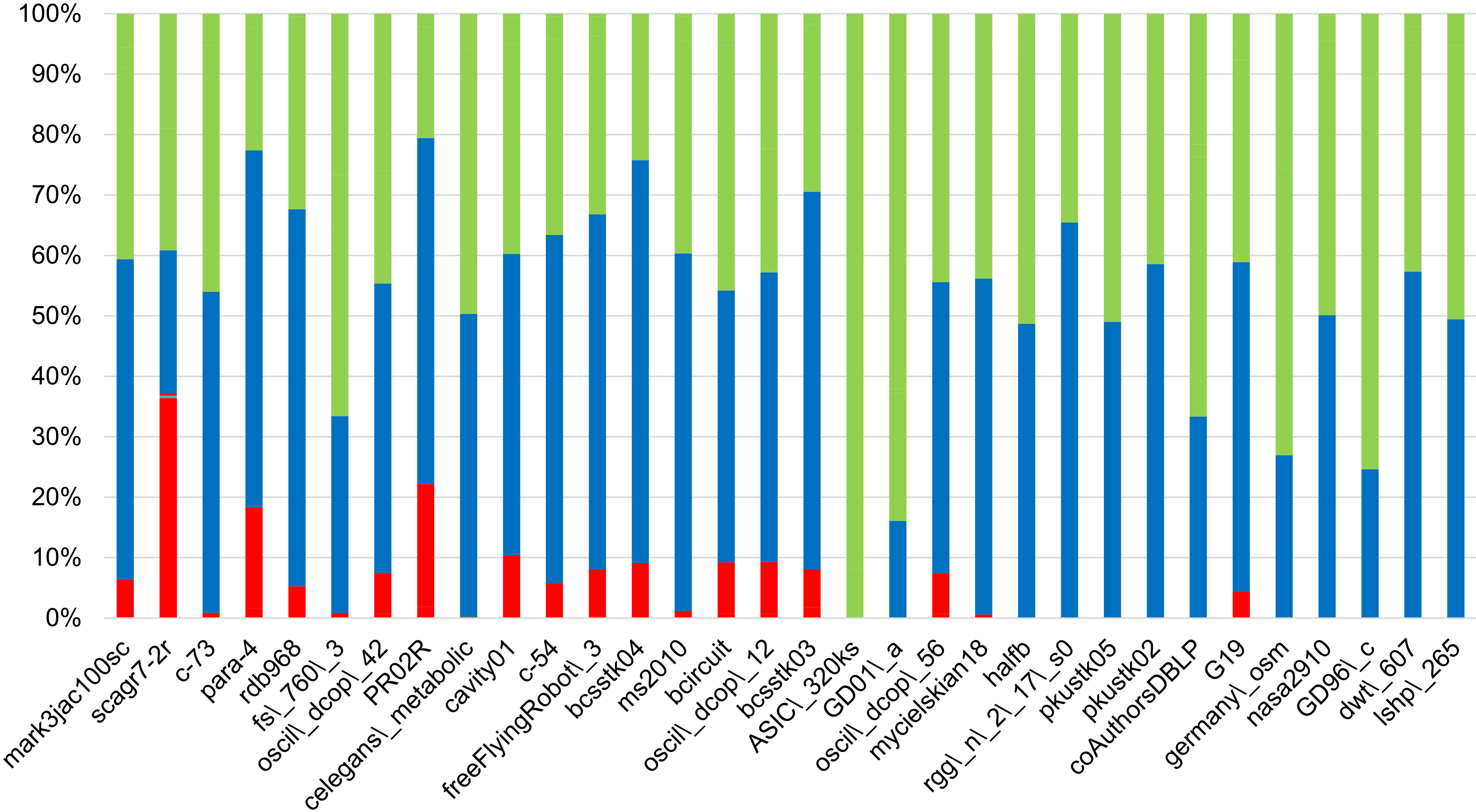
Accuracy

$$X_\tau = \frac{1}{9}$$

$$X_A = 0.111$$

$$|X_\tau - X_A| = 0.\underbrace{000}_{\text{3 significant decimal digits}}11$$

SpSpMV Accuracy

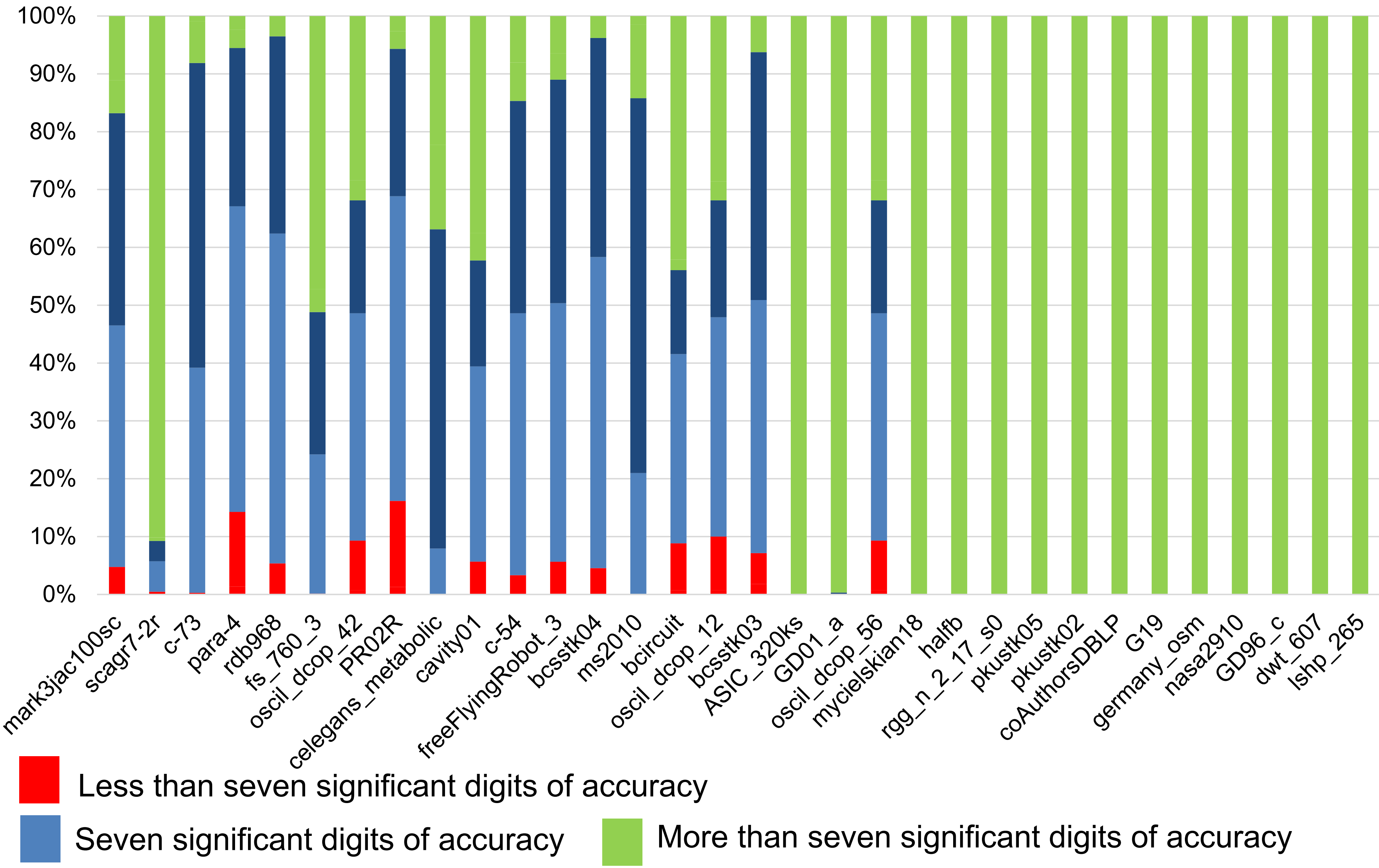


Less than six significant digits of accuracy

Six significant digits of accuracy

More than six significant digits of accuracy

MpSpMV Accuracy



Conclusion

New data driven mixed precision implementation

- ↓ execution time
- ↑ parallelism
- ↓ data movement
- ↑ accuracy compared to single precision

Results

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