Introduction

Below is a list of all logarithm problems in all years of Euclid/COMC/CSMC, and in years 2000-2021 of AMC12. Specifically, there were logarithm problems in all years of Euclid but 2005/2004/1998.

Euclid2021-7b

Suppose that $f(a) = 2a^2 - 3a + 1$ for all real numbers a and $g(b) = \log_{\frac{1}{2}} b$ for all $b > 0$. Determine all θ with $0 \leq \theta \leq 2\pi$ for which $f(g(\sin\theta)) = 0$.

Euclid2020-7b

Determine all pairs of angles (x, y) with $0^\circ \leq x < 180^\circ$ and $0^\circ \leq y < 180^\circ$ that satisfy the following system of equations:

$$\log_2(\sin x \cos y) = -\frac{3}{2}$$
$$\log_2\left(\frac{\sin x}{\cos y}\right) = \frac{1}{2}$$

Euclid2019-7a

Determine all real numbers x for which $2\log_2(x-1) = 1 - \log_2(x+2)$.

Euclid2018-8a

Determine all values of x such that $\log_{2x}(48\sqrt[3]{3}) = \log_{3x}(162\sqrt[3]{2})$.

Euclid2017-7b

Determine all pairs (a, b) of real numbers that satisfy the following system of equations:

$$\begin{aligned} \sqrt{a}+\sqrt{b} &= 8 \\ \log_{10} a+\log_{10} b &= 2 \end{aligned}$$

Give your answer(s) as pairs of simplified exact numbers.

Euclid2016-8b

Determine all real numbers $x > 0$ for which

$$\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$$

Euclid2015-9

Consider the following system of equations, in which all logarithms have base 10:

$$\begin{aligned}(\log x)(\log y) - 3\log 5y - \log 8x &= a \\ (\log y)(\log z) - 4\log 5y - \log 16z &= b \\ (\log z)(\log x) - 4\log 8x - 3\log 625z &= c\end{aligned}$$

(a) If $a = -4$, $b = 4$, and $c = -18$, solve the system of equations.

(b) Determine all triples (a, b, c) of real numbers for which the system of equations has an infinite number of solutions (x, y, z).

Euclid2014-7b

Determine all real numbers x for which

$$(\log_{10} x)^{\log_{10}(\log_{10} x)} = 10000$$

Euclid2013-8b

Determine all real values of x for which $\log_2(2^{x-1} + 3^{x+1}) = 2x - \log_2(3^x)$.

Euclid2012-8b

Determine all real values of x such that

$$\log_{5x+9}(x^2 + 6x + 9) + \log_{x+3}(5x^2 + 24x + 27) = 4$$

Euclid2011-7b

Determine all values of x for which $2^{\log_{10}(x^2)} = 3(2^{1+\log_{10} x}) + 16$.

Euclid2010-7b

Determine all points (x, y) where the two curves $y = \log_{10}(x^4)$ and $y = (\log_{10} x)^3$ intersect.

Euclid2009-9a

If $\log_2 x, (1 + \log_4 x)$, and $\log_8 4x$ are consecutive terms of a geometric sequence, determine the possible values of x.

(A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a constant. For example, 3, 6, 12 is a geometric sequence with three terms.)

Euclid2008-9

(a) The equation $2^{x+2}5^{6-x} = 10^{x^2}$ has two real solutions. Determine these two solutions.
(b) Determine all real solutions to the system of equations

$$\begin{array}{rcccl} x & + & \log_{10} x & = & y-1 \\ y & + & \log_{10}(y-1) & = & z-1 \\ z & + & \log_{10}(z-2) & = & x+2 \end{array}$$

and prove that there are no more solutions.

Euclid2007-7a

Determine all values of x for which $(\sqrt{x})^{\log_{10} x} = 100$.

Euclid2006-8a

If $\log_2 x - 2\log_2 y = 2$, determine y as a function of x, and sketch a graph of this function on the axes in the answer booklet.

Euclid2003-6b

Solve the system of equations:

$$\begin{aligned}\log_{10} x^3 + \log_{10} y^2 &= 11 \\ \log_{10} x^2 - \log_{10} y^3 &= 3\end{aligned}$$

Euclid2002-5a

What are all values of x such that

$$\log_5(x+3) + \log_5(x-1) = 1?$$

Euclid2001-7a

What is the value of x such that $\log_2(\log_2(2x-2)) = 2$?

Euclid2000-2b

If $\log_{10} x = 3 + \log_{10} y$, what is the value of $\dfrac{x}{y}$?

Euclid1999-6b

Determine the coordinates of the points of intersection of the graphs of $y = \log_{10}(x-2)$ and $y = 1 - \log_{10}(x+1)$.

CSMC2011-A6

In a magic square, the numbers in each row, the numbers in each column, and the numbers in each diagonal have the same sum. Given the magic square shown with $a, b, c, x, y, z > 0$, determine the product xyz in terms of a, b and c.

$\log a$	$\log b$	$\log x$
p	$\log y$	$\log c$
$\log z$	q	r

COMC2009-A7

Determine all angles θ with $0^\circ \le \theta \le 360^\circ$ such that $\log_2(-3\sin\theta) = 2\log_2(\cos\theta) + 1$.

COMC1998-A5

Compute the sum of the first 99 terms of the series

$$\log_a a - \log_a a^2 + \log_a a^3 - \log_a a^4 + \log_a a^5 - \log_a a^6 + \dots$$

COMC1996-A9

If $\log_{2n}(1944) = \log_n(486\sqrt{2})$, compute n^6.

AMC12A2019-15

Positive real numbers a and b have the property that

$$\sqrt{\log a} + \sqrt{\log b} + \log \sqrt{a} + \log \sqrt{b} = 100$$

and all four terms on the left are positive integers, where log denotes the base-10 logarithm. What is ab?

(A) 10^{52} **(B)** 10^{100} **(C)** 10^{144} **(D)** 10^{164} **(E)** 10^{200}

AMC12B2008-23

The sum of the base-10 logarithms of the divisors of 10^n is 792. What is n?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

AMC12B2004-17

For some real numbers a and b, the equation

$$8x^3 + 4ax^2 + 2bx + a = 0$$

has three distinct positive roots. If the sum of the base-2 logarithms of the roots is 5, what is the value of a?

(A) -256 **(B)** -64 **(C)** -8 **(D)** 64 **(E)** 256

AMC12B2000-23

Professor Gamble buys a lottery ticket, which requires that he pick six different integers from 1 through 46, inclusive. He chooses his numbers so that the sum of the base-ten logarithms of his six numbers is an integer. It so happens that the integers on the winning ticket have the same property—the sum of the base-ten logarithms is an integer. What is the probability that Professor Gamble holds the winning ticket?

(A) 1/5 **(B)** 1/4 **(C)** 1/3 **(D)** 1/2 **(E)** 1